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Classical Spin Glasses with Consideration of Relaxation Effects

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Abstract

The complex-classical short-range interaction Hamiltonian is used for the first time for solving spin glasses with consideration of relaxation effects. A system of recurrent equations is obtained on the nodes of the $1D$ lattice. An efficient mathematical algorithm is developed on the basis of these equations with consideration of extended Sylvester conditions which allows node-by-node construct a huge number of stable spin chains in parallel. As a result of the simulation, distribution functions of different parameters of a spin glass are constructed from the first principles of complex-classical mechanics. Also, the critical properties of spin glass such as catastrophes in the Clausius-Mossotti equation are studied depending on the external field. It is shown that the developed approach excludes these catastrophes, which allows to organize continuous parallel computation based on the whole-range values of the external field. A new representation of the partition function is suggested which, opposite to the usual definition, is a complex function with the derivatives defined everywhere, including at critical points.

1. Introduction

A wide class of phenomena which raise important and difficult calculation problems in physics, chemistry. material science, biology. nanoscience. evolution, organization dynamics, environmental and social structures, human logic systems. financial mathematics, etc, are mathematically well described in the framework of spin glass models [1, 2. 3, 4. 5, 6. 7, 8, 9]. In the literature. two types of mean field models have been developed. The first consists of true random-bond models, where the coupling between interacting spins is assumed to be independent random variables [10. 11. 12]. The solution of the model problems is obtained by the n-replica trick [10. 12] and requires an invention of sophisticated schemes of replica-symmetry breaking [12, 13]. In the second type models the bond-randomness is expressed in terms of some underlining hidden site-randomness and is thus of superficial nature. It is pointed out in works [14, 15. 16]. however. that this feature retains an important physical aspect of true spin-glasses. viz. they are random with respect to the positions of magnetic impurities. Recently it was shown that the critical properties in some types of media can be studied by the model of $3D$ spin glass on the scales of space-time periods of external standing electromagnetic fields [17].

In this paper we discuss in detail the statistical properties of classical $1D$ spin glasses which suppose that interactions between spins have a short-range character and that the system has a possibility to relax under the influence of an external field. Mathematically, we solve this problem in the framework of a complex-classical Heiseberg Hamiltonian, the meaning of which is similar to the idea of classical Newtonian mechanics generalization on complex-classical trajectories [18, 19, 20, 21, 22]. On the basis of these investigations the Clausius-Mossotti equation in external fields is generalized. Also, a new definition for the complex partition function is suggested which is defined everywhere, including at critical points.

2. Formulation of the Problem and Basic Formulas

It is well known that in isotropic media (as well as in the crystals with cubic symmetry) the dielectric constant ϵ_s is well described by the Clausius-Mossotti equation [23]:

$$\frac{\epsilon_s - 1}{\epsilon_s + 2} = \frac{4\pi}{3}\sum_m N_m^0 \alpha_m^0, \tag{1}$$

where N_m^0 is the concentration of particles (electrons, atoms, ions, molecules or dipoles) with given m types of polarizability and α_m^0 is the polarizability coefficient, correspondingly. From this equation follows that the static dielectric constant ϵ_s depends on the polarizability properties of the particles as well as on their topological order.

Taking into account the influence of the external electromagnetic fields, the equation for the dielectric constant may be formally written as:

$$\epsilon_{st}(\mathbf{g}) = \frac{1 + 2\Lambda(\mathbf{g})}{1 - \Lambda(\mathbf{g})}, \quad \Lambda(\mathbf{g}) = \frac{4\pi}{3}\Big[\sum_m N_m^0 \alpha_m^0 + \varrho(\mathbf{g})\Big]. \tag{2}$$

