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Necessary Conditions for Optimal Permissible Placement by the height of the Transitive Directed Tree with One Root (part second)

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Abstract

The present paper is the second part of the paper [1]. Here we have introduced a couple of additional concepts and have obtained some additional necessary conditions for the solution of the problem formulated in paper [1].
Keywords: transitive directed graph, optimal placement.

1. Introduction

The definitions of the following concepts: a transitive directed graph, a permissible placement of the directed graph, a height of the placement of the directed graph, an optimal placement by the height of the directed graph are given in paper [1].

From now on by the placement of the directed graph we will mean its permissible placement. All the results obtained in this paper are true for all the three definitions of the height in paper [1].

The following concepts are defined in paper [1]: a vertex directly following (preceding) the vertex, a basis of arcs, a transitive directed tree with one root, a branching vertex of the transitive directed tree, a branch of the directed tree, a structure, a partial placement, a height of the vertex (structure) in partial placement, an inner height, an (inner) incoming (outgoing) height, delta - Δ

In chapter 2 of the paper [1] the problem of optimal placement by the height for graphs was introduced. The problem is NP-complete [4]. Polynomial optimal algorithms for its??? solution are known only for certain special classes [2, 8-12]. In paper [1] the concept of the transitive directed tree with one root was introduced, the optimal permissible placement by the height problem was formulated for it, and necessary conditions were obtained for the problems solution. In the present paper certain important results have been obtained for the solution of the problem. In this paper we provided additional definitions for it.

2. The Formulation of the Problem

Let us recall our definitions of the transitive directed tree with one root and its optimal permissible placement problem introduced in paper [1].

The transitive directed graph the arc base of which is a directed tree with one root will be called a transitive directed tree with one root.

Problem: *An optimal placement by the height of transitive directed tree with one root.*
or the given transitive directed tree with one root $G=(V,U)$ find such a permissible placement the height of which is equal to the height of the directed tree $H(P,G)=\min_{P} H(P,G)$ where e minimum is taken by all the permissible placements of G.

Necessary New Definitions

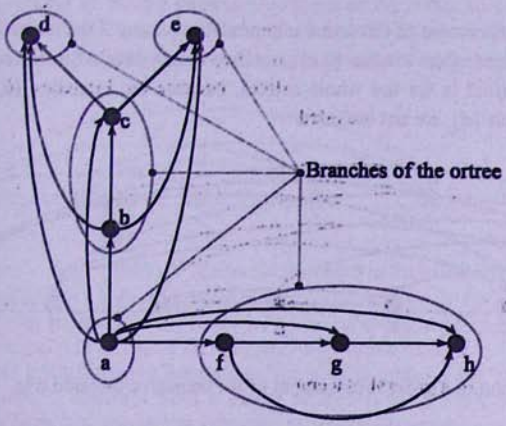
rencefoth, by ortree we will mean a transitive directed tree with one root.

Definition 1. *The branch S of the ortree will be called a direct descendent (or branch T is rectly preceding branch S), if the first vertex of branch S is the directly following vertex of e last vertex of T.*

Definition 2. *The branch of the ortree which has no descendents will be called a leaf.*

Definition 3. *The branch S of the ortree is the descendent of the branch T, if the origin of e branch S is the following vertex of the terminal point T.*

Example 1: the branch $\{d\}$ is the direct descendent of the branch $\{b,c\}$. $\{d\}$ is the scendent of $\{a\}$ but not a direct one. $\{d\}$ is a leaf.



Picture 1. A transitive directed tree with one root and its branches.

Definition 4. *The subtree of the given branch of the ortree will be called that given branch d the subgraph obtained from all the descendent branches of the given branch: that given anch will be called a stem of the subtree (or a basis of the subtree), and the branches following taken together will be called a subtree crown.*

Example 2: in picture 1 the subtree of the branch $\{b,c\}$ is the subgraph obtained from the anches $\{\{b,c\},\{d\},\{e\}\}$. $\{b,c\}$ is the stem of the subtree, branches $\{d\}$ and $\{e\}$ obtain the own.

Definition 5. *The main subtree of the given subtree of the ortree will be called a subtree of e direct descendent branch of the stem of the given subtree.*

Example 3: in picture 1 the subtree of the branch $\{b,c\}$ is a main subtree of the ortree, that of the subree of the branch $\{a\}$.

Definition 6. *(a subtree of the i-th branching level, a branch of the i-th branching level). The leaf is a subtree of zero branching level. It is also a branch of a zero branching level.*

The subtree of the first branching level is the subtree of the ortree the main subtrees o *which are subtrees of a zero branching level, that is, leaves. The stem of the subtree of the firs* *branching level will be called a branch of the first branching level.*

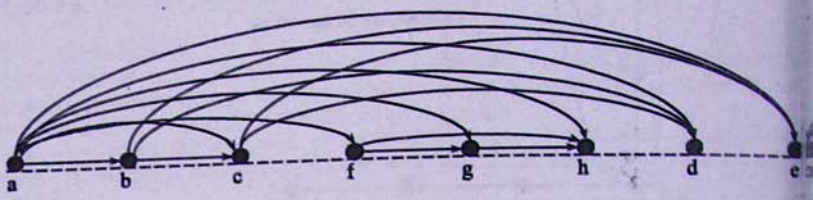
The subtree of the i-th branching level will be called a subtree the main subtrees of whic *are subtrees of 0,...,i-th branching level and there necessarily existed subtrees of i−1-t* *branching level.*

The stem of the subtree of the i-th branching level will be called a branch of i-th branchin *level.*

Example 4: the branches $\{d\}$, $\{e\}$, $\{f,g,h\}$ are the subrees and branches of a zer branching level, the subtree of branch $\{b,c\}$ is of the first branching level, the subtree of th branch $\{a\}$ – the second branching level.

Definition 7. *In the general placement the subcrown of the given subtree will be called* *subsequence formed by the first k branches of the sequence of the subtree crown branches pu in that placement (where k is less than the number of the crown branches or equal to it,* *particularly there may be also other branches belonging to another subtree among the branche* *of the subcrown.*

Example 5: in the general placement of the ortree mentioned in picture 2 the branches $\{d\}$ $\{e\}$ form a subcrown of the subtree of the branch $\{b,c\}$, but they do not obtain the subcrown fo the subtree of the branch $\{a\}$ (that is for the whole ortree), because the branches $\{b,c\}$ an $\{f,g,h\}$ placed before the branch $\{d\}$ are not included.



Picture 2. The illustration of a general placement of the transitive directed tree.

Definition 8. *In the general placement the partial subtree of the given subtree will be calle* *the stem of the subtree put in that placement or the stem and a continuous subcrown put direct* *after the stem taken together, that is, between the branches there mustn't be any branches* *other subtrees.*

Example 6: in the placement of picture 2 the branch $\{a\}$ forms a partial subtree of th ortree, the sequence of the branches $\{\{a\},\{b,c\}\}$ also forms a partial subtree of the ortree, bu $\{\{a\},\{b,c\},\{d\},\{e\}\}$ does not. The only partial subtree of the subtree of the branch $\{b,c\}$ is th branch $\{b,c\}$ itself, the sequence of the branches $\{\{b,c\},\{d\}\}$ does not form a partial subtree the subtree of the branch $\{b,c\}$.

The structure has been defined as an arbitrary set of branches put next to each other and a the lemmas about structures in [1] are true for the general definition of the structure.

But, henceforth, by the structure of the subtree (subcrown) we will mean a more concre structure.

Definition 9. *The structure of the given subtree (subcrown) is the sequence of the branch* *which in the general placement are put next to each other and belong to the given subtr*

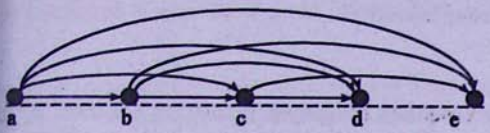
(subcrown) and there are branches belonging to other subtrees put directly before or after that structure.

Example 7: the $\{b,c\}$ and $\{\{d\},\{e\}\}$ are structures of the subtree of the branch $\{b,c\}$, the only branches $\{d\}$ and $\{e\}$ - are not.

Definition 10. *The boundary branch of the subtree in general placement is the first branch of the subtree the outgoing height of which in the partial placement of the stem of the subtree is smaller than the inner incoming height of the stem of the subtree.*

The general placement is noted in this definition since which one of the subtree branches will be the boundary branch depends on the mutual order of those branches in the general placement (and it does not depend on the other subtrees put in general placement). It must also be noted that the partial placement of the stem of the subtree is obtained from the general placement by removing all the branches of the general placement except the branches of the subtree and the preceding branches of the stem (as the subtree does not form a structure in the placement: we cannot use "a partial placement of the subtree" in the definition instead of the partial placement of the stem of the subtree).

Example 8: for the general placement of the ortree mentioned in picture 2 the boundary branch of the subtree of the branch $\{b,c\}$ is $\{d\}$, because in the partial placement of the branch $\{b,c\}$ mentioned in picture 3 the first branch with the outgoing height less than 4 is $\{d\}$.



Picture 3. The partial placement of the branch $\{b,c\}$ of the ortree.

Definition 11. *In the partial placement of the stem of the subtree the sequence formed by the stem of the subtree and the branches of the subtree placed up to the boundary branch (including that branch) will be called an integrity range of the subtree.*

The integrity range depends on the mutual order of the branches of the subtree in general placement. It does not form a partial subtree in the general placement as there may be other subtree branches between its branches in general placement.

Example 9: the sequence of branches $\{\{b,c\},\{d\}\}$ is the integrity range of the subtree of the branch $\{b,c\}$ for the general placement in picture 2.

4. Necessary Conditions for Optimal Permissible Placement by the Height of the Transitive Directed Tree with One Root

Theorem 1. *The positive structures of other subtrees placed within the branches of integrity range of the given subtree in optimal placement must be moved before the stem of the given subtree.*

Proof: Let us consider the first positive structure belonging to another subtree placed between the branches of the integrity range of the given subtree.

Let us consider the subcrown of the subtree, which is obtained from the branches of the given subtree crown placed before that considered positive structure.

The stem of the subtree together with the considered subcrown will be denoted by E. E is already a negative partial subtree and according to lemma 1 in paper [1] that positive structure must be moved before E.

Let's consider any arbitrary subcrown of the ortree.

Theorem 2. *In the optimal placement if the structures of the given subcrown of the subtree are all positive the negative structures of other subtrees placed between them or between them and the stem must be moved and put after the subcrown.*

Proof: As the subcrown structures are all positive the negative structures of other subtrees put between them must be moved in turn and put after the positive structures of the subcrown (according to paper [1] lemma 1). Due to lemma 3 in paper [1] the two positive structures taken together which in the result of replacement of negative structures belonging to another subtree appeared next to each other will now form a single positive structure and the positivity of the subcrown structures mentioned in our lemma will be preserved. According to the same lemma the unified structure obtained by the merge of negative structures of other subtrees which in the result of replacement appeared to be next to each other will also preserve the sign. In the result of the displacement the joined structure obtained by merging the adjacent structures having different signs and belonging to other subtrees can be both positive and negative. In case of being positive it is remained in its place and in case of being negative it is again removed after the structure of the subcrown considered. Thus, by applying lemma 1 and lemma 3 in paper [1] for the negative structures and the subcrown positive structures, all the remaining negative structures will be replaced after the subcrown in order.

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Մեկ արմատով տրանզիտիվ օրիենտացված ծառի ըստ բարձրության օպտիմալ թույլատրելի տեղադրման անհրաժեշտ պայմաններ (մաս երկրորդ)

Ա. Խաչատուրյան

Ամփոփում

Սույն հոդվածը հանդիսանում է [1] հոդվածի շարունակությունը: Այստեղ մենք դիտարկել ենք որոշ նոր հասկացություններ և ստացել [1] հոդվածում ձևակերպված խնդրի լուծման համար ևս մի քանի անհրաժեշտ պայմաններ:

Необходимые условия оптимальной допустимой расстановки по высоте транзитивно ориентированного дерева с одним корнем (часть вторая)

А. Хачатурян

Аннотация

Настоящая статья является продолжением статьи [1]. Здесь мы привели некоторые новые концепции и получили еще несколько необходимых условий для решения задачи сформулированной в статье [1].