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Approximation Algorithm for Wireless Network Interference Minimization

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Abstract

Interference minimization problem in wireless sensor and ad-hoc networks are considered. That is to assign a transmission radius to each node of network, to make it connected and at the same time to minimize the maximum number of overlapping transmission ranges on each node of network. Additional means of topology control besides the connectivity is blocking the long line connections at the receiver level. We propose a polynomial time approximation algorithm which finds connected network with at most $O((opt \cdot \log n)^2)$ interference where opt is the minimal interference of the given network, and n is the number of network nodes. The lower bound for this problem, where a general distance function is considered, has been proven to be $O(\log n)$. The algorithm is known which finds a network where maximum interference is bounded by $O(\sqrt{n})$.

1. Introduction

We consider interference minimization problem in energy limited wireless networks (wireless sensor and ad-hoc networks). On networks where changing or recharging the energy source of nodes is not possible the reduction of energy consumption is considered to increase the nodes operability time (networks' lifetime). One possible approach for doing this is interference reduction on network nodes. Wireless communication of two nodes which is experiencing the third one is interference on that node. High interference on a node (high number of nodes interfering on it) makes difficulty to determine and accept the signals dedicated to it. This makes the necessity to sender node to retransmit the signal which is extra energy consumption. Our work tends to reduce colliding transmission by reducing interference on network algorithmically.

1.1. Interference Minimization In Wireless Networks

Different models of Interference minimization problems have been proposed in literature [1-8]. In this paper we focus on interference minimization on receiver node. Formally, interference minimization problem we consider is the following: given a set of nodes on metric space, each node has transmission disk of a given radius. Interference on some point is the number of transmission disks including that point. Interference of network is defined as a highest interference among the all nodes forming the network. Interference minimization problem is to assign transmission ranges to each node and to select the proper subset of bidirectional links at these radii so that the network is connected through the selected bidirectional links and the interference on network is minimal. The main weakness of the distance model is the assumption that the radio coverage area is a perfect circle. This holds for free-space environment, and it does not consider the possible landscape properties, reflections and diffraction, and the transmission radius reduction at the nodes by the time. Note that in a radio communication an amount of energy is also consumed at the receiver node to receive and decode the transmitted signal. A notion here is that receiver reads the package header and then the body information, so that cancellation of a valid by transmission ranges symmetric link of connectivity in the network design stage assumes reading of only the headers in connections, achieving energy minimization in this way. The same time the most part of energy consumption is at the information transmission stage. It is known that the transmission power required by node i to correctly transmit data to node j is at least quadratic by the distance between i and j [20]. That is it is more convenient from the network capacity point of view that i sends the data to j along several short hops rather than using the direct long connection [19]. Nodes cannot abandon too many links to far-away neighbors without affecting the connectivity and they may not use an increasing number of links to nearby nodes without increasing the interference. In general this directly leads to a trade-off between the network connectivity and interference.

The task of computing a subgraph of the given network graph with certain properties, reducing the transmission power levels and thereby attempting to reduce interference and energy consumption is known as the Topology Control procedure. We provided a very general problem description at this point. *Interference minimization is one of the most studied problems on wireless and ad-hoc networks.*

Interference minimization problem on one dimensional network (where nodes are distributed along the line, so-called highway model) was considered in [1]. Authors showed that intuitive algorithm, which connects each node of a network with its closest right (except for the rightmost) and left (except for the leftmost) nodes can give a bad performance. An example of network where above algorithm has the worst performance is the exponential node chain, where distance between two consecutive nodes grows exponentially $(2^0, 2^1, \ldots, 2^{n-1})$. [1] Also gives two algorithms for line based case of interference minimization problem, one finds a network with at most $O(\sqrt{\Delta})$ interference (Δ is the interference of uniform radius network under consideration) and the another one approximates the optimum for the given network instance with factor of $O(\sqrt[4]{\Delta})$. Using ideas from [1], from computational geometry and ε-net theory, [2] proves the $O(\sqrt{\Delta})$ interference bound for maximum interference in two and more dimensional networks. A logarithmic lower bound for approximation of interference minimization problem under general distance function is proven in [3] by reduction of minimum set cover to the minimum interference problem.

