Mathematical Problems of Computer Science 36, 70–78, 2012.

New Representation for Non Ruin Probability of Insurance Model with Rent Contracts and Its Application for Assessment of Critical Risks

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Abstract

The paper considers the particular solution of one integro-differential equation of insurance risk theory. New representation for that solution is found, which is used for assessment of critical risks of the insurance companies that conduct purely rent operations. The critical risks are found in the case of a heavy traffic and regular tail variation of the insurance premiums distribution function.

Keywords: Insurance, rent, regular variation, heavy traffic, critical risks, integro-differential equation.

1. Introduction

In long-term collective insurance model with (negative insurance sums) rent contracts [1] the non-ruin probability $W(t,x)$ in time interval $(0,t)$ satisfies to the following integro-differential equation [1, 2, 3]:

$$\frac{\partial W(t,x)}{\partial t} = c\frac{\partial W(t,x)}{\partial x} - \lambda\left[W(t,x) - \int\limits_{-\infty}^{x} W(t,x-y)dF(y)\right], \tag{1}$$

where $x \geq 0$ is the initial capital of the insurance company and $F(\cdot), x \geq 0$ is the distribution function (DF) of insurance sums. It is known [1] that 1 is true for almost all $(t,x), t \geq 0, x \geq 0$.

In the present paper new representation for solution $W(t,x)$ is funded, which is used for assessment of critical Risks of insurance model in critical situations.

The concept of the insurance model with rent contracts is the following [1], [4].

The insurance company provides its clients with regular premiums which decrease the reserve at the rate $c < 0$. Without loss of generality, we assume that $c = -1$. The events of client deaths or contract interruptions follow at random moments $t_1, t_2, \ldots$. Each such event increases the reserve of the company by the amount of the unpaid rents $X_1, X_2, \ldots$ which are positive, independent and uniformly distributed random variables (RV) with DF F and mathematical expectation $a > 0$. The moments $t_1, t_2, \ldots$ form a Poisson point process of intensity $\lambda > 0$. Under these assumptions, the insurance amount $S(u) = \sum\limits_{0 \leq t_i \leq u} X_i$, which the company receives in the time interval $(0, u]$, is a generalized Poisson process with the intensity λ and jump DF F, and hence

$$P\{S(u) \le x\} = \sum_{n=0}^{\infty} \frac{e^{-\lambda u}(\lambda u)^n}{n!} F^{n^*}(x),$$

where F^{n^*} is the n-th convolution of the DF F, $F^{0^*}(x) = 1$ when $x \ge 0$ and $F^{0^*}(x) = 0$ when $x < 0$, and P is the probability. The reserve $\gamma(u) = x - u + S(u)$ of the company, where x is the initial capital, can become negative at some time moment, meaning a ruin[1] of the company. For company not be ruined in the time interval $(0, t]$ (or never), it is necessary that $u - S(u) \le x$ as $u \in (0, t]$ $(u \in (0, \infty))$. Therefore ([1], p. 164) $W(t, x) = 1, x > t$ and

$$W(t,x) = P\{r(t) \le x\} = 1 - \int_x^t \frac{x}{y} d_y P\{S(y) \le y - x\}, \quad 0 < x \le t, \tag{2}$$

where $r(t) = \sup_{0 \le u \le t} [u - S(u)]$. Let $r = \sup_{0 \le u < \infty} [u - S(u)]$ and $\psi(s) = \int_0^\infty e^{-sx} dF(x), \operatorname{Re} s \ge 0$. Then, for the companies never ruin probability it is true the following formula ([1], p. 164):

$$W(x) = P\{r \le x\} = 1 - \int_x^\infty \frac{x}{y} d_y P\{S(y) \le y - x\} = 1 - e^{\omega x}, \quad x > 0, \tag{3}$$

where ω is the greatest nonnegative root of the equation $\nu(s) \stackrel{def}{=} s - \lambda(1 - \psi(s)) = 0$.

Besides, it is known that $\omega = 0$ for $\rho_1 = \lambda a \le 0$ and $\omega > 0$ for $\rho_1 > 1$ (see [1], p. 52, Theorem 4).

In this paper we find new representation for $W(t, x)$ and use the obtained representation for finding of critical risks of insurance company.

We call "critical risks" to the asymptotic values of non-ruin probability $W(t, x)$, when the insurance company is in critical situation, that is the loading $\rho_1 \to 1$.

