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On the Number of Irreducible Linearised Coverings for Subsets in Finite Fields

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Abstract

We present lower and upper bounds for the number of irreducible linearised coverings of subsets in a finite field with characteristic greater than 2. In case of finite field with characteristic equal to 2, these bounds are obtained by Alexanian.

1. Introduction

Throughout this paper F_q stands for a finite field with q elements. and F_q^n for an n-dimensional linear space over F_q. If L is a linear subspace in F_q^n, then the set $\tilde{\alpha} + L \equiv \{\tilde{\alpha} + \tilde{x} | \tilde{x} \in L\}$. $\tilde{\alpha} \in F_q^n$ is a coset (or translate) of the subspace L and $\dim(\tilde{\alpha} + L)$ coincides with $\dim L$. An equivalent definition: a subset $N \subseteq F_q^n$ is a coset if whenever $\tilde{x}^1, \tilde{x}^2, \ldots, \tilde{x}^m$ are in N, so is any affine combination of them, i.e., so is $\sum_{i=1}^{m} \lambda_i \tilde{x}^i$ for any $\lambda_1, \lambda_2, \ldots, \lambda_m$ in F_q such that $\sum_{i=1}^{m} \lambda_i = 1$. It can be readily verified that any k-dimensional coset in F_q^n can be represented as a set of solutions of a certain system of linear equations over F_q of rank $n-k$ and vice versa. The number of k-dimensional linear subspaces in F_q^n equals to

$$\begin{bmatrix} n \\ k \end{bmatrix}_q = \frac{(q^n-1)(q^{n-1}-1)\cdots(q^{n-k+1}-1)}{(q^k-1)(q^{k-1}-1)\cdots(q-1)},$$

which is called Gauss coefficient.

Let N be a subset in F_q^n.

Definition 1. *A coset H in F_q^n is called maximal coset in$N$, if $H \subseteq N$and for any coset $H' \supseteq H$, $H' \not\subseteq N$.*

Definition 2. *A set of cosets $\{H_1, H_2, \ldots, H_m\}$ in F_q^n forms a linearised covering of N if $N = \bigcup_{i=1}^{m} H_i$. The length (or complexity) of the covering is equal to the number of cosets, i.e. m.*

Definition 3. *A linearised covering $D = \{H_1, H_2, \ldots, H_m\}$ of N is called irreducible linearised covering if H_i is a maximal coset in$N (i = 1, \ldots, m)$ and for any $D' \subset D$, D' does not form a covering for N.*

Definition 4. *We say that a set of cosets $A = \{H_1, H_2, \ldots, H_m\}$ in F_q^n forms an anti-chain if $i \neq j \Rightarrow H_i \not\subset H_j$ and $H_j \not\subset H_i$.*

It is shown in [1, pp. 13-15] that for the maximum length of anti-chain A in F_q^n the following inequality holds:

$$\max |A| \leq e^{4/3} q^{(n+1)^2/4} \tag{1}$$

Let $t(N)$stands for the number of all irreducible linearised coverings for N. We denote by

$$t(n) = \max_{N \subseteq F_q^n} t(N)$$

The purpose of this paper is to obtain lower and upper bounds for $t(n)$. For finite fields of char=2 this is done in [2].

2. Main Result

Theorem 1.

$$q^{\left(\lceil\frac{n}{2}\rceil-1\right)^2\lceil\frac{n}{2}\rceil q^{\lceil\frac{n}{2}\rceil-1}} \leq t(n) \leq q^{q^n \frac{(n+1)^2}{4}(1+\varepsilon_n)},$$

where $\lceil 0 \rceil$ is the integer part of a, and $\lim\limits_{n\to\infty} \varepsilon_n = 0$.

This theorem is a result of the following set of affirmations.

Let us consider the following equation over the field F_q^{2n}:

$$x_1x_2 + \ldots + x_{2n-1}x_{2n} = b. \tag{2}$$

where $b \in F_q$, and $b \neq 0$.
We denote by N_0 the set of solutions of 2. N_0 can be represented as a union of solutions of some systems of linear equations over F_q^{2n}as stated in the below lemma.