Our result: We present iterative algorithm for basic minimum interference problem which finds connected network with at most $O(opt \cdot \log^2 n)$ interference approximation ratio, where opt is interference of minimum interference connectivity network for the input instance of n nodes.

1.2. Interference Minimization In Cellular Networks

Below, besides the basic <u>connectivity</u> preserving model for WSN, we refer to one more model of interference minimization (relation to our work will be given in Section 5) which requires area <u>coverage</u>. This is cellular networks model that are heterogeneous networks consisting of two different types of nodes: base stations and clients. The base stations, acting as servers, are interconnected by an external fixed backbone network; clients are connected via radio links to base stations. Since communication over the wireless links takes place in a shared medium, interference can occur at a client if it is within transmission range of more than one base station. In order to prevent or control such collisions, coordination among the conflicting base stations is used. Commonly this problem is solved by segmenting the available frequency spectrum into channels to be assigned to the base stations in such a way as to prevent interference, in particular such that no two or a limited number of base stations with overlapping transmission range use the same channel. The further analysis is formed by the observation that interference effects occurring at a client depend on the number of base stations by whose transmission ranges it is covered. A scenario is assumed in which each base station can adjust its transmission power. The problem of minimizing interference then consists in assigning every base station a transmission power level such that the transmission disks of base stations cover some given area (set of clients) and the number of base stations covering any point (client) of the covered area is minimum (without requiring connectivity between base stations) [4].

Authors of [4] show the problem reduction to the minimum membership set cover combinatorial optimization problem and prove that in polynomial time the optimal solution can not be approximated closely than within a factor $O(\log n)$. On the other hand they give a polynomial time algorithm based on linear program relaxation technique, which asymptotically matches the lower bound.

2. Interference Minimization Problem

Wireless sensor/ad-hoc network consists of uniformly distributed set of nodes in a certain area. Nodes are equipped with energy source, computation and wireless communication devices and sensors.
We consider the scenario where the set $V = \{v_1,...,v_n\}$ of n wireless nodes is spatially distributed on a metric space with distance function $d : V \times V \to R^+$. The range assignment function $RA : V \to R^+$ assigns a suitable transmission range to each node of a network, i.e. each node has the maximum transition range that it can be assigned and the range assignment function RA assigns to each node a range between zero and its maximum.
Denote by $D(v_i, RA(v_i))$ the set of nodes which are in the transmission disk of node v_i which has transmission radius $RA(v_i)$, i.e. in the disk centered at v_i and having the radius $RA(v_i)$.
Bidirectional links simplify communication protocols of network nodes (e.g node v_i, sending a message to v_j, may directly receive an acknowledgment of message delivery) therefore only symmetric links between network nodes are considered. Assuming that nodes v_i and v_j can communicate if they are within each other's current transmission disks ($v_j \in D(v_i, RA(v_i))$ and $v_i \in D(v_j, RA(v_j))$).
Interference on node v_i is the number of transmission disks covering the node v_i, $I(v_i) = |\{D_j(v_j, RA(v_j)) / v_i \in D_j(v_j, RA(v_j)), j \neq i\}|$ and the overall interference of network is defined as the maximum interference among the all nodes: $I(V) = \max_{v_i \in V} I(v_i)$. At this point interference minimization problem can be defined as follows: For given set $V = \{v_1,...,v_n\}$, of distributed nodes, find a radius assignment function RA such that the resulting network is connected and the network interference is minimal. This is interference minimization problem by RA.

By $G_C = (V, E_C)$ we denote the network graph, where $(v_i, v_j) \in E_C$ if v_i and v_j can communicate with each other when their maximum radius transmission disks are considered. Next to the RA the topology control procedure applies the edge subset selection process in E_C. In this terms interference minimization problem can be formulated as finding a connected (spanning) subgraph (factor) $H \subset G_C$ such that interference $I(V)$ computed by the selected set of edges is minimal. Formally, having the subgraph $H(V, E_C^*)$ it is correct to further extract transmission radius for any node v_i as a distance between v_i and its farthest neighbor in H, $r(v_i) = \max_{(v_i, v_j) \in E_C^*} d(v_i, v_j)$, which avoids the unnecessary interference.

The following two sections contain some key definitions of technology we apply to the Topology Control for interference minimization.

3. Minimum Membership Set Cover problem (MMSC)

Set Cover problem is one of the core issues of combinatorial optimization [9, 10]. It is formulated as follows, given a set S and a collection C of subsets of S, find a subset C' of C as small as possible, such that the union of sets in C' covers S. It is well known that decision version of Set Cover is NP-complete and that in polynomial time the optimal solution can not be approximated closer than with logarithmic factor [9]. Several variants of Set Cover problem have been studied [4, 11-16]. One of the variants which we use in our work is Minimum Membership Set Cover [4]; given a set S and collection C of subsets of S, find a subset C' of C such that the union of sets in C' is S and occurrence of each element from S in selected subset C' is minimal.

MINIMUM MEMBERSHIP SET COVER (MMSC)

INPUT: A set S and collection C of subsets of S

OUTPUT: Subset $C' \subseteq C$ such that $\bigcup_{c \in C'} c = S$ and $\max_{s \in S} |\{c \in C' \setminus s \in c\}|$ is minimal.

The above problem is investigated in [4] motivated by interference minimization in cellular networks. [4] Contains the proofs of NP-completeness of decision version of **MMSC** problem and non-approximability of **MMSC** optimization problem by factor closer than $O(\ln n)$. Also, by using the linear program relaxation technique, [4] gives a polynomial time algorithm, which approximates the optimal solution with factor $O(\ln n)$. Below we present the integer program formulation of **MMSC** which later will be modified to fit to our requirements. Let $C' \subseteq C$ is some sub collection of C and to any subset $C_j \in C$ we have assigned variable $x_j \in \{0,1\}$ where $x_j = 1 \Leftrightarrow C_j \in C'$, then the integer program of **MMSC** could be written as:

MINIMIZE z

SUBJECT TO $\sum_{\{j/u_i \in C_j\}} x_j \geq 1 \quad u_i \in S \qquad (1)$

$\sum_{\{j/u_i \in C_j\}} x_j \leq z \quad u_i \in S \qquad (2)$

$x_j \in \{0,1\} \quad C_j \in C \quad (3)$

Easy to see that any $C' \subseteq C$ satisfying to (1) – (3) is optimal solution to **MMSC** problem.

4. Minimum Partial Membership Partial Set Cover Problem (MPMPSC)

Before describing our approximation algorithm for interference minimization problem we need to do one more important definition. The minimum membership set cover problem defined in previous section requires finding a sub collection $C' \subseteq C$ which is a cover for all the elements of S and maximum membership is counted within all the elements of S. Below we give a slightly different definition similar to **MMSC**. Let set $S = S_1 \cup S_2$ consists of two disjoints sets S_1 and S_2. As in case of **MMSC** a collection C of subsets of S is given. Then the new postulation is to find a sub collection $C'' \subseteq C$ such that the union of sets in C'' contains all the elements of S_1 and the maximum membership which is counted only within the elements of S_2 will be kept minimal.

MINIMUM PARTIAL MEMBERSHIP PARTIAL SET COVER (MPMPSC)

INPUT: A set $S = S_1 \cup S_2, S_1 \cap S_2 = \varnothing$ and collection C of subsets of S

OUTPUT: Subset $C'' \subseteq C$ such that $S_1 \subseteq \bigcup_{c \in C''} c$ and $\max_{s \in S_2} |\{c \in C'' \setminus s \in c\}|$ is minimal

Integer program for this problem will be:

MINIMIZE z

SUBJECT TO $\sum_{\{j/u_i \in C_j\}} x_j \geq 1 \quad u_i \in S_1 \qquad (1')$

$\sum_{\{j/u_i \in C_j\}} x_j \leq z \quad u_i \in S_2 \qquad (2')$

$x_j \in \{0,1\} \quad C_j \in C \quad (3')$

(1') – (2') comprise individual constraints twice less than in (1) – (2). In general, optimal solution of **MPMPSC** is smaller than in **MMSC**, but we apply the same level of approximation and then it is easy to show that the randomized rounding approach with relaxation of condition (3) for solving (1) – (3) presented in [4] is also suitable for (1') – (3') and gives the same approximation ratio, and we skip the proof or improvements here. To summarize, we have the following.

Theorem 1. *For* **MPMPSC** *there exists a deterministic polynomial-time approximation algorithm with an approximation ratio of* $O(\log(\max\{|S_1|,|S_2|\}))$.

Consider the complexity issue of **MPMPSC**. We will compare **MPMPSC** with standard Set Cover (**SC**). First we reduce the **SC** optimization version to the optimization version of **MPMPSC**. Then we justify the issue of reduction of the decision version of these problems.

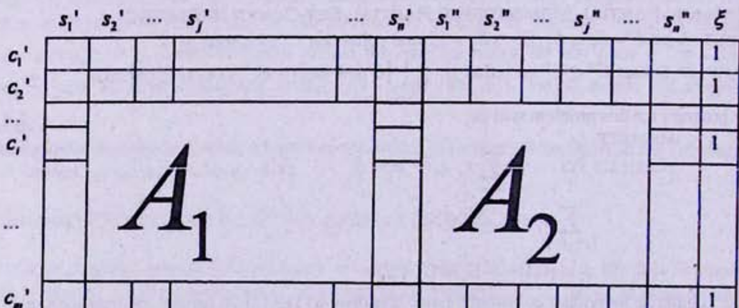
Let as bring the matrix descriptions of problems in question. A is a (0,1) matrix of m rows and n columns. Columns correspond to n elements of the initial set S of Set Cover problem, and rows represent the m subsets (elements of C) that will cover the set S. $a_{ij} \in A$ Equals to 1 iff $s_j \in S$ belongs to $c_i \in C$. In this terms **SC** optimization version looks for minimal number of rows which cover in total 1's in each column of matrix. **SC** Decision version for a given k asks if there exists a selection of at most k rows which cover the set S.

In **MPMPSC** optimization two (0,1) matrices A_1 and A_2 are given. n_1 columns of A_1 correspond to the elements of set S_1. n_2 Columns of A_2 correspond to the elements of set S_2. Rows represent the m subsets (members of C) that are composed of elements of sets S_1 and S_2. Optimization **MPMPSC** is to find the set of rows that cover the set S_1 so that maximal sub-column weight by part of A_2 is minimal. Decision version for a given k asks if there exists a selection of rows which cover the set S_1, so that sub-column weights by part of A_2 are at most k.

Theorem 2. ***MPMPSC*** *decision version is* NP *-complete.*

Proof. We prove the theorem in two steps. We first prove that **MPMPSC** is NP. Nondeterministic algorithm only needs to guess a subset of rows and then check that it covers the set and sub-column weights by part of A_2 are at most k. This takes polynomial time.

Next, we reduce the **SC** optimization to the **MPMPSC** optimization. Consider an individual instance of **SC**. This is set S, collection of subsets C and the corresponding binary matrix A. Compose an individual instance for **MPMPSC** in the following way. We take $S_1 = S$ and and $S_2 = S \cup \{\xi\}$, where ξ is an additional element. Then, $A_1 = A$, and $A_2 = A \| 1^m$, which means concatenation of A with an additional all 1 column for element $\xi \in S_2$.



The figure demonstrates individual instance of **MPMPSC** composed by the arbitrary instance of SC. The part A_1 is equivalent to the instance of **SC** and A_1 and A_2 are identical to each other.

Here is the transformation of an individual instance of **SC** to the individual instance of **MPMPSC**. If C' is a solution of **SC**, then the same set is a solution for **MPMPSC**. C' As the solution of **SC** consists of minimal number of rows that cover S. **MPMPSC** requires that its solution (set of rows) C'' covers the set S_1 (which is identical to S) and therefore the size of C'' is greater or equal to $|C'|$. In its turn and because of construction of column ξ maximal sum of sub column in condition of C'' is equal to $|C''|$ and so it is not less than $|C'|$. To optimize **MPMPSC** it is enough to select $C''=C'$.

Inversely, if C'' is a solution of **MPMPSC**, then the same set is a solution for **SC**. For considered individual instance of **MPMPSC** its solution C'' obeys "the maximal sub column weight is equal to the size of set cover". The minimum of this size achieved when the minimal set cover is used but then this is a solution for **SC**. This completes the proof of equivalence of two optimization problems: **SC** and composed individual instance **MPMPSC**.

It is very important that for all instances of **SC** we constructed appropriate **MPMPSC** instances so that optimization parameter of **SC** (size of minimal set cover) is equal to the optimization parameter of **MPMPSC** (minimax of sub column weights when S_1 is covered). This proves that instances of decision **SC** are mapped on part of instances of **MPMPSC** which finally proves that decision **MPMPSC** is NP-complete.

5. Approximation algorithm for minimum interference problem

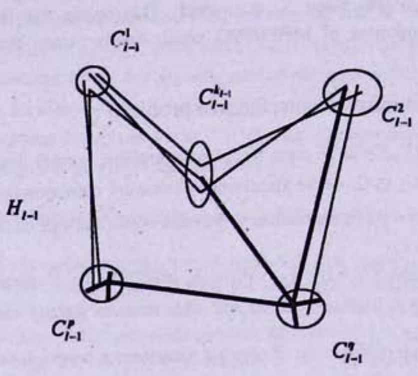
Algorithm takes network graph $G=(V,E)$ as an input and after logarithmic number $k \in O(\log n)$ of iterations returns connected subgraph $G_k \subseteq G$ where interference of network corresponding to graph G_k is bounded by $O((opt \cdot \ln n)^2)$, where $n=|V|$ is the number of network nodes and opt is interference of minimum interference connected network.

Algorithm starts its work with the graph $G_0=(V,E_0)$ where $E_0=\varnothing$. On the l^{th} iteration, $l \geq 1$, algorithm chooses some subset $F_l \subseteq E \setminus E_{l-1}$ of new edges and adds them to already chosen edge set $E_{l-1}=\bigcup_{i=1}^{l-1} F_i$. As a consequence of such enlargement of edge set, interference on graph vertices will increase in some value depending on F_l. Algorithm finishes the work if the graph $G_l=(V,E_l)$ is connected otherwise goes for the next iteration. Below we present the way how algorithm chooses the set of edges $F_l \subseteq E \setminus E_{l-1}$ on l^{th} iteration. Algorithms' quality, i.e the final maximal interference on nodes (its upper estimate) equals to the accumulated through the iterations interferences which we try to keep minimal.

Let $G_{l-1}=(V,E_{l-1})$ is the graph obtained at $(l-1)^{th}$ iteration, and let G_{l-1} has the set of connected components $C(G_{l-1})=\{C_{l-1}^1,\ldots,C_{l-1}^{k_{l-1}}\}$. We denote by $H_{l-1} \subseteq E \setminus E_{l-1}$ the set of all edges which have their endpoints in different connected components of G_{l-1}. In each step of algorithm a subset of H_{l-1} is selected to further reduce the number of connected components which finally brings us to a connected subgraph. In this way we build the collection $\mathrm{T}(C(G_{l-1}),H_{l-1})$ of special sets as follows.

Starting with H_{l-1} we add to the set $\mathrm{T}(C(G_{l-1}),H_{l-1})$ of l^{th} stage specific subsets $T^l(v_i,v_j)=\{C_{l-1}^p,C_{l-1}^q\} \cup D(v_i,d(v_i,v_j)) \cup D(v_j,d(v_i,v_j))$ defined by all $(v_i,v_j) \in H_{l-1}$, where C_{l-1}^p is the connected component where v_i belongs to, and C_{l-1}^q is where v_j belongs to. By selection of v_i and v_j we have that $p \neq q$. $T^l(v_i,v_j)$ is a composite set. It includes the two labels for components C_{l-1}^p and C_{l-1}^q, and all vertices which are incident to end points v_i and v_j of connection (v_i,v_j), which is the same maximal set of vertices that may receive interference increase after selecting the edge (v_i, v_j) as a new communication link. To know the real interference increase one have to check if v_i and/or v_j are selected first time at this step or if there is an increase of transmission radiuses by selecting (v_i, v_j) and that there are several new points covered by v_i and/or v_j first time at this step. So $T'(v_i, v_j)$ considers larger sets covered and counts larger interference than in reality. Formally, labels for connected components will compose the set S_1, and candidate vertices for interference – the set S_2 in terms of MPMPSC.

The figure below demonstrates connected components that are input to the stage l, and the set H_{l-1} of all cross component edges. Bold lines show an edge (v_i, v_j) between the arbitrary two components C^p_{l-1} and C^q_{l-1}, together with other links to vertices adjacent to the end points v_i and v_j by the distance $d(v_i, v_j)$ (these may be also points from other connected components which appeared only now by increased radius of end points of (v_i, v_j)).



After constructing $\mathrm{T}(C(G_{l-1}), H_{l-1})$ we solve the **MPMPSC** on the set $C(G_{l-1}) \cup V$ and by collection of subsets $\mathrm{T}(C(G_{l-1}), H_{l-1})$, where condition for elements from $C(G_{l-1})$ is to be covered and for elements from V is to have minimum membership (even zero).

Finally, based on solution $W(C(G_{l-1}), H_{l-1}) \subseteq \mathrm{T}(C(G_{l-1}), H_{l-1})$ of **MPMPSC** we build the set F_l of network graph edges, selected in l^{th} stage of algorithm as follows. We add to F_l all the edges $(v_i, v_j) \in H_{l-1}$ such that $T^l(v_i, v_j) \in W(C(G_{l-1}), H_{l-1})$.

The proof of algorithm performance is considered in next section.

6. Algorithm performance

Theorem 3: *The number of connected components is reduced at least by factor* 2 *on each stage of algorithm, which bounds the total number of iterations by* $O(\ln n)$.

Proof: For each connected component $C^p_{l-1} \in C(G_{l-1})$ of graph G_{l-1} the solution $W(C(G_{l-1}), H_{l-1})$ of **MPMPSC** solved at l^{th} iteration should contain at least one set $T^l(v_i, v_j) \in W(C(G_{l-1}), H_{l-1})$ such that $C^p_{l-1} \in T^l(v_i, v_j)$ (as $W(C(G_{l-1}), H_{l-1})$ is cover for the set $C(G_{l-1})$). And as each set $T^l(v_i, v_j) \in W(C(G_{l-1}), H_{l-1})$ contains exactly two connected components, then by adding the edge (v_i, v_j) to our solution, we merge those two connected components into one (connecting by the edge (v_i, v_j)). So every connected component merges with at least one other connected component, which reduces the number of connected components at least by factor 2.

Lemma 1: *Network corresponding to the graph $G^l = (V, F_l)$ (where F_l is the edge set obtained at l^{th} iteration of approximation algorithm) has interference in $O(opt^2 \cdot \ln n)$.*

Proof: Consider the set $C(G_{l-1}) = \{C^1_{l-1}, ..., C^{k_{l-1}}_{l-1}\}$ of connected components in l^{th} iterative step of algorithm. Let E_{opt} is the set of edges of some interference optimal connected network for our problem (edges of connected network with optimal interference opt). Then there is a subset $E^l_{opt} \subseteq E_{opt}$ which spans connected components $C(G_{l-1})$ and the network of the graph $G^l_{opt} = (V, E^l_{opt})$ has interference not exceeding the opt, which means;

Fact 1: The maximal vertex interference due to a spanner E^l_{opt} of $C(G_{l-1})$ is at most opt.

Now let us build the set collection $T_{opt}(C(G_{l-1}), E^l_{opt}) = \{T^l(v_i, v_j) / (v_i, v_j) \in E^l_{opt}\}$.

Fact 2: $T_{opt}(C(G_{l-1}), E^l_{opt})$ is sub collection of $T(C(G_{l-1}), H_{l-1})$ built on l^{th} iteration of algorithm and is cover for $C(G_{l-1})$.

$T_{opt}(C(G_{l-1}), E^l_{opt}, v_k) = \{T^l(v_i, v_j) \in T_{opt}(C(G_{l-1}), E^l_{opt}) / v_k \in T^l(v_i, v_j)\}$ will denote the collection of those sets from $T_{opt}(C(G_{l-1}), E^l_{opt})$ which contain v_k (at least one of v_i, v_j interferes on v_k). Now let v_{i_0} is some node which has interference on v_k.

Fact 3: due to *Fact 1* for each fixed vertex v_{i_0} the number of sets $T^l(v_{i_0}, v_j) \in T_{opt}(C(G_{l-1}), E^l_{opt}, v_k)$ will not exceed the opt.

And as the number of nodes, in a network graph $G^l_{opt} = (V, E^l_{opt})$, having interference on v_k is not exceeding the opt, then the cardinality of set $T_{opt}(C(G_{l-1}), E^l_{opt}, v_k)$ will not exceed the opt^2.

The *Facts 2* and *3* together show that the value of optimal solution of **MPMPSC** we solve on l^{th} iteration of algorithm is bounded by opt^2.

And approximation algorithm will find solution where each element is covered at most by $O(opt^2 \ln n)$ times, and as each set $T^l(v_i, v_j) \in T_{opt}(C(G_{l-1}), E^l_{opt}, v_k)$ which covers some element v_k increases interference on node v_k at most by 2 (both v_i and v_j may contain v_k on their transmission disks). Then the interference on v_k will be bounded by $O(opt^2 \ln n)$.

Theorem 4. *The network built by **MPMPSC** relaxation algorithm has at most $O(opt^2 \cdot \ln^2 n)$ interference.*

Proof. The proof is in combination of Theorem 1 and Lemma 1, and in applying $O(\ln n)$ steps each with additional interference $O(opt^2 \cdot \ln n)$.

7. Conclusion

This paper presents approximation algorithm for interference minimization problem in wireless networks with $O(opt \cdot \ln^2 n)$ approximation ratio. Algorithm is iterative and on each step of iteration the minimum partial membership partial set cover (**MPMPSC**) problem is solved. **MPMPSC** is an extension of **MMSC** and is possible to solve with the algorithm presented in [4]. The **MPMPSC** problem we solve at most $O(\log n)$ times has two constraints. The first is that the number of elements to be covered is no more than the number of elements the membership is to be minimized and the second is that each set contains exactly two elements which are from *to be covered* set. These two constraints will be considered in future to get better approximation of MPMPSC which on its turn will give a better approximation for minimum interference problem.

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Անլար ցանցերում ինտերֆերենսի մինիմիզացման մոտավոր ալգորիթմ

Հ. Ասլանյան

Ամփոփում

Դիտարկվել է անլար ցանցերում ինտերֆերենսի մինիմիզացման խնդիրը, որը հետևյալն է. ցանցի յուրաքանչյուր հանգույցին վերագրել հաղորդակցման շառավիղ այնպես, որ ցանցը լինի կապակցված և միևնույն ժամանակ ցանցում մաքսիմալ ինտերֆերենս ունեցող հանգույցի ինտերֆերենսը (հանգույցն ընդգրկող հաղորդակցման շրջանների քանակը) լինի մինիմալ: Այս խնդրի համար առաջարկվում է բազմանդամային ժամանակում աշխատող մոտավոր ալգորիթմ, որը գտնում է կապակցված ցանց, որում յուրաքանչյուր հանգույցի ինտերֆերենսը չի գերազանցում $O((opt \cdot \log n)^2)$-ն: Այստեղ opt-ը տրված n հանգույցներով ցանցի մինիմալ ինտերֆերենսն է: Հայտնի է, որ խնդիրը մետրիկական տարածությունում դիտարկելու դեպքում (որը աշխատանքում դիտարկված դեպքն է), $O(\log n)$-ը մոտարկման ստորին սահմանն է: Հայտնի է նաև բազմանդամային ալգորիթմ, որը գտնում է կապակցված ցանց առավելագույնը $O(\sqrt{n})$ ինտերֆերենսով: