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On Some Kinds of Constructive Fuzzy Logics*

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Abstract

Some modification of the Extended Fuzzy Constructive Logic [14] is considered. It is proved that this modification is actually equivalent to the Intuitionistic Fuzzy Logic introduced in [9].

l Introduction

In [9] the logical system of Intuitionistic Fuzzy Logic is introduced where some features of uuitionistic logic are brought into concordance with the general concept of fuzzy logic ([6], [7], [0], [11]).

However the considerations in [9] are not based on the intuitionistic [3], or constructive ıɾproach ([18], [20], [21]). They are based on a set-theoretical approach, where the concept of actual ıinity is admitted. In [14] the system in the Extended Fuzzy Constructive Logic is created, where ; logical and mathematical principles of Intuitionistic and Constructive Logic are combined with ; logical approach of Fuzzy Logic. The consideration in [14] is based on the constructive principles 8], [20], [21]).

Below some modification of the Extended Fuzzy Constructive Logic introduced in [14] is ɾnstructed; this modification is based on the set-theoretical principles. It will be proved that the ɪntioned modification is actually equivalent to the system of Intuitionistic Fuzzy Logic [9]. So the ɾnnection between logical systems considered in [9] and [14] is established.

Some Features of Extended Fuzzy Constructive Logic

Let us consider some definitions of notions used below.

A recursively enumerable fuzzy set (REFS) having the dimension $n \geq 1$ is defined as any ursively enumerable set of $(n+1)$- tuples $(x_1, x_2, ..., x_n, \varepsilon)$, where all x_i belong to the set of tural numbers $N = \{0,1,2,...\}$, and ε is a binary rational number $\frac{k}{2^m}$, such that $0 \leq \varepsilon \leq 1$ (cf. [5], 2]-[14], [16], [17]). The notion of *pseudonumber* is defined as in [15]. Elementary relations =,< tween pseudonumers, the operations +,− on pseudonumbers as well as Goedel numbering of eudonumbers are defined similarly to [15] in a natural way. *Specker's numbers* are defined ([8], 5]) as pseudonumbers generated by constructive increased bounded sequences of rational

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numbers. If α is an n-dimensional REFS than its *Specker's representation* is defined as the total recursive function ([2],[4]) (denoted by ψ_α) such that for any n-tuple $(x_1, x_2, ..., x_n)$ of natural numbers the value $\psi_\alpha(x_1, x_2, ..., x_n)$ is a Goedel number of Specker's number (which is also denoted by $\psi_\alpha(x_1, x_2, ..., x_n)$) satisfying the following conditions:

(1) if there is no ε such that $(x_1, x_2, ..., x_n, \varepsilon) \in \alpha$ then $\psi_\alpha(x_1, x_2, ..., x_n) = 0$;

(2) if there exists such ε, then $\psi_\alpha(x_1, x_2, ..., x_n)$ is the supremum of the numbers ε, such that $(x_1, x_2, ..., x_n, \varepsilon) \in \alpha$.

An n-dimensional REFS α is said to be *open* if it satisfies the following conditions (cf. [13], [14]):

1) All $(n+1)$-tuples having the form $(x_1, x_2, ..., x_n, 0)$ belong to α.

2) If $(x_1, x_2, ..., x_n, \varepsilon) \in \alpha$ and $0 \le \delta \le \varepsilon$ then $(x_1, x_2, ..., x_n, \delta) \in \alpha$.

3) For any $(n+1)$-tuple $(x_1, x_2, ..., x_n, \varepsilon) \in \alpha$, where $\varepsilon > 0$, there exists such $\delta > \varepsilon$ that $(x_1, x_2, ..., x_n, \delta) \in \alpha$.

We consider below only open REFSes. Clearly, every open REFS is completely defined by its Specker's representation. Two n-dimensional open REFSes α and β are said to be *equivalent* (cf. [13], [14]), if $\psi_\alpha(x_1, x_2, ..., x_n) = \psi_\beta(x_1, x_2, ..., x_n)$ for any natural numbers $x_1, x_2, ..., x_n$. The equivalence between α and β is denoted as usually, by $\alpha = \beta$.

This relation is the same as the relation of equivalence considered in [13] and [14]; it is different from the relation of equivalence considered in [5], [16], [17].

