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On Long Cycles in Digraphs with the Meyniel-type Conditions

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We shall assume that the reader is familiar with the standard terminology on directed graphs (digraphs) and use Bang-Jensen and Gutin [1] as reference for undefined terms. In this paper we consider finite digraphs without loops and multiple arcs. The subdigraph of D induced by a subset A of $V(D)$ is denoted by $\langle A\rangle$. We will denote the complete bipartite digraph with partite sets of cardinalities p, q by $K^*_{p,q}$.

Meyniel [11] proved the following theorem: If D is a strong digraph on $n \geq 2$ vertices and $d(x) + d(y) \geq 2n - 1$ for all pairs of non-adjacent vertices in D, then D is hamiltonian (see also [1], [5] and [12]).

Thomassen [14] (for $n = 2k+1$) and Darbinyan [7] (for $n = 2k$) proved: If D is a digraph on $n \geq 5$ vertices with minimum degree at least $n-1$ and with minimum semi-degree at least $n/2 - 1$, then D is hamiltonian (unless some extremal cases).

In each above mentioned theorems (as well as, in well know theorems Ghouila-Houri [10], Woodall [15]) imposes a degree condition on all pairs of non-adjacent vertices (on all vertices). Bang-Jensen, Gutin, Li, Guo and Yeo [2, 3] obtained sufficient conditions for hamiltonisity of digraphs in which degree conditions requiring only for some pairs of non-adjacent vertices. Namely, they proved the following theorems (in all three theorems D is a strong digraph on $n \geq 2$ vertices).

Theorem A [2]. If $min\{d(x), d(y)\} \geq n - 1$ and $d(x) + d(y) \geq 2n - 1$ for every pair of non-adjacent vertices x, y with a common in-neighbour, then D is hamiltonian.

Theorem B [2]. If $min\{d^+(x) + d^-(y), d^-(x) + d^+(y)\} \geq n$ for every pair of non-adjacent vertices x, y with a common out-neighbour or a common in-neighbour, then D is hamiltonian.

Theorem C [3]. If $min\{d^+(x) + d^-(y), d^-(x) + d^+(y)\} \geq n - 1$ and $d(x) + d(y) \geq 2n - 1$ for every pair of non-adjacent vertices x, y with a common out-neighbour or a common in-neighbour, then D is hamiltonian.

Note that Theorem C generalizes Theorem B. In [9, 13, 6, 8] it was shown that if the strong digraph D satisfies the condition of the theorem of Ghouila-Houri [10] (Woodall [15], Meyniel [11], Thomassen and Darbinyan [14, 7]), then D is pancyclic (unless some extremal cases, which are characterized). It is not difficult to check that the digraphs $K^*_{n/2,n/2}$ and $K^*_{n/2,n/2} - \{e\}$, where n is even and e is an arc of $K^*_{n/2,n/2}$, satisfy the conditions of Theorem A (B, C) and has no cycle of odd length. Moreover, if in Theorems A (B, C) the digraph D has no pair of non-adjacent vertices with a common in-neighbour and a common out-neighbour, D is a locally semicomplete digraph, and in [4], Bang-Jensen, Gutin and Volkmann acterize those strong locally semicomplete digraphs which are not pancyclic.

t is natural to set the following problem:

blem. Characterize those digraphs which satisfy the conditions of Theorem A (B, C), are not pancyclic.

To investigate that a given digraph D is pancyclic, in [9, 13, 6, 8] it was proved the stence of cycles of length $|V(D)|-1$ and $|V(D)|-2$, and then using the constructions of e cycles it was proved that D is pancyclic with some exceptions.

We prove three results which provide some support for the above Problem.

orem 1. Let D be a strong digraph on n vertices with minimum semi-degree at least If D satisfies the conditions of Theorem A, then either D contains a cycle of length $n-1$ is even and D is isomorphic to complete bipartite digraph $K^*_{n/2,n/2}$ or $K^*_{n/2,n/2}-\{e\}$, re e is an arc of $K^*_{n/2,n/2}$.

orem 2. Let D be a strong digraph on $n \geq 4$ vertices, which is not directed cycle of th n. If D satisfies the conditions of Theorem B, then either D contains a cycle of length 1 or n is even and D isomorphic to complete bipartite digraph $K^*_{n/2,n/2}$.

Note that Theorem 1 is sharp, in the sense that for all $n \geq 6$ there is a strong digraph D vertices which has minimum semi-degree one and satisfies the condition of Theorem 1, contain no cycle of length $n-1$. To see this, it is sufficient to consider the digraph $D_{n,m}$ h was defined in [13] (see also [1].p.300). When $m = n-1$, then $D_{n,m}$ has minimum -degree one and satisfies the conditions of Theorem 1 but has no cycle of length $n-1$.

We believe Theorem 2 can be generalized to the following

jecture. Let D be a strong digraph on $n \geq 4$ vertices. If D satisfies the conditions of orem C, then D contains a cycle of length $n-1$ maybe except some digraphs which has imple" characterization.

Support for the conjecture we prove the following.

orem 3. Let D be a strong digraph with $n \geq 2$ vertices, which is not directed cycle. If atisfies the conditions of Theorem C, then D contains a cycle of length $n-2$ or $n-1$.

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