For DF F we assume that it has the finite moment of order γ. Then the Laplace-Stieltjes transform (LST) $\psi(s), \operatorname{Re} s \ge 0$, admits asymptotical representation of the form

$$\psi(s) - 1 + as \sim A s^\gamma L\left(\frac{1}{s}\right), \quad s \downarrow 0, \quad A > 0 \quad (1 < \gamma \le 2), \tag{4}$$

where the measurable $L(x) > 0$ is a slowly varying function (SVF) at infinity [6].

If L is a SVF, then for any $\delta > 0$ we have $x^\delta L(x) \to 0$ and $x^{-\delta} L(x) \to 0$ as $x \to \infty$ [7]. Hence

$$s^{\gamma - 1} L\left(\frac{1}{s}\right) \xrightarrow[s \downarrow 0]{} 0, \quad 1 < \gamma \le 2, \tag{5}$$

which makes 4 meaningful. Moreover, 5 is true uniformly in a.

[1]The term "ruin" in the risk theory has arisen historically and precisely would be named this event, for example, "deficiency". Insufficiency of a reserve does not mean ruin of the insurance company in sense of stay of its operations or bankruptcies; the term "ruin" of the theory of risk should be understood as technical. If the balance of reserve fund is negative, it yet does not mean negativity of balance of the company as a whole as the company can have other sources of repayment of deficiency (for example, own means, loans, etc.). On the other hand, even at positive balance the company can experience financial difficulties if the part of actives in which the reserve fund is laid out, has low liquidity (the real estate objects, precious metals and so forth). Thus, it is not necessary to mix ruins probability with probability illiquidity (see [5]).

Note, that 4 is equivalent to the relation

$$1-F(x)\sim \frac{A(\gamma-1)}{\Gamma(\bullet)}x^{-\gamma}L(x),\ x\to\infty.$$

where $\Gamma(\bullet)$ is the Euler's Gamma function. It's mean that the DF F is heavy till function.

We use the following notations $B=\lambda A, \rho=|1-\rho_1|, F^*(x)=\frac{1}{a}\int_0^x[1-F(y)]dy$, $\psi^*(s)=\int_0^\infty e^{-sx}dF^*(x)$. Then $\psi^*(s)=\frac{1-\psi(s)}{as}, s>0$.

2. New Representation for Solution $W(t,x)$

Theorem 1 *Let $\{S(u):0\le u\le\infty\}$ be a separable random process with rearrangeable grouths, almost all sample distribution functions of which are nondecreasing step functions vanishing at $u=0$. Then there exists a distribution $U_y(x)=U(x,y)$ such that*

$$W(t,x)=\begin{cases}1-\int_x^t d_yU(x,y), & 0<x\le t,\\ 1, & x\ge t,\end{cases}=\begin{cases}1-U(x,t)+U(x,x), & 0<x\le t,\\ 1, & x\ge t,\end{cases}$$

$$W(x)=1-\int_x^\infty d_yU(x,y)=1-e^{-\omega x},\quad x>0,$$

where

$$\int_0^\infty e^{-sy}d_yU(x,y)=e^{-x\omega(s)},$$

and $z=\omega(s)$ is the unique root of the equation $\nu(z)=s$ in the domain $Re(s)\ge 0$.

Proof: Taking into account that $W(t,x)=1, x\ge t$, by 2 and 3 we get

$$W(t,x)=\begin{cases}1-\int_0^t \frac{x}{y}d_yP\{S(y)\le y-x\}, & 0<x\le t,\\ 1, & x\ge t,\end{cases}$$

and

$$W(x)=1-\int_x^\infty \frac{x}{y}d_yP\{S(y)\le y-x\},\quad x>0.$$

It is known (see [1], p. 67) that

$$\int_x^\infty e^{-sy}\frac{x}{y}d_yP\{S(y)\le y-x\}=e^{-x\omega(s)},\quad \text{Re } s>0. \tag{6}$$

and one can see that here $P\{S(y)\le y-x\}=0$ when $y<x$. Consequently, by 6 we obtain