We use the following operations on REFSes: (1) *union* $\alpha \cup \beta$ of n-dimensional REFSes α and β; (2) *intersection* $\alpha \cap \beta$ of n-dimensional REFSes α and β; (3) *Cartezian product* $\alpha \times \beta$ of n-dimensional REFS α and m-dimensional REFS β; (4) *projection* $\downarrow_i^n(\alpha)$ of an n-dimensional REFS α on i-th co-ordinate, where $0 \le i \le n$; (5) *generalization* $\uparrow_i^n(\alpha)$ of an n-dimensional REFS α on i-th co-ordinate, where $0 \le i \le n$; (6) *transposition* $T_{ij}^n(\alpha)$ of i-th and j-th co-ordinates in an n-dimensional REFS α, where $0 \le i, j \le n$; (7) *substitution of variables* $Sub^n{}_{i,j}(\alpha)$ in an n-dimensional REFS α (that is, the substitution of the variable x_j for the variable x_i).These operations are defined as in [13] and [14]. The definitions of $\cup, \cap \times, T_{ij}^n, \downarrow_i^n, \uparrow_i^n$ are similar to those given in [16] and [19]. By $\vee^n$ (correspondingly, $\wedge^n$) we denote n-dimensional open REFS α (correspondingly, β) such that for all $x_1, x_2, ..., x_n$ $\psi_\alpha(x_1, x_2, ..., x_n) = 1$ (correspondingly, $\psi_\beta(x_1, x_2, ..., x_n) = 0$).

On the basis of the mentioned operations the semantics of the Extended Fuzzy Constructive Logic (EFC-logic) is described in [14]. For the convenience of the reader we recall here the definitions of main notions connected with this semantics.

n-dimensional REFS- ideal Δ is defined as a non-empty set of open n-dimensional REFSes satisfying the following conditions:

(1) If $\alpha \in \Delta$ and $\beta \subseteq \alpha$ then $\beta \in \Delta$.

(2) If $\alpha \in \Delta$ and $\beta \in \Delta$ then $\alpha \cup \beta \in \Delta$.

(Note that the notion of set in [13], [14] and in this section is interpreted in constructive sense ([15], [18], [20], [21])).

Let Δ be a non-empty set of n-dimensional open REFSes. Let us consider the set Δ' of n-dimensional open REFSes β, such that there exist $\alpha_1 \in \Delta, \alpha_2 \in \Delta, ..., \alpha_k \in \Delta$ satisfying the condition: $\beta \subseteq \alpha_1 \cup \alpha_2 \cup ... \cup \alpha_k$. It is easy to see that the set Δ'obtained in such a way is a

REFS-ideal. In this case we say that Δ' is a *REFS-ideal generated by the set* Δ. n-dimensional REFS-ideal Δ is said to be *principal ideal* if there exists an n-dimensional REFS β, such that $\alpha \in \Delta$ if and only if $\alpha \subseteq \beta$. n-dimensional REFS- ideal is said to be *complete ideal* (correspondingly, *null ideal*) if all n-dimensional REFSes belong to Δ (correspondingly, only $\wedge^n$ belong to Δ)

The notion of *predicate formula* on the basis of logical operations $\&, \wedge, \supset, \neg, \forall, \exists$ is defined in a usual way ([2], [4]); we consider predicate formula without functional symbols and symbols of constant. The symbol T of truth and the symbol F of falsity are considered as elementary formulas. All auxiliary notions connected with predicate formulas are defined in a usual way ([2], [4]). We suppose that a sequence of all variables $x_1, x_2, \ldots$ included in considered predicate formulas is fixed. *Index majorant* of a formula A is defined as any number k such that every index i of a variable x_i (both free and bound) included in A is less or equal to k.

Now let us define the semantics of predicate formulas (as in [14]) in the framework of the Extended Fuzzy Constructive Logic (i.e. EFCL- semantics). Let A be a predicate formula which does not contain predicate symbols except $p_1, p_2, \ldots, p_l$ having the dimensions, correspondingly, $k_1, k_2, \ldots, k_l$; let k be the index majorant for A. *EFCL-assignment* for A is defined as an assignment φ which assigns a k_l- dimensional REFS-ideal to any predicate symbol p_l. An EFCL- assignment for A is said to be principal if all REFS-ideals assigned to $p_1, p_2, \ldots, p_l$ are principal REFS-ideals.