In (2) the symbol $\epsilon_{st}(\mathbf{g})$ designates the dielectric constant depending on the external field parameters $\mathbf{g} = (\Omega, h_0)$, where Ω and h_0 correspond to the frequency and amplitude of the external field. In addition, if the medium can be represented respectively as a model of disordered spin system (spin glass), then $\varrho(\mathbf{g})$ designates the coefficient of polarizability which is connected to the orientational effects of spins in an external field. It follows from the general considerations that we can simplify the problem and consider the spin glass medium as an ensemble of $1D$ spatial spin-chains (SSCs) of certain length L_x (see Fig. 1). The coefficient of polarizability $\varrho(\mathbf{g})$ is the mean value of polarization of an ensemble per spin, which should be complex in general and equal to:

$$\varrho(\mathbf{g}) = \frac{\bar{p}(\mathbf{g})}{N_x}, \quad \bar{p}(\mathbf{g}) = \int p(\mathrm{E}; \mathbf{g}) F(\mathrm{E}; \mathbf{g})\, d\mathrm{E}, \qquad Re\,\mathrm{E} \le 0, \tag{3}$$

where N_x denotes the number of spins in the chain, E designates the complex energy of the spin chain with the length L_x with taking into account of relaxation effects. $F(\mathrm{E}; \mathbf{g})$ designates its distribution function on the ensemble.

Thus, our aim is to calculate the polarizibility coefficient $\varrho(\mathbf{g})$ with consideration of relaxation effects occurring in a system of spins under the influence of external field.

We consider a classical ensemble of disordered $1D$ spatial spin-chains (SSC) with the length L_x, where for simplicity it is supposed that the interactions between spin chains are absent. Mathematically, spin-glass of this type can be described by the $1D$ Heisenberg spin-glass Hamiltonian [1, 2, 3]:

$$H(\{\mathbf{r}\};N_x) = -\sum_{i=0}^{N_x-1} J_{ii+1}S_iS_{i+1} - \sum_{i=0}^{N_x-1} h_iS_i, \tag{4}$$

where $\{\mathbf{r}\} = \mathbf{r}_0, \mathbf{r}_1...$ designates a set of spins' coordinates: ($\mathbf{r}_i$ is the coordinate of the i-th spin), S_i describes the i-th spin which is the unit length vector and has a random orientation, h_i is the external field which is orientated along the axis x:

$$h_i = h_0\cos(k_x x_i), \qquad x_i = i\cdot d_0, \quad k_x = 2\pi/L_x. \tag{5}$$

In addition, J_{ii+1} characterizes the random interaction constant between i and $i+1$ spins in (4) and can have positive as well as negative values (see [1, 4]).

For further investigations, (4) is convenient to write in spherical coordinates (Fig. 1):

$$H(\{\mathbf{r}\};N_x) = -\sum_{i=0}^{N_x-1}\{J_{ii+1}[\cos\psi_{i+1}\cos(\varphi_i-\varphi_{i+1}) + \tan\psi_i\sin\psi_{i+1}] + h_0\cos(2\pi i/N_x)\tan\psi_i\}\cos\psi_i. \tag{6}$$

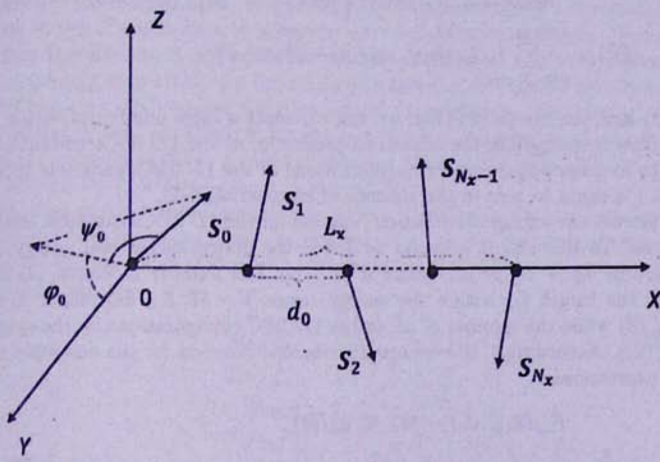

Figure 1: A stable $1D$ spatial spin-chain with random interactions and length $L_x = d_0N_x$, where d_0 is the distance between nearest-neighboring spins, N_x designates the number of spins in the chain. The spherical angles φ_0 and ψ_0 describe the spatial orientation of the S_0 spin, the pair of angles (φ_i, ψ_i) defines the spatial orientation of the S_i spin.

Equations (6) for stationary points of the Hamiltonian will play a central role in the consecutive calculations of the problem:

$$\frac{\partial H}{\partial\psi_i} = 0, \qquad \frac{\partial H}{\partial\varphi_i} = 0. \tag{7}$$

where $\Theta_i = (\psi_i, \varphi_i)$ are the angles of the i-th spin in the spherical coordinates, (ψ_i is the polar and φ_i the azimuthal angle). Using expression (4) and equations (7), it is easy to find the following system of trigonometric equations:

$$\sum_{\nu=i-1;\,\nu\neq i}^{i+1} J_{\nu i}[\sin\psi_\nu - \tan\psi_i \cos\psi_\nu \cos(\varphi_i - \varphi_\nu)] + h_i = 0,$$
$$\sum_{\nu=i-1;\,\nu\neq i}^{i+1} J_{\nu i}\cos\psi_\nu \sin(\varphi_i - \varphi_\nu) = 0, \qquad J_{\nu i} \equiv J_{i\nu}. \tag{8}$$

If the interaction constants $J_{i-1\,i}$, $J_{i\,i+1}$, as well as the angles $(\psi_{i-1}, \varphi_{i-1})$, (ψ_i, φ_i) are known, it is possible to explicitly calculate the pair of angles $(\psi_{i+1}, \varphi_{i+1})$. Correspondingly, the i-th spin will be in the ground state if the following conditions are satisfied (Sylvester conditions) at the stationary point $\Theta_i^0 = (\psi_i^0, \varphi_i^0)$:

$$A_{\psi_i\psi_i}(\Theta_i^0) > 0, \qquad A_{\psi_i\psi_i}(\Theta_i^0)\,A_{\varphi_i\varphi_i}(\Theta_i^0) - A^2_{\psi_i\varphi_i}(\Theta_i^0) > 0, \tag{9}$$

where $A_{\alpha_i\alpha_i}(\Theta_i^0) = \partial^2 H_0/\partial\alpha_i^2$, $\quad A_{\alpha_i\beta_i}(\Theta_i^0) = A_{\beta_i\alpha_i}(\Theta_i^0) = \partial^2 H_0/\partial\alpha_i\partial\beta_i$, in addition:

$$A_{\psi_i\psi_i}(\Theta_i^0) = \Big\{ \sum_{\nu=i-1;\,\nu\neq i}^{i+1} J_{\nu i}[\cos\psi_\nu \cos(\varphi_\nu - \varphi_i^0) + \tan\psi_i^0 \sin\psi_\nu] +$$
$$h_0 \cos(2\pi i/N_x)\tan\psi_i^0\Big\}\cos\psi_i^0, \qquad A_{\psi_i\varphi_i}(\Theta_i^0) = 0,$$
$$A_{\varphi_i\varphi_i}(\Theta_i^0) = \sum_{\nu=i-1;\,\nu\neq i}^{i+1} J_{\nu i}\cos\psi_\nu \cos(\varphi_\nu - \varphi_i^0)\cos\psi_i^0. \tag{10}$$

Using equations (8) and conditions (9)-(10), we can calculate a huge number of stable $1D$ SSCs, which will allow investigating the statistical properties of the $1D$ SSCs ensemble. It is supposed that the average polarization (magnetization) of the $1D$ SSCs ensemble (polarizability of $1D$ SSC) is equal to zero in the absence of an external field.

