An Algorithm of Digital Image Interpolation

Lilit G. Minasyan

State Engineering University of Armenia

Abstract

The effective algorithms for reducing and increasing the size of images using the orthogonal wavelet-like transformation are presented. The experimental results show an improvement in terms of PSNR in comparison to the well-known image interpolation algorithms.

Keywords: interpolation, orthogonal transform, image resizing algorithm.

1. Introduction

Image resizing is one of the important operations, for instance, in the field of medical image processing, computer graphics, image database, *etc.* The problem may occur when the user needs to display an image at different resolutions depending on the resolution of a display device [1]. We often need to perform the operations involving image resizing such as zoom in, zoom out and crop operations on huge amount of images [2].

The aim of image resizing is to magnify or reduce the size of an image while preserving the details and visual quality of original image. Image resizing is reduced to the problem of interpolation. The interpolation of digital images is also used to change the image scanning from one pixel grid to another, the correction of lens distortion, perspective shift, or rotation of image.

There are many interpolation algorithms. One of the simplest methods is the nearest neighbor interpolation. The methods such as the bilinear and bicubic interpolations maintain a better image quality than the nearest neighbor interpolation when resizing is done to enlarge the image. However, these methods are often not suitable for image reduction. Most of image resizing algorithms are based on the principle of spatial interpolation and are not adapted to an image content. They may fail to preserve image details, especially during reduction [3].

It should be noticed that not all the image resizing techniques are expected to work well for all types of images, and different methods work well on different types of images. Therefore, even if one considers the same image, the results may vary significantly depending on the interpolation algorithm. Since, in general, any interpolation process is only an approximation, an image quality will deteriorate each time it is being interpolated. In this case, the result depends strongly upon the image itself. In many cases, it is advisable to use an adaptive image interpolation algorithm. The adaptive methods depend on the object interpolation. Moreover, they are changed during processing depending on sharpness of edges, smooth areas and other features of the processed image. Non-adaptive algorithms handle all parts of the image equally.

In [4] the synthesis of orthogonal wavelet-like transform is presented. In [4], [5] the image compression algorithm based on this transform is described which has a high compression ratio and good perceptual quality. In [6] a class of transforms as well as an algorithm for zooming in of the image with the use of these transformations is presented.

In this work, the algorithms for image zooming in and zooming out using the transformation proposed in [4] are described. The comparison with the results of the well-known non-adaptive image interpolation algorithms is also provided.

The paper is organized as follows. In Sections 2 and 3 we first give a short description of the conventional image interpolation algorithms as well as non-adaptive and adaptive methods, respectively. Section 4 describes a general image resizing concept in a transform domain. In Section 5 we present an image interpolation method in our proposed new transform domain. The experimental results are also illustrated in this section.

2. Non-adaptive Algorithms for Image Enlargement

It is known that digital image may be identified with a rectangular or square matrix of values of the brightness of its pixels. Here we bring a short description of the well-known non-adaptive image interpolation methods.

2.1 Nearest Neighbor Method

Among the non-adaptive algorithms, the simplest method of image resizing is the nearest neighbor interpolation. For each point of interpolation, the algorithm assigns (or replicates) the value of the nearest neighbor pixel. As a result, the rows and columns of the matrix are repeated twice. In the case of repeated non-uniform application of the algorithm to the image, it later becomes checkered. The algorithm requires a minimal processing time [7].

2.2 Two-dimensional Spline Interpolation of Images

Bilinear interpolation

Bilinear spline is a two-dimensional generalization of 1D linear spline of the function of two variables defined on a rectangular grid [8].

Suppose we want to interpolate the value of the function $f(x,y)$ at the point $P(x,y)$ belonging to the interior of the rectangle with vertices $P_{11}(x_1,y_1)$, $P_{12}(x_1,y_2)$, $P_{21}(x_2,y_1)$ $P_{22}(x_2,y_2)$, $(x_1 < x_2, y_1 < y_2)$. At first step, the value of $f(x,y)$ is interpolated linearly at the intermediate points $Q_1(x,y_1)$, $Q_2(x,y_2)$ (Figure 1), and then at the point $P(x,y)$. Thus, the function $f(x,y)$ is calculated with the following formula:

$$f(x,y) \approx f(x,y_1)\frac{y-y_2}{y_1-y_2} + f(x,y_2)\frac{y-y_1}{y_2-y_1}, \tag{1}$$

where

$$f(x,y_j) \approx f(x_1,y_j)\frac{x-x_2}{x_1-x_2} + f(x_2,y_j)\frac{x-x_1}{x_2-x_1}, j=1,2.$$

The algorithm is characterized by its simplicity and speed. It should be noticed that it disadvantages are the low level of accuracy, non-smoothness of interpolated spline a interpolation nodes.