Now we define (as in [14]) the *EFCL-interpretation* $\Pi_{\varphi,\kappa}(A)$ of a formula A concerning a given EFCL-assignment φ for A and a given index majorant k for A. This notion is defined by induction on the construction of A. Let A be an elementary formula having the form $p_r(\xi_1, \xi_2, \ldots, \xi_t)$, where $\xi_1, \xi_2, \ldots, \xi_t$ are variables having the indices, correspondingly, $j_1, j_2, \ldots, j_t$; let k be a number such that $j \le k$ for $1 \le i \le t$. Let Δ be a t-dimensional REFS-ideal assigned to p_r in an EFCL-assignment φ. For constructing of $\Pi_{\varphi,\kappa}(p_r(\xi_1, \xi_2, \ldots \xi_t))$ we construct the k-dimensional REFS-ideal Δ', generated by all REFSes $\alpha \times V^{k-t}$, where $\alpha \in \Delta$. After this we construct the k-dimensional REFS-ideal Δ'' generated by all REFSes γ obtained from the REFSes belonging to Δ' by the system of transformations T^k_{ij} and Sub^k_{ij} transferring the variables with indices $j_1, j_2, \ldots, j_t$ on the places, correspondingly, $\xi_1, \xi_2, \ldots, \xi_r$. The REFS-ideal obtained in such a way, is $\Pi_{\varphi,\kappa}(p_r(\xi_1, \xi_2, \ldots \xi_t))$. The REFS-ideals $\Pi_{\varphi,\kappa}(T)$ and $\Pi_{\varphi,\kappa}(F)$ are principal ideals, generated correspondingly, by $\vee^k$ and $\wedge^k$. The REFS-ideal $\Pi_{\varphi,\kappa}(B \& C)$ (correspondingly, $\Pi_{\varphi,\kappa}(B \vee C)$) is defined as the set of open k-dimensional REFSes, having the form $\beta \cap \gamma$ (correspondingly, $\beta \cup \gamma$), where $\beta \in \Pi_{\varphi,\kappa}(B), \gamma \in \Pi_{\varphi,\kappa}(C)$. The REFS-ideal $\Pi_{\varphi,\kappa}(B \supset C)$ is defined as the set of all open k-dimensional REFSes ω such that $\beta \cap \omega \in \Pi_{\varphi,\kappa}(C)$ for any $\beta \in \Pi_{\varphi,\kappa}(B)$. The REFS-ideal $\Pi_{\varphi,\kappa}(\neg B)$ is defined as $\Pi_{\varphi,\kappa}(B \supset F)$. $\Pi_{\varphi,\kappa}(\exists x_i(B))$ is defined as the set of all open k-dimensional REFSes ω satisfying the following condition: $\omega \subseteq \uparrow^k_i(\beta)$ for some $\beta \in \Pi_{\varphi,\kappa}(B)$. $\Pi_{\varphi,\kappa}(\forall x_i(B))$ is defined as the set of all open k-dimensional REFSes ω such that $\uparrow^k_i(\omega) \in \Pi_{\varphi,\kappa}(D)$.

Note that all sets of REFSes mentioned in the definitions considered above are REFS-ideals (see [14], Lemmas 3.1-3.8)

A predicate formula A is said to be strongly EFCL-valid (correspondingly, weakly EFCL-valid) if for any EFCL-assignment φ (correspondingly, for any principal EFCL-assignment φ) and

for any great enough index majorant k for A the following condition holds: $\Pi_{\varphi,k}(A)$ is a complete k-dimensional REFS-ideal. The following theorems are proved in [14]: (1) every formula deducible in the constructive (intuitionistic) predicate calculus is strongly EFCL-valid; (2) some formulas having the form
$(A \vee \neg A), (A \supset (B \vee C)) \supset ((A \supset B) \vee (A \supset C)), \forall x(A \supset P(x)) \supset (A \supset \forall x P(x))$,
(where A does not contain free x) are not weakly EFCL-valid (see [14], Theorems 5.1 and 5.2).

3. Modification of Extended Fuzzy Constructive Logic

In this section we create some modifications of Extended Fuzzy Constructive Logic which will be denoted by ST-EFCL ("Set-theoretical Extended Fuzzy Constructive Logic"). We shall admit in this section the "classical" set-theoretical point of view. In particular, instead of REFS-ideals we shall consider "Set-theoretical REFS-ideals" ("ST- REFS-ideals") which are defined in the same way as REFS-ideals with only difference: the notion of set is considered from the "classical" set-theoretical point of view (however the notion of REFS is actually not changed). (ST- REFS-ideals are called in [1] "classical fuzzy set ideals" or "CFS-ideals".)

The notion of ST-REFS-ideal generated by a given set of n-dimensional REFSes, the notion of complete ST-REFS-ideal, the notion of null ST-REFS-ideal are defined similarly to the corresponding notions defined above for REFS-ideals.

We shall use the "classical" set-theoretical notion of real number. *Standard real function* having a dimension n is defined as a function f (in the set-theoretical interpretation of the notion of function) such that for any natural numbers $x_1, x_2, ..., x_n$ the value $f(x_1, x_2, ..., x_n)$ is a "classical" real number such that
$0 \le f(x_1, x_2, ..., x_n) \le 1$. Clearly, for any n-dimensional ST-REFS-ideal Δ there exists an n-dimensional standard real function f (which is said to be an α majorant of Δ), such that for any natural numbers $x_1, x_2, ..., x_n$ $f(x_1, x_2, ..., x_n) = \sup_{\alpha \in \Delta} \Psi_\alpha(x_1, x_2, ..., x_n)$. The majorant of ST-REFS-ideal Δ will be denoted by Δ_f.

We say that n-dimensional ST-REFS-ideals Δ and Ω are quasi-equivalent if

$$f_\Delta(x_1, x_2, ..., x_n) = f_\Omega(x_1, x_2, ..., x_n)$$

for any $x_1, x_2, ..., x_n$.

Below ST-REFS-ideals will be considered up to their quasi-equivalence.

Primitive n-dimensional REFS generated by natural numbers $z_1, z_2, ..., z_n$ and rational number a such that $0 \le a \le 1$, is defined as an open REFS α such that

$$\Psi_\alpha(x_1, x_2, ..., x_n) = \begin{cases} a & if \quad x_1 = z_1, x_2 = z_2, ..., x_n = z_n; \\ 0 & \quad in \quad other \quad cases. \end{cases}$$

Such a REFS will be denoted below by $W(a, z_1, z_2, ..., z_n)$.

Now let f be an n-dimensional standard real function. Let us consider a ST-REFS- ideal Δ generated by the set of all REFSes $W(\alpha, z_1, z_2, ..., z_n)$ such that $z_1, z_2, ..., z_n$ are any natural numbers, a is a rational number, $a < f(z_1, z_2, ..., z_n)$ if $f(z_1, z_2, ..., z_n) > 0$, and $a = 0$ if and only if $(z_1, z_2, ..., z_n) = 0$. The majorant of such Δ will be equal to f. Hence the following Lemma is true.

Lemma 3.1. *For any n-dimensional standard real function f there exists a ST-REFS-ideal such that its majorant is equal to f.*

The ST-REFS ideal Δ obtained by the construction described above for an n-dimensional standard real function f will be denoted by Δ_f.

For any standard real function f we consider the class of ST-REFS-ideals having the majorant f. All these ST-REFS-ideals are quasi-equivalent to Δ_f.

It is easily seen that the transformations of ST-REFS-ideals considered below preserve the relation of quasi-equivalence between ST-REFS-ideals. So for the description of these transformations it is sufficient to describe considered transformations for Δ_f-ideals.

Theorem 3.1. *If f and g are n-dimensional standard real functions then the following statements hold.*

(1) The set of all open REFSes representable in the form $\omega=\alpha\cup\beta$ (correspondingly, $\omega=\alpha\cap\beta$), where $\alpha\in\Delta_f$, $\beta\in\Delta_g$ is an ST-REFS-ideal which is quasi-equivalent to Δ_h, such that $h(x_1,x_2,...,x_n)=\max(f(x_1,x_2,...,x_n),g(x_1,x_2,...,x_n))$

(correspondingly, $h(x_1,x_2,...,x_n)=\min(f(x_1,x_2,...,x_n),g(x_1,x_2,...,x_n))$).