Lemma 1. *The union of solutions of systems*

$$\begin{cases} x_2 = \alpha_1 \\ \ldots \\ x_{2n} = \alpha_n \\ \sum\limits_{i=1}^{n} \alpha_i x_{2i-1} = b \;\; (b \neq 0) \end{cases} \tag{3}$$

for all $\tilde{\alpha} = (\alpha_1, \ldots, \alpha_n) \in F_q^n$ and $\tilde{\alpha} \neq (0, \ldots, 0)$ gives N_0.

Proof. Suppose $\tilde{\beta} = (\beta_1, \ldots, \beta_{2n})$ is a solution of (2), so we have $\beta_1\beta_2 + \ldots + \beta_{2n-1}\beta_{2n} = b$. Since we have $b \neq 0$, then $(\beta_2, \beta_4 \ldots, \beta_{2n}) \neq (0, \ldots, 0)$. It is obvious that $\tilde{\beta}$ is a solution of 3 for $(\alpha_1, \ldots, \alpha_n) = (\beta_2, \beta_4 \ldots, \beta_{2n})$. Now let us proof the opposite: suppose $\tilde{\beta} = (\beta_1, \ldots, \beta_{2n})$ is a solution of system (3) for some $\tilde{\alpha} = (\alpha_1, \ldots, \alpha_n) \in F_q^n$, $\tilde{\alpha} \neq (0, \ldots, 0)$. Putting $\alpha_i = \beta_{2i}$, and $x_{2i-1} = \beta_{2i-1}$in the last equation of (3) we make sure that $\tilde{\beta} = (\beta_1, \ldots, \beta_{2n})$ satisfies equation (2). So the lemma is proved. ■

Hereafter by$\left\{\tilde{\alpha}^1, \tilde{\alpha}^2, \ldots, \tilde{\alpha}^{q^n-1}\right\}$ we denote the set of all vectors $\tilde{\alpha} \in F_q^n$, $\tilde{\alpha} \neq (0, \ldots, 0)$, and for every $\tilde{\alpha}^j = \left(\alpha_1^j, \ldots, \alpha_n^j\right)$, $j = 1, \ldots, q^n - 1$ we denote by $N(\tilde{\alpha}^j)$ the set of solutions of system (3) for $(\alpha_1,\dots,\alpha_n) = \left(\alpha_1^j,\dots,\alpha_n^j\right)$. It is clear that for every $\tilde{\alpha}^j$ the equations in system (3) are linearly independent, so the rank of (3) equals $n+1$. Consequently, every $N(\tilde{\alpha}^j)$represents a coset with dimension $2n-(n+1) = n-1$. It is also obvious that $N(\tilde{\alpha}^i)\cap N(\tilde{\alpha}^j) =$ whenever $i \neq j$. So, we come to a conclusion, that $N_0 = \bigcup_{j=1}^{q^n-1} N(\tilde{\alpha}^j)$ and

$|N_0| = \left|\bigcup_{j=1}^{q^n-1} N(\tilde{\alpha}^j)\right| = q^{n-1}(q^n-1) = q^{2n-1} - q^{n-1}.$

Lemma 2. *If C is a coset inN_0, then* $\dim C \le n-1$.

Proof. Let us denote by B the linear subspace of the vectors $\tilde{x} = (x_1,\dots,x_{2n})$such that $x_{2i} = 0$, $i = 1,\dots,n$. For every vector $\tilde{\alpha}^j = \left(\alpha_1^j,\dots,\alpha_n^j\right)$, $j = 1,\dots,q^n-1$ we construct a vector $b_j \in F_q^{2n}$, such that $b_j = \left(0,\alpha_1^j,0,\alpha_2^j,\dots,0.\alpha_n^j\right)$. Let us denote by $B_j \equiv B + b_j$. Observe that $N(\tilde{\alpha}^j)\subset B_j$ and $N(\tilde{\alpha}^j) = N_0 \cap B_j$. Consider a coset C of linear subspace F such that $C \subseteq N_0$. Let $\dim C = \dim F = k$. One can easily check that $C = \bigcup_{j=1}^{q^n-1}(C\cap N(\tilde{\alpha}^j)) = \bigcup_{j=1}^{q^n-1}(C\cap B_j)$ (since $C\cap N(\tilde{\alpha}^j) = C\cap B_j$). On the other hand every non-empty $C\cap B_j$ is a coset of linear subspace $B\cap F$. Indeed, if we have $z\in C\cap B_j$. It follows that $C = z+F$ and $B_j = z + B$ therefore $C\cap B_j = z + (F\cap B)$. So, we can state that $|C\cap B_j| = |F\cap B| = q^p$for any $j = 1,\dots,q^n-1$, such that $C\cap B_j \neq$, and the number of non-empty $C\cap B_j$ is q^{k-p}. Without loss of generality, let us suppose that these non-empty sets are $C\cap B_1,\dots,C\cap B_{q^{k-p}}$. Let also $\sum_{i=1}^{n}\alpha_i^j x_{2i-1} = b$ indicates the last equation of system (3) related with coset B_j. Now consider the following system of linear equations:

$$\begin{cases} x_{2i} = 0 & i = 1,\dots,n \\ \sum_{i=1}^{n}\alpha_i^j x_{2i-1} = 0 & j = 1,\dots,q^{k-p} \end{cases} \tag{4}$$

It is clear that the set of solutions D of (4) is a linear subspace in B. Let us show that $|C\cap B_j| \le |D|$. If $C\cap B_j =$ there is nothing to show. Suppose $C\cap B_j \neq \emptyset$. As we know $C\cap N(\tilde{\alpha}^j) = C\cap B_j$, and since the odd coordinates of vector $-b_j$ are zeros, the equality $\sum_{i=1}^{n}\alpha_i^j x_{2i-1} = b$ does not change for the coset $-b_j + (C\cap B_j)$. and its vectors satisfy that equation. Easy to see that $-b_j + (C\cap B_j) = d + (B\cap F)$ for some $d\in B$. Therefore, for all the vectors of subspace $B\cap F$ the sum $\sum_{i=1}^{n}\alpha_i^j x_{2i-1}$ is constant, and since $0\in B\cap F$ we come to a conclusion that for all vectors of $B\cap F$, $\sum_{i=1}^{n}\alpha_i^j x_{2i-1} = 0$. Consequently, $B\cap F \subseteq D$. So, $\dim D \ge \dim(B\cap F) = p$. Any equation of (4) in the form $\sum_{i=1}^{n}\alpha_i^j x_{2i-1} = 0$ can not be linearly represented with the equations $x_{2i} = 0$, therefore, besides the first n equations, there also exist at least $k-p+1$ linearly independent equations in (4). Hence, the rank of (4) is not less than $n+k-p+1$. So we get $\dim D \le 2n-(n+k-p+1) = n-k+p-1$. On the other hand $\dim D \ge p$, therefore $k\le n-1$. This was to be proved. ■

Lemma 3. *Let $\lambda_j \in F_q$ $(j = 1,\dots,q^n-1)$ such that $\sum_{j=1}^{q^n-1}\lambda_j = 1$, $\tilde{\alpha}^1,\dots,\tilde{\alpha}^{q^n-1}$are all non-zero $n-$dimensional vectors and $\lambda_1\tilde{\alpha}^1 + \lambda_2\tilde{\alpha}^2 + \dots + \lambda_{q^n-1}\tilde{\alpha}^{q^n-1} \neq (0,0,\dots,0)$, then*

$$\lambda_1 N\left(\tilde{\alpha}^1\right) + \lambda_2 N\left(\tilde{\alpha}^2\right) + \dots + \lambda_{q^n-1} N\left(\tilde{\alpha}^{q^n-1}\right) = N\left(\lambda_1\tilde{\alpha}^1 + \lambda_2\tilde{\alpha}^2 + \dots + \lambda_{q^n-1}\tilde{\alpha}^{q^n-1}\right) \tag{5}$$

Proof. Let $\tilde{\beta}^j = \left(\beta_1^j, \ldots, \beta_{2n}^j\right)$ be a solution of (3) corresponding the vector $\tilde{\alpha}^j$, i.e. $\tilde{\beta}^j \in N\left(\tilde{\alpha}^j\right)$. So, for all $j = 1, \ldots, q^n - 1$ take place

$$\begin{cases} \beta_2^j = \alpha_1^j \\ \ldots \\ \beta_{2n}^j = \alpha_n^j \\ \sum_{i=1}^{n} \alpha_i^j \beta_{2i-1}^j = b \end{cases} \tag{6}$$

