

Mathematical Problems of Spin-Glass Systems in One-Dimension

Ashot S Gevorkyan,

Institute for Informatics and Automation Problems, NAS of Armenia

Spin glass models are very useful for investigations of a number of important and difficult applied problems of physics, chemistry, material science, biology, evolution, organization dynamics, hard-optimization, environmental and social structures, human logic systems, financial mathematics etc (see for example [1-10]). It is possible to divide the considered mean-field models of spin-glasses into two types. The first consists of the true random-bond models, where the coupling between interacting spins are taken to be independent random variables [11-13]. The solution of these models was obtained by n -replica trick [11-13] and required the invention of sophisticated schemes of replica-symmetry breaking [13-14]. In the models of second type, the bond-randomness is expressed in terms of some underlining hidden site-randomness and is thus of a superficial nature. It has been pointed out in the works, however, that this feature retains an important physical aspect of true spin-glasses, viz. that they are random with respect to the positions of magnetic impurities. The main way of investigation of spin-glasses systems is a method of numerical simulation. But, numerical studies of spin-glass problem is difficult to accomplish and, in general, only for a small number of spins, the numerical simulation is effectively solvable. For development of new ideas and creation of program packages for numerical simulation of spin-glass systems the investigation of 1D model is very important. The modified 1D model can be useful also for analysis of series of real problems.

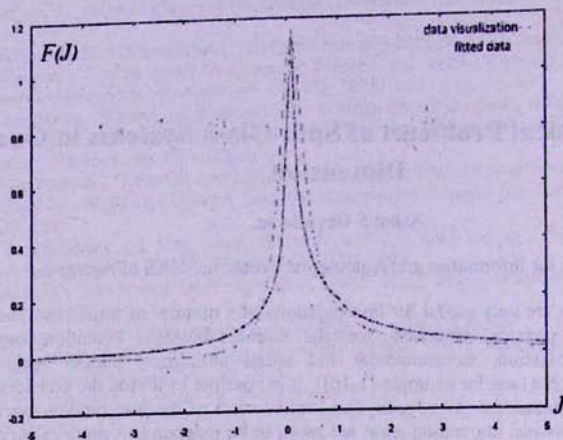


Fig. 1. The red line shows visualization of numerical data while the green line shows analytic approximation by Cauchy function.

The statistical properties of ensemble of disordered 1D steric spin-chains (SSC) of various lengths are investigated. Using 1D spin-glass type classical Hamiltonian, the recurrent transcendental equations for stationary points and corresponding conditions for the construction of stable 1D SSCs are found. The ideal ensemble of spin-chains (non-interacting spin-chains) is analyzed and the latent interconnections between random angles and interaction constants for each set of three nearest-neighboring spins are found. It is analytically proved and by numerical calculation is shown that the distribution of interaction constant satisfy Levy's alpha-stable distribution law [15] (see Fig. 1 and reference [16]). Energy distribution in ensemble is calculated depending on different conditions of possible polarization of spin-chains. It is specifically shown that the dimensional effects in the form of set of local maximums in the energy distribution arise when the number of spin-chains $M \ll N_x^2$ (where N_x is number of spins in a chain) while in the case when $M \sim M \ll N_x^2$ the energy distribution has one global maximum and ensemble of spin-chains satisfies the Birkhoff's ergodic theorem. An effective algorithm for parallel simulation of the problem which includes calculation of different statistic parameters of 1D SSCs ensemble has been elaborated.

Finally, it is important to note that the developed scheme of solution of 1D SSCs problem can be used for analysis of different applied problems, also it can be useful for creation of effective program package for simulation of 3D spin-glass problem.

- [1] K. Binder and A. P. Young, Spin glasses: Experimental facts, theoretical concepts, and open questions. Rev. Mod. Physics, 58(4), 801-976 (1986).
- [2] M. Me'zard, G. Parisi, M. A. Virasoro, Spin Glass Theory and Beyond (World Scientific, Singapore, 1987)
- [3] A. P. Young (ed.), Spin Glasses and Random Fields (World Scientific, Singapore, 1998)
- [4] R. Fisch and A. B. Harris, Phys. Rev. Let., 47, 620 (1981).

- [5] C. Ancona-Torres, et al., Phys. Rev. Lett., **101**, 057201 (2008).
- [6] A. Bovier, Statistical Mechanics of Disordered Systems: A Mathematical Perspective, Cambridge Series in Statistical and Probabilistic Mathematics, p 308 (2006).
- [7] Y. Tu, J. Tersoff and G. Grinstein, Phys. Rev. Lett., **81**, 4899 (1998).
- [8] K. V. R. Chary, G. Govil, NMR in Biological Systems: From Molecules to Human (Focus on Structural Biology 6), Springer, p. 511, (2008).
- [9] E. Baake, M. Baake and H. Wagner, Phys. Rev. Lett., **78**(3), 559-562 (1997.)
- [10] A. S. Gevorkyan et al., New Mathematical Conception and Computation Algorithm for Study of Quantum 3D Disordered Spin System Under the Influence of External Field, Trans. On Comput. Sci., VII, LNCS 132-153, Springer-Verlage, 10.1007/978-3-642-11389-58
- [11] D. Sherrington and S. Kirkpatrick, Phys. Rev. Lett., **35**, 1972 (1975).
- [12] B. Derrida, Phys. Rev., **B 26**, 2613 (1981).
- [13] G. Parisi, Phys. Rev. Lett. **43**, 1754 (1979).
- [14] A. J. Bray and M. A. Moore, Phys. Rev. Lett., **41**, 1068 (1978).
- [15] J. P. Nolan, Stable Distributions: Models for Heavy Tailed Data (2009-02-21). en.wikipedia.org/Stable_distribution.
- [15] A. S. Gevorkyan, H. G. Abajyan and H. S. Sukiasyan, Ideal Ensemble of Disordered 1D Steric Spin-Chains in External Field, J. Computational Science (in press).