(2) The set of all open REFSes ω such that $\alpha\cap\omega\in\Delta_g$ for any $\alpha\in\Delta_f$ is an ST-REFS-ideal which is quasi-equivalent to Δ_h, such that

$$h(x_1,x_2,...,x_n)=\begin{cases}1 & \text{if } f(x_1,x_2,...,x_n)\le g(x_1,x_2,...,x_n);\\ g(x_1,x_2,...,x_n) & \text{if } f(x_1,x_2,...,x_n)>g(x_1,x_2,...,x_n).\end{cases}$$

(3) If j is a d-dimensional standard real function, where $d<n$, then the set of all open REFSes ω representable in the form $\omega=\alpha\times\vee^{n-d}$, where $\alpha\in\Delta_j$, is quasi-equivalent to ST-REFS-ideal Δ_h such that $h(x_1,x_2,...,x_n)=j(x_1,x_2,...,x_d)$ (here h has fictitious variables $x_{d+1},x_{d+2},...,x_n$).

(4) The set of all open REFSes ω representable in the form $T^n_{ij}(\alpha)$, where $1\le i,\ j\le n$, $\alpha\in\Delta_f$, is an ST-REFS-ideal Δ_h, such that

$$h(x_1,x_2,...,x_{i-1},x_i,x_{i+1},...,x_{j-1},x_j,x_{j+1},...,x_n)=f(x_1,x_2,...,x_{i-1},x_j,x_{i+1},...,x_{j-1},x_i,x_{j+1},...,x_n)$$

(5) The set of all open REFSes ω representable in the form $Sub^n_{ij}(\alpha)$ where $1\le i,\ j\le n$, $\alpha\in\Delta_f$, is an ST-REFS-ideal Δ_h, such that

$$h(x_1,x_2,...,x_{i-1},x_i,x_{i+1},...,x_{j-1},x_j,x_{j+1},...,x_n)=f(x_1,x_2,...,x_{i-1},x_j,x_{i+1},...,x_{j-1},x_j,x_{j+1},...,x_n)$$

(6) The set of all open n-dimensional REFSes ω satisfying the condition $\omega\subseteq\uparrow^n_i(\alpha)$ for some $\alpha\in\Delta_f$, is an ST-REFS-ideal quasi-equivalent to Δ_h, such that

$$h(x_1,x_2,...,x_{i-1},x_i,x_{i+1},...,x_n)=\sup_{x_i\in N} f(x_1,x_2,...,x_{i-1},x_i,x_{i+1},...,x_n).$$

(7) The set of all open n-dimensional REFSes ω satisfying the condition $\uparrow^n_i(\omega)\in\Delta_f$ is an ST-REFS-ideal quasi-equivalent to Δ_h, such that

$$h(x_1,x_2,...,x_{i-1},x_i,x_{i+1},...,x_n)=\inf_{x_i\in N} f(x_1,x_2,...,x_{i-1},x_i,x_{i+1},...,x_n).$$

Proof. The statements (1)-(7) are easily proved using corresponding definitions. Let us give, for example, the proof of the statement (2).

Obviously, the set of all open n-dimensional REFSes ω such that $\alpha\cap\omega\in\Delta_g$ for any $\alpha\in\Delta_f$ is an ST-REFS-ideal Δ'. Let us consider some n-tuple $(z_1,z_2,...,z_n)$ such that $f(z_1,z_2,...,z_n)\le g(z_1,z_2,...,z_n)$. If a is any rational number, $0\le a\le 1$, then for any $\alpha\in\Delta_f$ the set $\omega = W(a, z_1, z_2, ..., z_n)$ satisfies the condition: $\Psi_{\alpha\cap\omega}(z_1, z_2, ..., z_n) \le f(z_1, z_2, ..., z_n) \le g(z_1, z_2, ..., z_n)$. Hence $\alpha \cap \omega \in \Delta_g$ for any $\alpha \in \Delta_f$. So $\omega \in \Delta'$. If $f(z_1, z_2, ..., z_n) > g(z_1, z_2, ..., z_n)$, and $a < g(z_1, z_2, ..., z_n)$, then for any $\alpha \in \Delta_f$ the set $\omega = W(a, z_1, z_2, ..., z_n)$ satisfies the condition $\Psi_{\alpha\cap\omega}(z_1, z_2, ..., z_n) < g(z_1, z_2, ..., z_n)$, and $\omega \in \Delta_g$ as in preceding case.