$$\int_0^\infty e^{-sy}\frac{x}{y}d_yP\{S(y)\le y-x\}=e^{-x\omega(s)}.$$

Besides, $e^{-x\omega(0)} = e^{-x\omega}$, and hence denoting

$$\varphi(x,s) = \int_0^\infty e^{-sy} \frac{xe^{x\omega}}{y} d_y P\{S(y) \le y - x\} = e^{-x(\omega(s)-\omega)}, \tag{7}$$

we obtain $\varphi(x,0) = 1$. Let us show that the function $\varphi(x,s)$ is completely monotone (CM). i.e. $(-1)^n \varphi_s^{(n)} \ge 0$ [8]. To this end, we note that e^{-xs} obviously is CM and show that the derivative of the function $\omega(z) - \omega > 0$ is CM. In fact, $\omega(s)$ is a solution of the equation $\nu(s) = s$, and therefore setting

$$\beta(s) = 1 + \frac{s}{\lambda} - \frac{\omega(s)}{\lambda} = \psi(\omega(s))$$

we obtain

$$\omega(s) = s + \lambda - \lambda\beta(s), \tag{8}$$

$$\psi(s + \lambda - \lambda\beta(s)) = \beta(s). \tag{9}$$

By 9 the function $\beta(s)$ is the LST of some nondecreasing function (see [8], p. 497). Without loss of generality, we assume that $\beta(s)$ is the LST of its own DF (otherwise we can divide $\beta(s)$ by $\beta(0) > 0$ and get $\beta(0) = 1$). Hence, the function $\beta(s)$ is CM (see [8], p. 495, Theorem 1), or, which is the same

$$(-1)^k \beta^{(k)}(s) \ge 0, \quad k \ge 0, \quad s \ge 0. \tag{10}$$

From 8, it follows that $(\omega(s) - \omega)' = 1 - \lambda\beta(s)$, and consequently

$$(-1)^n \left((\omega(s) - \omega)'\right)^{(n)} = (-1)^n (1 - \lambda\beta'(s))^{(s)}$$

$$= \lambda(-1)^{n+1} \left(\beta''(s)\right)^{(n-1)} = \lambda(-1)^{n+1} \left(\beta^{(n+1)}(s)\right) \ge 0, \quad n \ge 1.$$

Besides, by 10 (for $k = 1$) we obtain $(\omega(s) - \omega)' = 1 - \lambda\beta'(s) \ge 0$ (for $n = 0$). Thus. $(\omega(s) - \omega)'$ is CM. Hence. the function $\varphi(x,s)$ is CM since it is a superposition of the CM function e^{-xs} and $\omega(s) - \omega > 0$, the derivative of which is CM (see [8], p 497). Consequently (see [8], p. 495, Theorem 1) the function $\varphi(s,x)$ has the following form

$$\varphi(s,x) = \int_0^\infty e^{-sy} d_y G(x,y).$$

Now, taking into account the representation 7 of the function $\varphi(x,s)$ and using a uniqueness theorem we get

$$\frac{x}{y} d_y P\{S(y) \le y - x\} = d_y\{e^{-x\omega} G(x,y)\}.$$

Thus, the LST of the function $U(x,y) = e^{-x\omega} G(x,y)$ is $e^{-x\omega(s)}$, and the proof is complete.

Remark 1 *If a random process $\{S(u) : 0 \le u \le \infty\}$ has a density, then (see [1], p. 51)*

$$U(x,y) = \int_0^y \frac{x}{u} \frac{\partial P\{S(u) \le u - x\}}{\partial x} du.$$

Remark 2 *From the proof of theorem it is obvious that the $W(t,x)$ has the LST $e^{-x\omega(s)}$.*

3. Application for Assessment of Critical Risks

Denote

$$L_2^{\gamma-1}(t)=\frac{1}{t^{\gamma-1}}\frac{1}{M^{(\gamma-1)}(t)},$$

where $M^{(\gamma-1)}(t)$ is the inverse function of $t^{\gamma-1}/L(t)$. Then [9]

$$\omega\sim\left(\frac{\rho}{B}\right)^{1/(\gamma-1)}L_2^{(\gamma-1)}\left(\frac{B}{\rho}\right)\quad\text{as}\quad\rho_1\downarrow 1.$$

For $\rho\uparrow 1$ we use the same notation $\omega\sim(\rho/B)^{1/(\gamma-1)}L_2^{(\gamma-1)}(B/\rho)$ for quantities of the rate $(\rho/B)^{1/(\gamma-1)}L_2^{(\gamma-1)}(B/\rho)$ and assume that $\rho/B\to 0$ as $\rho_1\uparrow 1$.

Let the functions $\nabla(s)$ and $\Delta(s)$ respectively be the unique solutions of the equations $z^\gamma-z=s$ and $z^\gamma+z=s$ $(s\geq 0)$, satisfying the conditions $\nabla(0)=1$ and $\Delta(0)=0$[2] [9], then the following result is true [9].

Lemma 1 *If the condition 4 is fulfilled and* $\alpha(\rho)\underset{\rho\to\vartheta}{\longrightarrow}0$, *then*

$$\omega(\alpha(\rho)s)\sim\beta(\rho)A(s)L_0,\quad\rho_1\to 1,$$

where

$$\beta(\rho)=\begin{cases}\left(\frac{\rho}{B}\right)^{1/(\gamma-1)}, & \alpha(\rho)=o(\rho\omega),\ \rho_1\downarrow 1,\\ \frac{\alpha(\rho)}{\rho}, & \alpha(\rho)=o(\rho\omega),\ \rho_1\uparrow 1,\\ \left(\frac{\rho}{B}\right)^{1/(\gamma-1)}, & \alpha(\rho)\sim o(\rho\omega),\ \rho_1\uparrow\downarrow 1,\\ \left(\frac{\alpha(\rho)}{B}\right)^{1/\gamma}, & \rho\omega=o(\alpha(\rho)),\ \rho_1\uparrow\downarrow 1.\end{cases}$$

$$A(s)=\begin{cases}1, & \alpha(\rho)=o(\rho\omega),\ \rho_1\downarrow 1,\\ s, & \alpha(\rho)=o(\rho\omega),\ \rho_1\uparrow 1,\\ \nabla(s), & \alpha(\rho)\sim\rho\omega,\ \rho_1\downarrow 1,\\ \Delta(s), & \alpha(\rho)\sim\rho\omega,\ \rho_1\uparrow 1,\\ s^{1/\gamma}, & \rho\omega=o(\alpha(\rho)),\ \rho_1\uparrow\downarrow 1.\end{cases}$$

$$L_0=\begin{cases}L_2^{(\gamma-1)}\left(\frac{B}{\rho}\right), & \alpha(\rho)=o(\rho\omega),\ \rho_1\downarrow 1,\\ 1, & \alpha(\rho)=o(\rho\omega),\ \rho_1\uparrow 1,\\ L_2^{(\gamma-1)}\left(\frac{B}{\rho}\right), & \alpha(\rho)\sim\rho\omega),\ \rho_1\uparrow\downarrow 1,\\ L_2^{(\gamma)}\left(\frac{B}{\alpha(\rho)}\right), & \rho\omega=o(\alpha(\rho)),\ \rho_1\uparrow\downarrow 1.\end{cases}$$

Besides, formula 3 and Lemma 1 imply the following

Theorem 2 *If the condition 4 is fulfilled and* $\vartheta(\rho)\to 0$ *as* $\rho_1\to 1$, *then the limit*

$$\lim_{\rho_1\downarrow 1}P\{\vartheta(\rho)\tau\leq x\}=1-e^{-x},\quad x>0.$$

exists if and only if

$$\vartheta(\rho)\sim\left(\frac{\rho}{B}\right)^{1/(\gamma-1)}L_2^{(\gamma-1)}\left(\frac{B}{\rho}\right)\sim\omega.$$

[2] Equation $z^\gamma+z=s$ and its solution $\Delta(s)$ are introduced by Danielian [10]. Then, there are also considered by Sahakyan [11] and Chitchyan [12].

Let $\alpha(\rho) \to 0$ as $\rho \to 0$ and let $\beta(\rho)$ be that of Lemma 1. Then the following statement is true in case of convergence $t\alpha(\rho) \to \tau$, $0 < \tau \le \infty$.

Theorem 3 *Let the condition 4 be fulfilled and let $\vartheta(\rho) \sim \beta(\rho)L_0$ as $\rho \to 0$ and $t \to \infty$, so that $t\alpha(\rho) \to \tau$, $0 < \tau \le \infty$. Then there exists the limit*

$$\lim_{(\rho,t)} P\{\vartheta(\rho)r(t) \le x\} = 1 - \int_0^\tau d_\nu U_0(x,\nu) = 1 - U_0(x,\tau) + U_0(x,0),\ x > 0,$$

where (ρ, t) denotes the pair of passages $\rho \to 0$, $t \to \infty$ and

$$\int_0^\infty e^{-s\nu} d_\nu U_0(x,\nu) = e^{-sA(s)},\ s \ge 0.$$

In all other cases $\lim_{(\rho,t)} P\{\vartheta(\rho)r(t) \le x\} = 1, x > 0$, where $\vartheta(\rho) = o(\beta(\rho)L_0)$ and $\beta(\rho)L_0 = o(\vartheta(\rho))$.

Proof: By Theorem 1,

$$P\{\vartheta(\rho)r(t) \le x\} = P\left\{r(t) \le \frac{x}{\vartheta(\rho)}\right\} = \begin{cases} 1 - \int_x^t d_y U\left(\frac{x}{\vartheta(\rho)}, y\right), & 0 < x \le t\vartheta(\rho), \\ 1, & x \ge t\vartheta(\rho), \end{cases} \tag{11}$$

where

$$\int_0^\infty e^{-sy} d_y U(x,y) = e^{-x\omega(s)}. \tag{12}$$

Changing the integration variable as $y = \nu/\alpha(\rho)$ in 11 and 12, we obtain

$$P\{\vartheta(\rho)r(t) \le x\} = \begin{cases} 1 - \int_{x\frac{\alpha(\rho)}{\vartheta(\rho)}}^{t\alpha(\rho)} d_\nu U\left(\frac{x}{\vartheta(\rho)}, \frac{\nu}{\alpha(\rho)}\right), & 0 < x \le t\vartheta(\rho), \\ 1, & x > t\vartheta(\rho). \end{cases}$$

and

$$\int_0^\infty e^{-s\nu} d_\nu U\left(\frac{x}{\vartheta(\rho)}, \frac{\nu}{\alpha(\rho)}\right) = e^{-x\frac{\omega(\alpha(\rho))}{\vartheta(\rho)}}. \tag{13}$$

Besides, by Lemma 1, $\omega(\alpha(\rho)s) \sim \beta(\rho)A(s)L_0$ as $\rho_1 \to 1$. Hence, by 13 and the known theorem on generalized continuity (see [8], p. 488) we conclude that a nondegenerate limit

$$U_0(x,y) = \lim_{\rho\to 0} U\left(\frac{x}{\vartheta(\rho)}, \frac{x}{\alpha(\rho)}\right)$$

exists if and only if $\vartheta(\rho) \sim \beta(\rho)L_0$. The remaining cases $\vartheta(\rho) = o(\beta(\rho)L_0)$ and $\beta(\rho)L_0 = o(\vartheta(\rho))$ are degenerate, i.e. in these cases $\lim_{(\rho,t)} P\{\vartheta(\rho)r(t) \le x\} = 1$. $x > 0$. Thus, passing to the limits in formula 13 we obtain

$$\int_0^\infty e^{-s\nu} d_\nu U_0(x,\nu) = e^{-xA(s)}.$$

Further, $\omega(s) = s + \lambda(1 - \psi(\omega(s))) \geq s$, and hence we obtain that if $\vartheta(\rho) \sim \beta(\rho)L_0$, then

$$\frac{\alpha(\rho)s}{\vartheta(\rho)} \sim \frac{\alpha(\rho)s}{\beta(\rho)L_0} \leq \frac{\omega(\alpha(\rho)s)}{\beta(\rho)L_0} \sim \frac{\beta(\rho)L_0A(s)}{\beta(\rho)L_0} = A(s)$$

for any s and $\rho < \rho_0$. Thus,

$$\frac{\alpha(\rho)}{\beta(\rho)L_0} \leq \frac{A(s)}{s}, \quad \rho < \rho_0. \tag{14}$$

and

$$\frac{\alpha(\rho)}{\vartheta(\rho)} \sim \frac{\alpha(\rho)}{\beta(\rho)L_0} \to 0 \quad \text{as} \quad \rho \to 0, \tag{15}$$

since 14 is true for any fixed s. Taking into account that $t\alpha(\rho) \to \tau$ $(0 < \tau \leq \infty)$, by 15 we get

$$\lim_{(\rho,t)} P\{\vartheta(\rho)r(t) \leq x\} = 1 - \int_0^\tau d_\nu U_0(x,\nu). \tag{16}$$

and this relation is true for any $x > 0$ since $x \leq t\vartheta(\rho) = t\alpha(\rho)\frac{\vartheta(\rho)}{\alpha(\rho)} \underset{(\rho,t)}{\longrightarrow} \infty$.

Remark 3 *For any fixed t and any $\vartheta(\rho)$ such that $\vartheta(\rho) \to 0$ as $\rho \to 0$:*

$$W_t(x) = \lim_{\rho\to 0} P\{\vartheta(\rho)r(t) \leq x\} = 1, \quad x > 0.$$

In [9] it is proofed that in cases $\theta(\rho) \sim \omega$ and $\omega = o(\theta(\rho))$ when $\rho_1 \to 1$ for $U_0(x,t)$ there exist the density function $f(x,t)$ for which the following representation is found:

$$f(x,t) = \frac{t}{\gamma\pi x}\sum_{n=0}^{\infty}\frac{(-1)^n(t-\delta(x))^n}{n!}\Gamma\left(\frac{n+1}{\gamma}\right)x^{-\frac{n+1}{\gamma}}\sin\frac{n+1}{\gamma}\pi, \; 1 < \gamma \leq 2,$$

were $\delta(\tau) = \begin{cases} \tau sign(1-\rho_1), & \theta(\rho) \sim \omega \text{ or } \theta(\rho) = o(\omega), \rho \to 0, \\ 0, & \omega = o(\theta(\rho)), \rho \to 0 \end{cases}$.

Therefore theorem 3 will take the following form:

Theorem 4 *1) $W_\tau(x) = 1 - e^{-x}$ in case of $\theta(\rho) = o(\omega)$ when $\rho_1 \downarrow 1$;*

2) $W_\tau(x) = \begin{cases} 0 & x < \tau \\ 1 & x \geq \tau \end{cases}$ in case of $\theta(\rho) = o(\omega)$ when $\rho_1 \uparrow 1$.

3) $W_\tau(x) = 1 - \int_0^\tau f(v,x)\,dv$ in case of $\theta(\rho) \sim \omega$ and $\omega = o(\theta(\rho))$ when $\rho_1 \to 1$.

In all other cases $W_\tau(x) = 1$, $x > 0$.

Example

Consider the particular case when $\gamma = 2$. Then under the conditions of theorem 4, when $\rho_1 \to 1$. $W_\tau(x) = 1 - F_x(2\tau)$ if $\omega = o(\theta(\rho))$ and $W_\tau(x) = 1 - e^{sign(1-\rho_1)\frac{x}{2}-\frac{\tau}{4}} f_{x/\sqrt{2}}(\tau) * e^{-\frac{\tau}{4}}$ if $\theta(\rho) \sim \omega$, where $F_x(u)$ is DF with density $f_x(u) = \frac{x}{\sqrt{2\pi u^3}}\exp\left\{-\frac{x^2}{2u}\right\}$, $u > 0$ (see [8]).

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Ռենտաների պայմանագրերով ապահովագրական մոդելի չսնանկացման հավանականության նոր ներկայացում և նրա կիրառությունը կրիտիկական ռիսկերի գնահատման նպատակով

Ա. Մարտիրոսյան

Ամփոփում

Հոդվածում դիտարկվում է ապահովագրական ռիսկերի տեսության մեկ ինտեգրոդիֆերենցիալ հավասարման մասնակի լուծումը: Այդ լուծման համար գտնված է նոր ներկայացում, որն օգտագործվել է միայն ռենտաների գործարքներով զբաղվող ապահովագրական ընկերության կրիտիկական ռիսկերի գնահատման համար: Կրիտիկական ռիսկերը գտնված են ծանրաբեռնվածության և կանոնավոր փոփոխվող պոչերով ապահովագրական վճարների բաշխման դեպքում:

Новое представление для вероятности неразорения страховой модели с договорами связанными с рентами и ее применением при оценке критических рисков

А. Мартиросян

Аннотация

В работе рассмотрено частное решение одного интегродифференциалного уравнения теории страхового риска. Для этого решения найдено новое представление, которое принимается при оценке критических рисков страховых компаний, занимающихся операциями, связанными с обычной рентой. Критические риски найдены в случаях критической загрузки и при правильном изменении хвостов распределений страховых выплат.