Bicubic spline. The use of bilinear splines often does not provide a required quality obtained after image processing. The discontinuity of the derivatives is one of the confounding factors and influences the result. In such cases, it is desirable to use bicubic splines [10]. The have continuous first and mixed second derivatives. To construct a bicubic spline the function values, its first derivatives as well as the value of the mixed second derivative at the nodes of th mesh are required. Similarly, as in the case of bilinear spline interpolation, one can first perform interpolation along one variable, and then, along another one.

Figure 1 shows the results of 4- times increase of a part of image *Lena* with size 64x64 using the nearest neighbor algorithms, the methods of bilinear and bicubic splines.

a) Original *Lena*

b) c) d)

Figure 1. Results after applying different interpolation methods: b) nearest neighbor, c) bilinear spline, d) bicubic splines.

Non-adaptive interpolation algorithms result in stair-wise boundaries, blurring and boundary halo.

The methods for the nearest neighbor and bilinear spline lead to a stair-wise effect on image boundaries. These methods are a little susceptible to the boundary halo, and differ only by a different balance between the stair-wise effect and blurring. The methods for Lanczos and bicubic spline show a small level of stair-wise effect, but lead to significant blurring of the image [10]-[12].

3. Adaptive Algorithms for Image Magnification (resizing)

The most common adaptive algorithms are the following: the method of genuine fractals [13], PhotoZoom (standard), PhotoZoom (GUI), PhotoZoom (text), SmartEdge. The most commonly used fractal algorithms are genuine fractals. In general, they process an image analogous to the files of vector graphics. Theoretically, the algorithm is zooming in an image without loss, but in practice there are noises in the form of small-scale texture. In some cases, the results are not better than that of bicubic interpolation [10].

To find the luminance value of the pixel being processed, the adaptive methods using the algorithms for detection of boundaries take into account the pixel values of its surrounding area with some appropriate weights. These algorithms depend on the neighborhood of a pixel. Their flexibility allows to get sharper images with fewer defects compared to non-adaptive methods [10]-[12]. These algorithms recover boundaries by using S-splines. They require more processing time and usually are more expensive.

The algorithm SmartEdge is the only one resulting in a clear smooth boundary. However, the SmartEdge algorithm is publicly unavailable.

4. Interpolation Method with the Use of Transformations

Figure 2 shows a block diagram of the image interpolation algorithm in a transform domain. The scheme uses the following abbreviations: DOT stands for discrete orthogonal transform, IDOT stands for inverse discrete orthogonal transform.

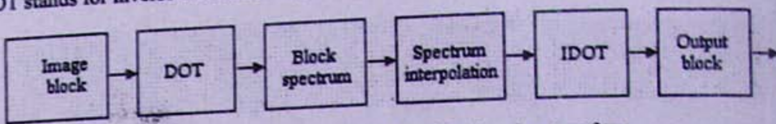

Figure 2. Image interpolation algorithm based on transform.

5. The Proposed Image Interpolation Method Based on the New System of Basis Functions

Let

$$\varphi_0(x)=\begin{cases}\cos^2\pi x, x\in\left[0,\frac{1}{2}\right],\\ 0, \quad x\in\left[\frac{1}{2},1\right],\end{cases}\quad \varphi_1(x)=\begin{cases}0 \quad ,x\in\left[0,\frac{1}{2}\right],\\ \cos^2\pi x, x\in\left[\frac{1}{2},1\right],\end{cases}\quad \varphi_n(x)=\varphi_{ki}(x)=\begin{cases}\sin^2 2^k\pi x, x\in\left[\frac{i-1}{2^k},\frac{i}{2^k}\right],\\ 0, \quad x\in\left[\frac{i-1}{2^k},\frac{i}{2^k}\right],\end{cases}$$

$n=2^k+i$, $k=0,1,2,\ldots,i=1,2,\ldots,2^k$.

Denote by T_N - $N\times N$ ($N=2^k$) a matrix obtained by uniform discretization and Gram Schmidt orthogonalization of the first N basis functions (1). The plots of the first f orthogonal basis functions are presented below on Figure 3.

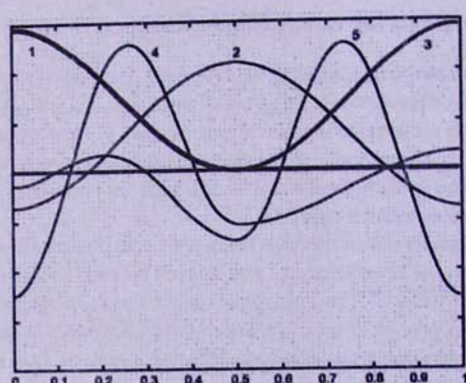

Figure 3. Plots of the first five orthogonal basis functions.

[4]-[6] the discrete orthogonal wavelet-like transforms for image compression and filtration are described in terms of the new obtained transforms. These algorithms have a high compression ratio while providing a good visual quality.

The following steps describe briefly the image interpolation process:

1. Applying 2D transform T_N to block M, we get its 2D spectrum S:

$$S = T_N \cdot M \cdot T_N'$$

where M is the matrix of luminance values of the image block of size $N \times N$, T_N is the transform matrix, T_N' is a matrix transposed to T_N.

2. Denote by S_1 a matrix consisting of the first $n \times n$ $(n = 2^m, m < k)$ elements of the upper left corner of the 2D matrix S.
3. Using 2D inverse transform T_{N_1} of dimension $n \times n$ to block S_1 we obtain the restored image block M_1 of size $n \times n$:

$$M_1 = T_{n}' \cdot S_1 \cdot T.$$

In the case of zooming in, in step 2 we shall replace the matrix S_1 by the following matrix:

$$S_1 = \begin{bmatrix} S & Z \\ Z & Z \end{bmatrix},$$

where S is the spectrum of the block M and Z is the zero matrix of size S.

The quality assessment has a subjective nature and, generally, it is done visually. However, we bring also the numerical estimates which give some assessment of quality. Suppose we have an image of M and its approximation M_1 of the size of $n \times m$ pixels. Denote by

$$MSE = \frac{1}{n \times m} \sum_{i=1}^{n} \sum_{j=1}^{m} [M(i,j) - M_1(i,j)]^2, \; PSNR = 10 \cdot \lg\left(\frac{255^2}{MSE}\right).$$

MSE determines an accuracy according to the mean-squared error. The larger is the $PSNR$ (peak signal-to-noise ratio), the smaller is the error of approximation.

To assess the quality of image interpolation after zooming in by *kxk* times, we took every *k*-column of the *k*-th row starting from the first row, then we have increased the image size and compared it with the original one. That is, the PSNR value is considered here in this sense.

When zooming an image out, in order to estimate the error, the smallest image is zoomed in up to the size of the original one and is compared with the original image. Below in Figure 4 the image *Barbara* and its spectrum are presented.

Barbara Spectrum of *Barbara*

Figure 4. ¼ part of spectrum 1/16 part

Table 1. PSNR values of 2 times zoomed in images.

Image/size	Method						
	dct2	nearest	bilinear	bicubic	lanczos3	Prop. trans	haar
	PSNR						
Cameraman, 256	22.91	22.43	23.91	23.70	23.44	24.98	22.44
Lena, 256	25.07	24.46	26.20	25.94	25.65	27.51	24.47
Lena, 512	29.49	28.29	30.21	30.13	29.94	33.13	28.2[illegible]
Barbara, 512	22.01	22.22	23.88	23.34	22.85	23.96	22.2[illegible]

Table 2. PSNR values of 4 times zoomed in images.

Image/size	Метод						
	dct2	nearest	bilinear	bicubic	lanczos3	Prop. trans	haar
	PSNR						
Cameraman, 256	18.58	18.14	19.69	19.24	18.98	20.67	18.1[illegible]
Lena, 256	20.31	19.79	21.42	20.98	20.72	23.07	19.7[illegible]
Lena, 512	23.37	22.70	24.47	24.05	23.78	27.15	22.7[illegible]
Barbara, 512	19.57	19.34	20.95	20.42	20.12	20.87	19.3[illegible]

Table 3. PSNR value of 2 times zoomed out images.

Images	Метод						
	dct2	nearest	bilinear	bicubic	lanczos3	Prop. trans	haar
	PSNR						
Cameraman, 256	27.40	22.37	24.70	26.33	26.88	26.82	25.49
Lena, 256	30.08	24.47	27.19	29.02	29.63	29.70	27.71
Lena, 512	36.02	28.29	31.42	34.15	35.23	35.16	31.56
Barbara, 512	25.55	22.22	24.54	25.35	25.54	25.81	25.64

Figure 5 shows parts of images of *Lena* and *Barbara* of sizes 64x64 and the increased ones according to the above algorithm, to the size of 256x256.

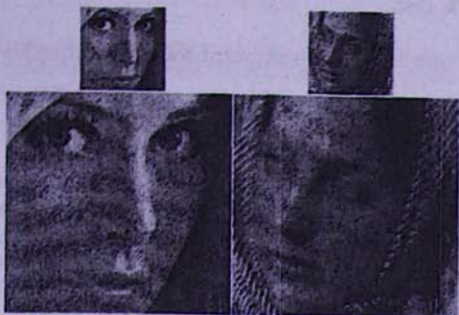

Figure 5. Images of *Lena* and *Barbara* of sizes 64x64, 256x256.

References

[1] J. Mukherjee, S. K. Mitra, "Image resizing in the compressed domain using subband DCT," *IEEE Trans. Circuits and Systems for Video Technology*, vol. 12, no. 7, pp. 620-627, 2002.

[2] S. A. Martucci, "Image resizing in the discrete cosine transform domain," *IEEE Proc. Int. Conf. Image Processing*, vol. 2, pp. 244-247, 1995.

[3] J. Scott, R. Tutwiler and M. Pusateri, "Hyper-spectral content aware resizing," *IEEE Conf. Applied Imagery Pattern Recognition Workshop*, pp. 1-7, 2008.

[4] S. Khachatryan, L. Minasyan, "On synthesis and application of an orthogonal discrete transform for image compression," *Journal of Engineering Academy of Armenia*, T4, № 1. Yerevan, pp.124-129, 2007 (in Russian).

[5] S. Khachatryan and L. Minasyan, "Design of orthogonal transform on partion of unity method and their application to image compression, " *Int. Workshop on Local and Non-Local Approximation in Image Processing, LNLA'2008*, Tuusula, Finland, pp. 145-152, 2008.

[6] S. Khachatryan and L. Minasyan, "On algorithms of digital image interpolation," *Mathematics in high School*, T. 6, № 2, pp. 32-41, 2010.

[7] R. Gonzalez and R. Woods, *Digital Image Processing*, 2nd Ed. Upper Saddle River, New Jersey: Prentice-Hall, Inc., 2002.

[8] T. W. Parks and C. S. Burrus. *Digital Filter Design*. New York: John Wiley & Sons, pp. 209-213, 1987.

[9] A. V. Oppenheim and R. W. Schafer. "Discrete-time signal processing". *Upper Saddle River*, NJ: Prentice-Hall, pp. 450-454, 1999.

[10] R. Keys, "Cubic convolution interpolation for digital image processing," *IEEE Trans. on Signal Processing*, Acoustics, Speech, and Signal Processing 29 (6), pp.1153-1160, 1981.

[11] K. Turkowski and S. Gabriel, "Filters for common resampling Tasks". In Andrew S. Glassner. *Graphics Gems I*. Academic Press. pp. 147–165. ISBN 9780122861659, 1990.

[12] W. Burger, M. J. Burge, *Principles of digital image processing: core algorithms*. Springer. pp. 231–232. ISBN 9781848001947, 2009.

[13] G. Schaub, "Genuine Fractals 4.1; Resampling with GF might make the megapixel race moot," March, Shutterbug, 2006.

[14] J. Canny, "A computational approach to edge detection," *IEEE Transactions on PAMI*, 8(6), pp. 679–698, 1986.

Թվային պատկերների միջարկման մի ալգորիթմի մասին

Լ. Մինասյան

Ամփոփում

Առաջարկված են պատկերների չափերի փոքրացման և մեծացման արդյունավետ ալգորիթմներ՝ վեյվլեթ տիպի օրթոգոնալ ձևափոխության կիրառմամբ:
Կատարված են համեմատություններ պատկերների միջարկման հայտնի ալգորիթմների արդյունքների հետ:

Об одном алгоритме интерполяции цифрового изображения

Л. Минасян

Аннотация

Предложены эффективные алгоритмы уменьшения и увеличения размеров изображений с применением ортогонального вейвлетообразного преобразования. Приведены сравнения с результатами известных алгоритмов интерполяции изображений.