Now we can construct the energy distribution function for the $1D$ SCCs ensemble subject to external influence. To this aim it is useful to divide the dimensionless real energy axis E into the regions $0 > \mathrm{E}_0 > ... > \mathrm{E}_n$, where $n >> 1$. The number of stable $1D$ SSC configurations with the length L_x within the energy range $[\mathrm{E} - \delta\mathrm{E}, \mathrm{E} + \delta\mathrm{E}]$, $\delta\mathrm{E} \ll 1$, will be denoted by $M_{L_x}(\mathrm{E})$ while the number of all stable $1D$ SSC configurations by the symbol $M_{L_x}^{full} = \sum_{j=1}^{n} M_{L_x}(\mathrm{E}_j)$. Accordingly, the energy distribution function for the ensemble may be defined by the expressions:

$$F_{L_x}(\mathrm{E}, \mathbf{g}; d_0) = M_{L_x}(\mathrm{E}, \mathbf{g})/M_{L_x}^{full}, \tag{11}$$
$$\lim_{n\to\infty}\sum_{j=1}^{n} F_{L_x}(\mathrm{E}_j, \mathbf{g}; d_0)\,\delta\mathrm{E}_j = \int_{-\infty}^{0} F_{L_x}(\mathrm{E}, \mathbf{g}; d_0)\,d\mathrm{E} = 1,$$

where the second equality is the normalization condition of the distribution function.

3. Solution of Equations for Stationary Points

Using following designations:

$$\xi_{i+1} = \cos\psi_{i+1}, \qquad \eta_{i+1} = \sin(\varphi_i - \varphi_{i+1}), \tag{12}$$

it is possible to solve equations (8) in an explicit form [24]:

$$\xi_{i+1}^2 = C_2^2 J_{ii+1}^{-2} \eta_{i+1}^{-2}, \qquad \eta_{i+1}^2 = C_2^2 A B^{-1}, \tag{13}$$

where

$$A = J_{ii+1}^2 \cos^2\psi_i + C_3 + 2C_1 \sin^2\psi_i [C_1 \pm \sqrt{J_{ii+1}^2 - C_1^2 - C_2^2} \cot\psi_i],$$

$$B = J_{ii+1}^4 \cos^4\psi_i + 2C_3 J_{ii+1}^2 \cos^2\psi_i + (C_1^2 + C_2^2 \sin^2\psi_i)^2,$$

$$C_1 = J_{i-1i}[\sin\psi_{i-1} - \tan\psi_i \cos\psi_{i-1}\cos(\varphi_i - \varphi_{i-1})] + h_0 \cos(2\pi i/N_x)\cos\psi_i,$$

$$C_2 = J_{i-1i}\cos\psi_{i-1}\sin(\varphi_i - \varphi_{i-1}), \quad C_3 = -C_1^2 + C_2^2 \sin^2\psi_i.$$

Let us recall that the analysis of equations (8) allowed us to find a new condition that the spin-spin interaction constant should satisfy :

$$J_{ii+1}^2 \geq C_1^2 + C_2^2, \tag{14}$$

which plays an important role in the further calculations.

In our recent work [17] we showed that even for weak external fields there arise such values of polarization that lead to catastrophe in equation (2). In order to solve this issue, it is necessary to consider relaxation effects occurring in a $1D$ SSCs ensemble under the influence of an external field. Mathematically, consideration of a complex Hamiltonian can be one of the effective ways to solve the above-mentioned problem. Note that the idea of complex Hamiltonian is often used for solution of classical and semiclassical problems near zero scattering angles [25]. In the specified cases the divergence problems are successfully solved by using the so-called complex-classical trajectories. We consider that spin-chains, as a matter of fact, are classical trajectories where the analogue of time is the sequence of nodes. It is obvious that in a complex-classical trajectory (spin chain) it is possible to take into account the relaxation effects in the spin system.

We thus propose that Hamiltonian (4) is a complex function where the constants J_{ii+1} and angles between spins have complex values. The system of recurrent equations which will allow calculate spin chains with consideration of relaxation effects can be written in the following form:

$$\begin{aligned}
Re\{\bar{\xi}_{i+1}^2 - \bar{C}_2^2 \bar{J}_{ii+1}^{-2} \bar{\eta}_{i+1}^{-2}\} &= 0,\\
Im\{\bar{\xi}_{i+1}^2 - \bar{C}_2^2 \bar{J}_{ii+1}^{-2} \bar{\eta}_{i+1}^{-2}\} &= 0,\\
Re\{\bar{\eta}_{i+1}^2 - \bar{C}_2^2 \bar{A}\bar{B}^{-1}\} &= 0,\\
Im\{\bar{\eta}_{i+1}^2 - \bar{C}_2^2 \bar{A}\bar{B}^{-1}\} &= 0,\\
Im\{\bar{A}_{\bar{\psi}_{i+1}\bar{\psi}_{i+1}}(\bar{\Theta}_{i+1})\} &= 0,\\
Im\{\bar{A}_{\bar{\varphi}_{i+1}\bar{\varphi}_{i+1}}(\bar{\Theta}_{i+1})\} &= 0,\\
Im\{\bar{J}_{ii+1}^2 - \bar{C}_1^2 - \bar{C}_2^2\} &= 0.
\end{aligned} \tag{15}$$

Let us recall that all functions in (15) in the complex region are analytically extended: $\bar{\sigma} = \sigma^r + i\sigma^I$, where σ^r and σ^I are the real and imaginary parts of the function, correspondingly. Note that the first four equations in (15) are found from the complex extension of equations (13) by separating the real and imaginary parts. The next three equations are found from the zeroing condition of imaginary parts of Sylvester conditions (9) and inequality (14). The condition of local minimum energy for spins requires to satisfy the following inequalities:

$$Re\,\{\tilde{A}_{\tilde{\psi}_i\tilde{\psi}_i}(\bar{\Theta}_i^0)\} > 0, \quad Re\,\{\tilde{A}_{\tilde{\vartheta}_i\tilde{\vartheta}_i}(\bar{\Theta}_i^0)\} > 0, \quad Re\,\{\tilde{J}_{i\,i+1}^2 - \tilde{C}_1^2 - \tilde{C}_2^2\} \geq 0. \qquad (16)$$

System of equations (15) with the constraint conditions of inequalities (16) allow conduct computation and construct stable spin-chains with consideration of relaxation effects which occur as a result of energy exchange between spins inside the chain and excitation of their internal degrees of freedom. Simulation of system (15) can be realized under various scenarios. In particular, if we assume that relaxation occurs only between the spins in chains without excitation of internal degrees of freedom, then the following conditions must be satisfied:

$$|\bar{\xi}_{i+1}^2| \leq 1, \qquad |\bar{\eta}_{i+1}^2| \leq 1. \qquad (17)$$

When the relaxation goes on two degrees of freedom, obviously conditions (17) are not satisfied.

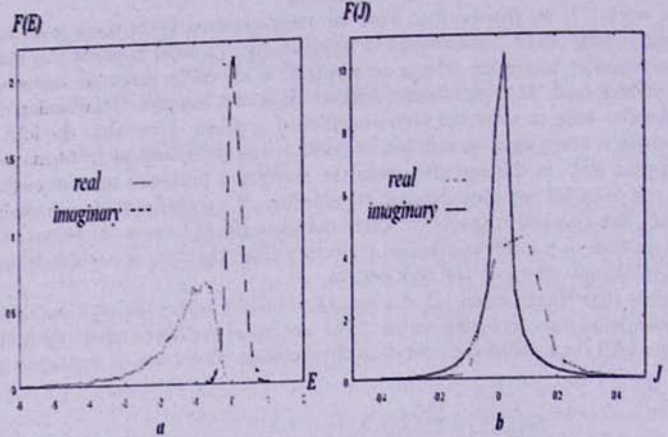

Figure 2: a) The distribution of the energy (for the real and imaginary parts, correspondingly) of an ensemble consisting of $1D$ SSCs of the length $L_x = 25\,d_0$. b) The distribution of spin-spin interaction constants (for the real and imaginary parts, correspondingly). These distributions essentially differ from the Gauss distribution and correspond to the class of Lévy's alpha-stable distributions [26].

4. Statistical Properties of an Ensemble

We have investigated the behavior of the average value of the ensemble polarization depending on the external field. Using definition (3), we have calculated mean values of polarizations $\bar{p}_\eta^{(o)}(\gamma)$ on all coordinates $(\eta = x, y, z)$, where the index $(o = r, i)$ designates the real and imaginary parts.

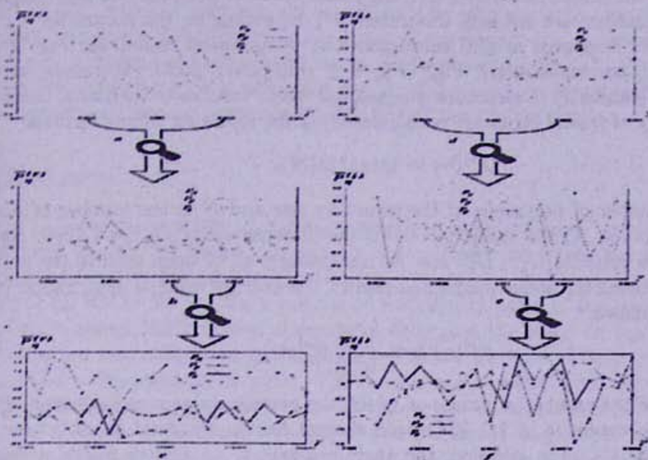

Figure 3: In (*a, b, c*) self-similar curves of the real part of polarization are shown. In (*d, e, f*) self-similar curves of the imaginary part of polarization are shown. Mean values of both the real and imaginary parts of the polarization are strongly frustrated on all coordinates (x, y, z) depending on the external field.

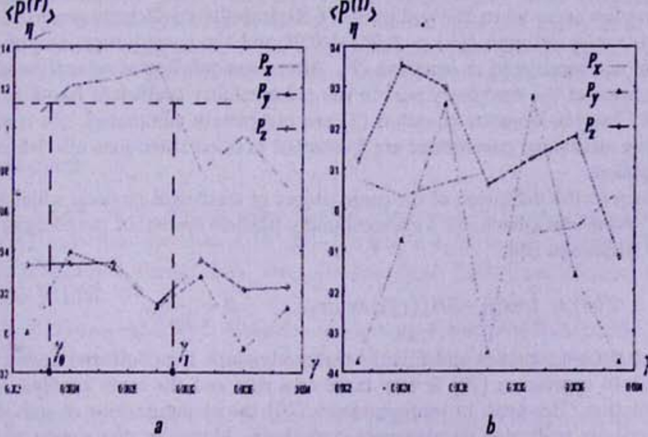

Figure 4: In *a* curves of the real part of polarizations are shown on different coordinates (x, y, z) after averaging on the spin-chains ensemble and fractal structures. The usual Clausius-Mossotti equation (2) (without consideration of relaxation effects) has a catastrophe (figure *a*) for two values of external field (γ_0, γ_1). In *b* curves of the imaginary part of polarizations are shown after full averaging by spin chains ensemble and fractal structures.

The numerical simulation has shown that the mean values of both the real and imaginary parts of the polarizations are strongly frustrated [27] depending on the parameter γ. This frustration does not disappear at grid convergence of computation region. see Fig. 3 *a*, *b*, *c* (real part) and. correspondingly, Fig. 3 *d*, *e*, *f* (imaginary part). Moreover, at each separation the self-similarity of structure is conserved which testifies to its fractal character. The dimensionality of fractal structure is calculated by the following simple formula:

$$D_{\eta}^{(\sigma)}(\gamma) = \ln(n)/\ln(N), \tag{18}$$

where n is the number of partitions of the structure size and N is the number of placing of the initial structure. At the value $\gamma = 0.003$ the dimensionality $D_x^{(r)} \approx 1.2095$. Similar calculations can be done for $D_y^{(r)}$, $D_z^{(r)}$ etc. At increasing γ all of them tend to the unit.

Taking into account the above-mentioned results, the average value of polarization (magnetization) is as follows:

$$< \bar{p}_{\eta}^{(\sigma)}(\gamma) > \simeq \frac{1}{n}\sum_{i=1}^{n} \bar{p}_{\eta}^{(\sigma)}(\gamma_i). \tag{19}$$

where n stands for the number of points at which the average value of polarization $\bar{p}_{\eta}^{(\sigma)}(\gamma_i)$ (averaged over the ensemble of $1D$ SSCs spin chains) has an extremal value, where $\gamma_i \in [\gamma - \delta\gamma, \gamma + \delta\gamma]$, $\delta\gamma << 1$, in addition, the angle brackets $< . >$ denote fractal averaging, i.e. the arithmetic mean. As it follows from Fig. 4 *a* , *b*, the mean value of polarization $< \bar{p}_{\eta}^{(\sigma)}(\gamma) >$ has a set of phase transitions of first order depending on γ after averaging on fractals.

It is important to note that in the system critical phenomena may occur connected with catastrophes in the Clausius-Mossotti equation (2) (Fig. 4 *a*) when the real part of the denominator in the equation tends to one. The analysis of a large class of spin glasses shows that catastrophes occur when the real part of polarizability coefficient connected with orientational effects varies between $\varrho(\gamma) \propto 0.025 \div 0.05$ and the contribution coming from relaxation effects is not considered in equation (2). After consideration of relaxation effects which lead to formation of the imaginary part in the polarizability coefficient (see Fig. 4 *b*), divergences in the Clausius-Mossotti equation (2) are completely eliminated. As is shown on Fig. 4. the above-mentioned parameters are frustrated in other directions also where the external field is applied.

Finally. we return to the definition of the main object of statistical physics, which is the partition function. As is well known. for a classical many-particle system in the configuration space it is defined as follows [28]:

$$Z(\beta) = \int \exp\{-\beta H(\{\mathbf{r}\})\} d\mathbf{r}_1 d\mathbf{r}_2 ..., \qquad \beta = \frac{1}{k_B T}. \tag{20}$$

where k_B is the Boltzmann constant and T is the thermodynamic temperature. Anyway, the number of integrals in expression (20) is very large as a rule and the main problem lies in their correct calculation. However. in representation (20) the configurations of spin chains which are not physically realizable do obviously contribute. Moreover. the weight of these configurations is not known in general and it is unclear how it can be defined. With this in mind and also taking into account the ergodicity of the spin glass in the above-mentioned sense. we can define the partition function in the form:

$$Z_*(\beta; N_s) = \Big\langle \int \exp\{\beta \mathcal{H}(\mathbf{E}.\mathbf{p})\} \mathcal{F}(\mathbf{E}.\mathbf{p}; \mathbf{g}.\mathcal{N}_5) \lceil \mathbf{E} \lceil \mathbf{p} \Big\rangle. \qquad \mathcal{R} \rceil \mathbf{E} \leq \imath. \tag{21}$$

where $F(\mathbf{E}, \mathbf{p}; \mathbf{g}, N_x)$ is the distribution function of an ensemble, where $\mathbf{p}$ denotes polarization of 1D SSC which is complex. $d\mathbf{p} = dp_x^{(r)}dp_y^{(r)}dp_z^{(r)}dp_x^{(i)}dp_y^{(i)}dp_z^{(i)}$. In addition, $\mathcal{H}(\mathbf{E}, \mathbf{p})$ designates spin chain Hamiltonian in the space of the energy and polarization, $(\mathbf{E}, \mathbf{p})$ and symbol $< ... >$ designates averaging by fractal structures like (19).

Thus, according to the new definition, the partition function is a complex function and its derivatives have regular behaviors respectively at the critical points.

5. Conclusion

In order to solve the problem of critical phenomena in spin glasses under external fields, we examined for the first time the possibility of its description in the framework of a complex Hamiltonian. We have studied a short-range interaction model of the spin glass which consists of 1D SSCs. We use the condition of stationarity point of the Hamiltonian on the nodes, which allows find a system of recurrent equations (8) based on the fact that stable spin chains are essentially classical trajectories, where the role of time in the context of this problem is the sequence of nodes. These equations together with Sylvester conditions (9) allow step by step construction of stable spin-chains as classical trajectories. The generalization of classical trajectories on the complex classical trajectories leads to a system of equations (15) which satisfy inequalities (16). The solutions of equations (15) for both angles and spin-spin coupling constants are complex since all parameters of the problem are complex. As a result, it helps to avoid a catastrophe in equation (2) and build up a reliable numerical algorithm for solving the spin glass problem taking into account relaxation effects. The developed approach allows us to generalize the Clausius-Mossotti equation and makes it suitable for qualitative and quantitative study of the behavior of the medium's dielectric constant including the cases where critical phenomena occur in the medium.

Finally, it is important to note that the presented approach allows us to construct in the framework of main conceptions of probability foundations a new correct form of the partition function (21), which is a complex function with the derivatives that do not diverge at the critical points.

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Դասական սպինային ապակիները հաշվի առնված ռելաքսացիոն երևույթները

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Ամփոփում

Աշխատանքում ուսումնասիրված է արտաքին դաշտի առկայությամբ տարբեր երկարությամբ $1D$ չկարգավորված տարածական սպինային շղթաների (SUC) համույթի վիճակագրական հատկությունները՝ հաշվի առնելով ռելաքսացիոն երևույթները: Առաջին անգամ օգտագործվել է կոմպլեքս-դասական Համիլտոնիանը: Պարբերական $1D$ ցանցի հանգույցներում ստացվել է ռեկուրենտ եռանկյունաչափական հավասարումների համակարգը, որոնք Սիլվեստրի պայմանների հետ միասին անալիտիկորեն շարունակվում են կոմպլեքս տարածության մեջ և հնարավորություն են տալիս հանգույց առ հանգույց հաշվել սպինի ուղղորդվածությունը՝ հաշվի առնելով սպինային շղթաներում ռելաքսացիոն երևույթները:

Ուսումնասիրված են նաև սպինային համույթում տեղի ունեցող որոշակի կրիտիկական երևույթներ, ինչպիսիք են Կլաուզիուս-Մոսսոտիի (Կ-Մ) հավասարման մեջ աղետները՝ կախված արտաքին դաշտի մեծությունից:

Առաջարկված է վիճակագրական գումարի նոր ներկայացում վերջավոր թվով ինտեգրալային արտահայտության տեսքով՝ էներգիայի և բևեռացվածության տարածությունում:

Классические спиновые стекла с учетом релаксационных эффектов

А. С. Геворкян А. Г. Абаджян

Аннотация

В данной работе исследованы статистические свойства ансамбля неупорядоченных $1D$ пространственных спин цепочек (ПСЦ) с определенной длиной во внешнем поле с учетом релаксационных эффектов. Для решения этой проблемы впервые был использован короткодействующий комплексно-классический Гамильтониан и разработан эффективный математический алгоритм, который, с учетом расширенных условий Сильвестра, позволяет параллельно, шаг за шагом построить большое количество стабильных $1D$ ПСЦ. Функции распределения различных параметров спинового стекла построены на основе анализа результатов расчета $1D$ ПСЦансамбля. Показано, что распределения разных параметров спинового стекла по-разному ведут себя в зависимости от внешнего поля. Показано, что обобщенный комплексно-классический подход исключает возможность возникновения катастроф в уравнении Клаузиуса-Моссотти, что позволяет организовать непрерывные вычисления на всем интервале значений внешнего поля, включая критические точки. На основе проведенных исследований предложен новый, более точный способ построения статистической суммы системы, которая в отличие от обычных представлений, является комплексной функцией. Статистическая сумма, и ее производные аналитичны повсюду включая критические точки.