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Dynamic Geometry of Some Polynomials

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In 1836, Gauss showed that all the roots of P', distinct from the multiple roots of the polynomial P itself, serve as the points of equilibrium for the field of forces created by identical particles placed at the roots of P (provided that r particles are located at the root of multiplicity r). The following equality to zeros provides a quick proof of Gauss-Lucas theorem (see for example [1] or [2]). Thus was appeared the branch of mathematics, which after the book of Morris Marden[3], was called Geometry of Polynomials. The polynomial conjectures of Sendov and Smale are two challenging problems of this branch[4,5,6].

One of the beautiful theorems of mathematics is Marden's theorem [3,7]. It gives a geometric relationship between the zeros of a third-degree polynomial with complex coefficients and the zeros of its derivative. A more general version of this theorem, due to Linfield [8].

This article focuses on the dynamic behavior of critical points in the case of moving one of the roots of cubic polynomial on a given trajectory. The equations of the curves, where the critical points moves, are obtained. Discovered new geometric properties of positions of the zeros and critical points of a complex polynomial of degree three. The case of multiple roots of the given polynomial is considered as well.

References

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