So, if $a < h(z_1, z_2, ..., z_n)$ where $h(z_1, z_2, ..., z_n)$ has the form noted in the definition of the statement (2), then $W(a, z_1, z_2, ..., z_n) \in \Delta'$. Hence $\Delta_h \subseteq \Delta'$ because Δ_h is generated by the sets $W(a, z_1, z_2, ..., z_n)$ having the form noted above. The reverse statement $\Delta' \subseteq \Delta_h$ is obvious because no REFS α belonging to Δ' can satisfy the inequality $\Psi_\alpha(x_1, x_2, ..., x_n) > g(x_1, x_2, ..., x_n)$ in the case $f(x_1, x_2, ..., x_n) > g(x_1, x_2, ..., x_n)$.

Other statements (1)-(7) are proved in a similar way.

Now we define the semantics of the modified EFC-logic mentioned in the Introduction. This logic will be denoted as "ST-EFC-logic" ("set-theoretical extended fuzzy constructive logic").

Let A be a predicate formula which does not contain predicate symbols except $p_1, p_2, ..., p_l$ having the dimensions, correspondingly, $k_1, k_2, ..., k_l$; let k be index majorant for A. *ST-EFCL-assignment* for A is defined as an assignment φ which assigns to any predicate symbol p_i a k_i-dimensional ST-REFS-ideal.

We consider ST-REFS-ideals up to their quasi-equivalence; in connection with this assumption we can suppose that any ST-REFS-ideal assigned to some p_i has the form Δ_h for some h.

ST-EFCL-interpretation $\Pi^*_{\varphi,k}(A)$ of a formula A concerning a given ST-EFCL-assignment φ for A and a given index majorant k for A is defined similarly to the notion of $\Pi_{\varphi,k}(A)$ given above with the only difference: ST-REFS-ideals are considered instead of REFS-ideals. In connection with the assumption mentioned above and using the corresponding points in the formulation of Theorem3.1 we obtain that ST-EFCL-interpretation $\Pi^*_{\varphi,k}(A)$ of any predicate formula A has the form Δ_h for some standard real function h. The transformations of function h generated by logical operations are the same as it is noted in the corresponding points of Theorem3.1.

We say that a predicate formula A is *ST-EFCL-valid* if for any ST-EFCL-assignment φ and for any great enough index majorant k for A the ST-EFCL-interpretation $\Pi^*_{\varphi,k}(A)$ has the form Δ_h where the function h is identically equal to 1.

Corollary of Theorem 3.1. A predicate formula A is ST-EFCL-valid if and only if the sequent $\Rightarrow A$ is valid in the sense of Intuitionistic Fuzzy Logic described in [9].

Indeed, the logical transformations of standard real functions described in the corresponding points of Theorem3.1 are the same as the transformations of models of IF (concerning logical operations) described in [9] (see [9], pp. 855-856 and p. 851).

Some statements of this paper are formulated (without proofs) in [1].

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Կոնստրուկտիվ ոչ պարզորոշ տրամաբանությունների որոշ տարատեսակների մասին

Հ. Բոլիբեկյան և Ի. Զասլավսկի

Ամփոփում

Դիտարկվում է ընդլայնված ոչ պարզորոշ կոնստրուկտիվ տրամաբանության [14] որոշ վերափոխություն: Ապացուցվում է, որ այդ վերափոխությունը փաստորեն համարժեք է [9] աշխատանքում սահմանված Ինտուիցիոնիստական ոչ պարզորոշ տրամաբանությանը:

О некоторых разновидностях нечетких конструктивных логик

О. Болибекян и И. Заславский

Аннотация

Рассматривается модификация расширенной нечеткой конструктивной логики [14]. Доказывается, что данная модификация фактически эквивалентна и интуиционистской нечеткой логике, определенной в [9].