Let us multiply left and right sides of all the equations in 6 with λ_j, and sum the first equations of systems (6) over all the values of $j = 1, \ldots, q^n - 1$, then sum the seconds, thirds, and so on. We get

$$\begin{cases} \sum_{j=1}^{q^n-1} \lambda_j \beta_2^j = \sum_{j=1}^{q^n-1} \lambda_j \alpha_1^j \\ \ldots \\ \sum_{j=1}^{q^n-1} \lambda_j \beta_{2n}^j = \sum_{j=1}^{q^n-1} \lambda_j \alpha_n^j \\ \sum_{i=1}^{n} \beta_{2i-1}^j \left(\sum_{j=1}^{q^n-1} \lambda_j \alpha_i^j\right) = \sum_{j=1}^{q^n-1} \lambda_j b = b \end{cases} \tag{7}$$

As we have that $\left(\sum_{j=1}^{q^n-1} \lambda_j \alpha_1^j, \ldots, \sum_{j=1}^{q^n-1} \lambda_j \alpha_n^j\right) \neq (0.0, \ldots, 0)$. so (7) is a system of type (3) corresponding the vector $\tilde{\alpha} = \left(\sum_{j=1}^{q^n-1} \lambda_j \alpha_1^j, \ldots, \sum_{j=1}^{q^n-1} \lambda_j \alpha_n^j\right)$ and vector $\lambda_1 \tilde{\beta}^1 + \ldots + \lambda_{q^n-1} \tilde{\beta}^{q^n-1}$ satisfies it and the lemma is proved.

■

Suppose $C \subseteq N_0$ is a coset of linear subspace F with dimension $n-1$. As we already know $C = \bigcup_{j=1}^{q^{n-1}} \left(C \cap N\left(\tilde{\alpha}^j\right)\right)$, where every non-empty $C \cap N\left(\tilde{\alpha}^j\right) = C \cap B_j$ is a coset of $H = B \cap F$. Let $\dim\left(C \cap N\left(\tilde{\alpha}^j\right)\right) = p$, this means that there are exactly q^{n-1-p} non-empty cosets $C \cap N\left(\tilde{\alpha}^j\right)$. Assume these cosets are $C \cap N\left(\tilde{\alpha}^1\right), \ldots, C \cap N\left(\tilde{\alpha}^{q^{n-1-p}}\right)$, i.e. $C = \bigcup_{j=1}^{q^{n-1-p}} \left(C \cap N\left(\tilde{\alpha}^j\right)\right)$. Let $\varphi_B : F_q^{2n} \to F_q^{2n}/B$ is a canonical homomorphism. It is clear, that as a result of this homomorphism, the images of vectors of C form $(n-1-p)$-dimensional coset. Let $L = \{B_1, \ldots, B_{q^{n-1-p}}\}$ be the coset in F_q^{2n}/B, corresponding the coset C. Obviously, $B \cap N_0 =$, so $B \notin L$. Let $\{B_1, \ldots, B_{n-p}\}$ be a basis in L, i.e. every element in L can be represented as $\lambda_1 B_1 + \ldots + \lambda_{n-p} B_{n-p}$, where $\sum_{i=1}^{n-p} \lambda_i = 1$, $\lambda_i \in F_q$, $i = 1, \ldots, n-p$. Let D be the subspace of solutions of the following system:

$$\begin{cases} x_2 = 0 \\ \ldots \\ x_{2n} = 0 \\ \sum_{i=1}^{n} \alpha_i^j x_{2i-1} = 0, \quad j = 1, \ldots, q^{n-1-p} \end{cases} \tag{8}$$

Observe, that $H \subseteq D$. Suppose $LF(2n-1)$ is the linear space of linear forms over variables $\{x_1, x_3, \dots, x_{2n-1}\}$, then by lemma 2 we have that functions $\left\{\sum_{i=1}^{n} \alpha_i^j x_{2i-1}\right\}$ $j = 1, \dots, q^{n-1-p}$ form a coset in $LF(2n-1)$ and 0 is not in that coset. This means that the rank of system of equations $\sum_{i=1}^{n} \alpha_i^j x_{2i-1} = 0$ $j = 1, \dots, q^{n-1-p}$ is $n-p$. Therefore, the rank of (8) equals to $2n-p$, and $\dim D = p$. On the other hand we have $H \subseteq D$ and $\dim H = p$, so we get $H = D$, this means that the coset L identifies Huniquely. In every coset B_j, $j = 1, \dots, n-p$ let us choose a coset of H, which elements satisfy the equation $\sum_{i=1}^{n} \alpha_i^j x_{2i-1} = b$. Let $C_1, C_2, \dots, C_{n-p}$ be these cosets. Obviously, all cosets in C(of the form $C \cap N(\bar{\alpha}^j)$) are linear combinations of cosets $C_1, C_2, \dots, C_{n-p}$.

Summarizing all the above said, we can state that maximal coset in N_0 is $(n-1)$-dimensional, and every maximal coset in N_0 is constructed in the following way:

1. Choose an $(n-1-p)$-dimensional coset $L = \{B_1, \dots, B_{q^{n-1-p}}\}$ in space F_q^{2n}/B $(B \notin L)$, where B is the linear subspace of the vectors $\bar{x} = (x_1, \dots, x_{2n})$such that $x_{2i} = 0$, $i = 1, \dots, n$ and $0 \le p \le n-1$.
2. If $\{B_1, \dots, B_{n-p}\}$ is a basis in L, then for all $i = 1, \dots, n-p$ we choose cosets C_i in $N_0 \cap B_i$of p-dimensional linear subspace D which is uniquely identified by L.
3. The union $C = \bigcup_{j=1}^{q^{n-1-p}} C_j$is $(n-1)$-dimensional coset in N_0, where $C_1, C_2, \dots, C_{q^{n-1-p}}$ are all the cosets in form $\lambda_1 C_1 + \dots + \lambda_{n-p} C_{n-p}$. $\sum_{i=1}^{n-p} \lambda_i = 1$, $\lambda_i \in F_q$, $i = 1, \dots, n-p$.

From the construction follows that if we have a coset $C \subseteq N_0$ such that $\dim C = k < n-1$, then it is embedded in some $n-1$-dimensional coset in N_0. Indeed, it is enough to mention that any $(k-p)$-dimensional coset $L \subseteq F_q^{2n}/B$, $B \notin L$can be enlarged to $(n-1-p)$-dimensional coset $L' \subseteq F_q^{2n}/B$, such that $B \notin L'$. So we come to a conclusion that every maximal coset in N_0 is $(n-1)$-dimensional. Thus the abbreviated linearised covering of N_0 consists only from $(n-1)$-dimensional cosets in N_0. Let us count the number of various maximal cosets in N_0. In first step of the construction we can choose L in $\begin{bmatrix} n \\ p+1 \end{bmatrix}_q (q^{p+1}-1)$ different ways. Every coset C_i in $N_0 \cap B_i$ in step 2 can be chosen in q^{n-1-p} ways, and all the cosets $C_1, \dots, C_{n-p}$ in $(q^{n-1-p})^{n-p}$ ways. We finally get that the number of all $(n-1)$-dimensional cosets in N_0 is equal to

$$\sum_{p=0}^{n-1} \begin{bmatrix} n \\ p+1 \end{bmatrix}_q \left(q^{p+1}-1\right) q^{(n-p)(n-p-1)}.$$

Now let us count the number of irreducible linearised coverings for the subset N_0.

Lemma 4.

$$t(N_0) \ge q^{(n-1)^2 n q^{n-1}}, \tag{9}$$

where $N_0 \subseteq F_q^{2n}$ is the set of solutions of equation (2) ($b \neq 0$).

Proof. Observe, that $\dim\left(F_q^{2n}/B\right) = n$, and F_q^{2n}/B is isomorphic to F_q^n. Let $\varphi : F_q^{2n}/B \to F_q^n$ implements that isomorphism and let $\varphi(B) = \tilde{\beta}$. There exists a system of linear equations over F_q^n, with rank n, which solution gives the vector $\tilde{\beta}$. Suppose the system is the following:

$$\begin{cases} l_1(\tilde{x}) = 0 \\ l_2(\tilde{x}) = 0 \\ \dots \\ l_n(\tilde{x}) = 0 \end{cases} \tag{10}$$

Let us denote by H_i the coset of solutions of the equation $l_i(\tilde{x}) + 1 = 0$. Obviously, vector $\tilde{\beta}$ does not contained in H_i for any $i = 1, \dots, n$. On the other hand, for every $i = 1, \dots, n$, there exists a vector $\tilde{\alpha}^i$, such that $\tilde{\alpha}^i \in H_i$ and $\tilde{\alpha}^i \notin \bigcup_{\substack{j=1 \\ j \neq i}}^{n} H_j$, otherwise we would have that $l_i(\tilde{x}) + 1$ can be linearly represented by functions $l_1(\tilde{x}) + 1, \dots, l_{i-1}(\tilde{x}) + 1, l_{i+1}(\tilde{x}) + 1, \dots, l_n(\tilde{x}) + 1$which contradicts the fact that the rank of 10 equals n. For a specific $i \in \{1, \dots, n\}$ let us choose as$L$(described in step 1 of constructing $(n-1)$-dimensional cosets in N_0) the coset $\varphi^{-1}(H_i)$. So we have $\dim L = n-1$. This means that the dimension of subspace D is 0. Let $\{B_1, \dots, B_n\}$ be the basis in L. and $B_1 = \varphi^{-1}(\tilde{\alpha}^i)$. For constructing the $(n-1)$-dimensional coset. we need to choose a single element in each $N(\tilde{\alpha}^i) \subset B_i$, $i = 1, \dots, n$. Let us fix a vector $\tilde{\chi} \in N(\tilde{\alpha}^1)$. In every coset $N(\tilde{\alpha}^i)$, $i = 2, \dots, n$ the number of choices of a single vector equals to q^{n-1}. So the number of cosets $C_{\tilde{\chi}}$ containing vector $\tilde{\chi} \in N(\tilde{\alpha}^1)$ equals $(q^{n-1})^{n-1} = q^{(n-1)^2}$. As every $C_{\tilde{\chi}}$ covers its specific vector $\tilde{\chi} \in N(\tilde{\alpha}^1)$, it cannot be absorbed by other $C_{\tilde{\chi}'}$. Thus, for a fixed value $i \in \{1, \dots, n\}$, there are $q^{(n-1)^2 q^{n-1}}$different coverings. As every coset H_i has at least one vector, that is not being covered by other cosets H_j, $j = 1, \dots, n$, $j \neq i$, it is guaranteed that for each $i \in \{1, \dots, n\}$, we will have a coset B_1, which is not contained in L for other i-s. So we get at least $\left(q^{(n-1)^2 q^{n-1}}\right)^n$ different irreducible coverings, and the lemma is proved. ■

Now we can prove the main theorem.

Proof (Main Theorem). Substituting n with $\frac{n}{2}$ in (9) in case of $n \equiv 0(\text{mod}2)$ and with $\frac{n-1}{2}$ if $n \equiv 1(\text{mod}2)$ we get the lower bound for $t(n)$

$$t(n) q^{\left(\lceil\frac{n}{2}\rceil - 1\right)^2 \lceil\frac{n}{2}\rceil q^{\lceil\frac{n}{2}\rceil - 1}}.$$

Obviously, maximal cosets in a linearised covering for any subset Nform an anti-chain. Thus using (1) we have that the number of maximal cosets is not greater than $e^{4/3} q^{(n+1)^2/4}$.

Observe that the maximum number of cosets in any irreducible linearised covering does not exceed q^n, so we can state that

$$t(n) \leq \sum_{k=1}^{q^n} \binom{e^{4/3} q^{(n+1)^2/4}}{k},$$

where $\binom{m}{s}$ is the binomial coefficient C_m^s. Finally we get

$$t(n) \leq q^n \binom{e^{4/3} q^{(n+1)^2 1/4}}{q^n} \leq q^n \left(e^{4/3} q^{(n+1)^2/4}\right)^{q^n} = q^{q^n \frac{(n+1)^2}{4}(1+\varepsilon_n)},$$

where $\lim_{n \to \infty} \varepsilon_n = 0$. The theorem is proved.

References

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Վերջավոր դաշտերի ենթաբազմությունների գծայնացվող փակուղային ծածկույթների քանակի մասին

Հ. Նուրիջանյան

Ամփոփում

Աշխատանքում ներկայացված է 2-ից մեծ բնութագրիչով վերջավոր դաշտի ենթաբազմությունների փակուղային գծայնացվող ծածկույթների քանակի վերին և ստորին սահմանները: 2 բնութագրիչ ունեցող վերջավոր դաշտի դեպքում, այդ վերին և ստորին սահմանները ստացված են Ալեքսանյանի կողմից: