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Maxwell Electrodynamics Subjected to Quantum Vacuum Fluctuations

Ashot S. Gevorkya and Araksya A. Gevorkyan[1]

Institute for Informatics and Automation Problems of NAS of RA.
[1]Yerevan State University
g_ashot@sci.am

Abstract

The propagation of electromagnetic waves in vacuum is considered taking into account quantum fluctuations in the limits of Maxwell-Langevin (ML) equations. For a model of "white noise" fluctuations, using ML equations, the second order partial differential equation is found which describes the quantum distribution of virtual photons in vacuum. It is proved that in order to satisfy observed facts, the Lamb Shift etc, the virtual photons should be quantized in nonperturbed vacuum. For a model of the reverse harmonic quantum oscillator, the quantum distribution of photons is obtained precisely. It is shown, that the quantized virtual photons having negative energies, in toto (approximately 85 percent) are condensed on the energy level absolute value of which is minimal. It is proved that the extension of Maxwell electrodynamics with inclusion of vacuum quantum field fluctuations may be constructed on $6D$ space-time continuum with $2D$ compactified subspace. The problem of propagation of various types electromagnetic waves in vacuum is investigated. Their influence on the refraction index of vacuum is studied.

1. Introduction

It is obvious that in the nearest future all advanced technologies in one or other will be connected with quantum physics, with all of its seeming absurdities.

Although the standard approaches of the quantum mechanics and quantum electrodynamics (QED) describe well many different quantum effects meeting in the nature, nevertheless often unsatisfied desire stay to interpret experiment not only statistically (especially in a case of QED), but also to understand more deeply an essence of the quantum phenomena occurring during a finite times, depending on concrete conditions of experiment. By all appearances on the many difficult questions we can receive more clear answers if we study the phenomena of vacuum quantum fluctuations more sequentially. Let's remind, that the vacuum fluctuations though in the sum have large values of energy, their role in the nature is insufficiently obvious because of their extremely uniform density. Nevertheless, there are certain conditions at which the uniformity of the background electromagnetic zero-point energy is slightly disturbed and leads to physical effects. One is the slight perturbation of the lines seen from transitions between atomic states known as the Lamb Shift [1]. Other phenomenon, the unique attractive quantum force arising between closely-spaced metal plates is named Casimir Effect [2]. Also it is shown that an extract of electrical energy from the vacuum is possible at least in principle [3]. As is well-known, up to now all attempts to unite gravity with the other forces (electromagnetic, strong and weak nuclear forces) in the limits of unified theory haven't led to success. To explain this problem some of researchers assume that gravitation is not a fundamental interaction, but rather a secondary or residual effect associated with other (non-gravitational) fields [4]. In particular, it has been assumed that gravity might be an induced effect brought about by changes in the zero-point energy of the vacuum, due to the presence of mass.

It is important to note, that all of aforementioned effects and many others too, are being received in the limits of some type of classical electrodynamics which is accepted to call a stochastic electrodynamics (SED). Recall that SED is a term for a collection of many different approaches where the fluctuations enter as one of the postulates on the homogeneous boundary conditions of Maxwell's equations [5, 6, 7, 8, 9, 10].

In this article a new representation is developed for investigation of dynamical and statistical properties of quantum vacuum under the influence of external electromagnetic field. In contrast to the SED, in considered case the quantum vacuum fluctuations are being introduced in the Maxwell's equations. In other words we postulates a system of stochastic differential equations (SDE), later named Maxwell-Langevin SDEs. This radically changes the logic of building theory and allows finding the equation for quantum distribution of vacuum depending from the $2D$ equilibrium vacuum's coordinates, and parametrically from the $4D$ space-time coordinates. Finally, using the quantum distribution function the macroscopic parameters, type of refraction indexes of vacuum are constructed.

2. Formulation of the problem. Stochastic equations of fields

As well-known, the system of Maxwell equations in empty space has a following kind:

$$\frac{1}{c}\frac{\partial \mathbf{h}_0}{\partial t} + \operatorname{rot}\mathbf{e}_0 = 0, \qquad \operatorname{div}\mathbf{h}_0 = 0,$$
$$\frac{1}{c}\frac{\partial \mathbf{e}_0}{\partial t} - \operatorname{rot}\mathbf{h}_0 = 0, \qquad \operatorname{div}\mathbf{e}_0 = 0, \tag{1}$$

where vectors $\mathbf{e}_0(\mathbf{r},t)$ and $\mathbf{h}_0(\mathbf{r},t)$ describes electrical and magnetic fields correspondingly.

Maxwell's electrodynamics expansion, taking into account the quantum fluctuations of vacuum, can be presented in particular by the following model of stochastic differential equations (Maxwell-Langevin SDEs):

$$\frac{1}{c}\frac{\partial \mathbf{b}}{\partial t} = \boldsymbol{\eta}_b(\mathbf{r},t) - \operatorname{rot}\mathbf{d}, \qquad \operatorname{div}\mathbf{b} = \varrho_b(\mathbf{r},t),$$
$$\frac{1}{c}\frac{\partial \mathbf{d}}{\partial t} = \boldsymbol{\eta}_d(\mathbf{r},t) + \operatorname{rot}\mathbf{b}, \qquad \operatorname{div}\mathbf{d} = \varrho_d(\mathbf{r},t), \tag{2}$$

where $\boldsymbol{\eta}_b(\mathbf{r},t)$ and $\boldsymbol{\eta}_d(\mathbf{r},t)$ the fluctuating fields are characterize the different type of processes in $4D$ Minkowskii space-time $(\mathbf{r},t)$, which are connected with the random changes of fields and the charged currents, $\varrho_b(\mathbf{r},t)$ and $\varrho_d(\mathbf{r},t)$ random densities of magnetic and electric charges correspondingly.

For the free (without charged particles and fields) or nonperturbed vacuum, from Eq.s (2) obviously may be found the following Langevin type SDEs:

$$\frac{1}{c}\frac{\partial \boldsymbol{\theta}_b}{\partial t} + \operatorname{rot}\boldsymbol{\theta}_d = \boldsymbol{\eta}_b(\mathbf{r},t), \qquad \operatorname{div}\boldsymbol{\theta}_b = \varrho_b(\mathbf{r},t),$$

$$\frac{1}{c}\frac{\partial \boldsymbol{\theta}_d}{\partial t} - \operatorname{rot}\boldsymbol{\theta}_b = \boldsymbol{\eta}_d(\mathbf{r},t), \qquad \operatorname{div}\boldsymbol{\theta}_d = \varrho_d(\mathbf{r},t), \tag{3}$$

where $\boldsymbol{\theta}_b$ and $\boldsymbol{\theta}_b$ describes stochastic vacuum fields. Further we will discuss topological various fields, as a scalar fields also and whirling fields.

The solutions of SDEs (2) in the case of presence of external electromagnetic field it is useful to represent in the following kinds:

$$\mathbf{b}(\mathbf{r},t:\{\xi_b\}) = \mathbf{h}_0(\mathbf{r},t)[1+\xi_b(\mathbf{r},t)], \qquad \mathbf{d}(\mathbf{r},t:\{\xi_d\}) = \mathbf{e}_0(\mathbf{r},t)[1+\xi_d(\mathbf{r},t)], \tag{4}$$

where $\xi_b(\mathbf{r},t)$ and $\xi_d(\mathbf{r},t)$ are stochastic topologically scalar fields.

Substituting (4) into (2) with taking into account (1) we can find the following system of SDE:

$$\begin{aligned}
\frac{\mathbf{h}_0}{c}\frac{\partial \xi_b}{\partial t} &= (\xi_b-\xi_d)\operatorname{rot}\mathbf{e}_0 - \boldsymbol{\nabla}\xi_d\times\mathbf{e}_0 + \boldsymbol{\eta}_b(\mathbf{r},t),\\
\frac{\mathbf{e}_0}{c}\frac{\partial \xi_d}{\partial t} &= (\xi_b-\xi_d)\operatorname{rot}\mathbf{h}_0 + \boldsymbol{\nabla}\xi_b\times\mathbf{h}_0 + \boldsymbol{\eta}_d(\mathbf{r},t),\\
\mathbf{h}_0\cdot\boldsymbol{\nabla}\xi_b &= \varrho_b(\mathbf{r},t),\\
\mathbf{e}_0\cdot\boldsymbol{\nabla}\xi_d &= \varrho_d(\mathbf{r},t).
\end{aligned} \tag{5}$$

Now multiplying the first equation in (5) on the field, $\mathbf{h}_0$ and correspondingly the second one on the field, $\mathbf{e}_0$ we can receive:

$$\begin{aligned}
\frac{\partial \xi_b}{\partial t} &= A_b(\mathbf{r},t)\,(\xi_b-\xi_d) + B_b(\mathbf{r},t)\frac{\partial \xi_d}{\partial r} + \bar{\eta}_b(\mathbf{r},t),\\
\frac{\partial \xi_d}{\partial t} &= A_d(\mathbf{r},t)\,(\xi_b-\xi_d) + B_d(\mathbf{r},t)\frac{\partial \xi_b}{\partial r} + \bar{\eta}_d(\mathbf{r},t),
\end{aligned} \tag{6}$$

where $(\xi_b,\xi_d)\in(-\infty,+\infty)$ we will name the equilibrium vacuum's coordinates.
In Eq.s (6) the following notation are made:

$$\begin{aligned}
A_b(\mathbf{r},t) &= \frac{c}{h_0^2}(\mathbf{h}_0\cdot\operatorname{rot}\mathbf{e}_0) = -\frac{\partial \ln h_0^2}{\partial t}, \qquad A_d(\mathbf{r},t) = \frac{c}{h_0^2}(\mathbf{e}_0\cdot\operatorname{rot}\mathbf{h}_0) = \frac{\partial \ln e_0^2}{\partial t},\\
B_b(\mathbf{r},t) &= \frac{c}{h_0^2}\cdot\mathbf{h}_0(\mathbf{e}_0\times\mathbf{n}), \qquad B_d(\mathbf{r},t) = -\frac{c}{e_0^2}\cdot\mathbf{e}_0(\mathbf{h}_0\times\mathbf{n}).
\end{aligned} \tag{7}$$

in addition:

$$\mathbf{h}_0\boldsymbol{\eta}_b = h_0\eta_b = (h_0^2/c)\bar{\eta}_b, \qquad \mathbf{e}_0\boldsymbol{\eta}_d = e_0\eta_d = (e_0^2/c)\bar{\eta}_d, \qquad \mathbf{n} = \mathbf{r}/r.$$

At projection of the first equation in (5) on the field, $\mathbf{e}_0$ and correspondingly of the second one on the field, $\mathbf{h}_0$, may be received the following SDEs:

$$\mathbf{e}_0(\mathbf{e}_0\times\mathbf{n})\frac{\partial \xi_d}{\partial r} = -\mathbf{e}_0\boldsymbol{\eta}_b(\mathbf{r},t), \qquad \mathbf{h}_0(\mathbf{h}_0\times\mathbf{n})\frac{\partial \xi_b}{\partial r} = \mathbf{h}_0\boldsymbol{\eta}_d(\mathbf{r},t). \tag{8}$$

From equations (8) in particular follows, that when $\mathbf{e}_0 \parallel \mathbf{n}$ the random field, $\boldsymbol{\eta}_b(\mathbf{r},t)\perp\mathbf{e}_0$ (or along a $\mathbf{h}_0$ is directed) and correspondingly when $\mathbf{h}_0\parallel\mathbf{n}$ in this case the random field, $\boldsymbol{\eta}_d(\mathbf{r},t))\perp\mathbf{h}_0$ (or along a $\mathbf{e}_0$ is directed).

The third and fourth equations in (5) correspondingly may be transformed and written in the following kinds:

$$(\mathbf{h}_0\cdot\mathbf{n})\frac{\partial\xi_b}{\partial r}=\varrho_b(\mathbf{r},t),\qquad(\mathbf{e}_0\cdot\mathbf{n})\frac{\partial\xi_d}{\partial r}=\varrho_d(\mathbf{r},t).\tag{9}$$

The equations (9) may be represented in the kind:

$$\begin{aligned}\frac{\partial\xi_b}{\partial r}&=\zeta_b(\mathbf{r},t),\qquad\zeta_b(\mathbf{r},t)-\frac{\varrho_b(\mathbf{r},t)}{\mathbf{h}_0\cdot\mathbf{n}},\\ \frac{\partial\xi_d}{\partial r}&=\zeta_d(\mathbf{r},t),\qquad\zeta_d(\mathbf{r},t)=\frac{\varrho_d(\mathbf{r},t)}{\mathbf{e}_0\cdot\mathbf{n}},\end{aligned}\tag{10}$$

where $\zeta_b(\mathbf{r},t)$ and $\zeta_d(\mathbf{r},t)$ it can be interpreted as fluctuating fields of free vacuum, or in other words the mentioned fields do not depend on an external field.

Now taking into account Eq.s (10) the system of equations (6) we can write in the kind:

$$\begin{aligned}\frac{\partial\xi_b}{\partial t}&=A_b(\mathbf{r},t)\,(\xi_b-\xi_d)+f_b(\mathbf{r},t),\\ \frac{\partial\xi_d}{\partial t}&=A_d(\mathbf{r},t)\,(\xi_b-\xi_d)+f_d(\mathbf{r},t),\end{aligned}\tag{11}$$

where the following notations are made:

$$\begin{aligned}f_b(\mathbf{r},t)&=\bar{\eta}_b(\mathbf{r},t)+\frac{c}{e_0^2}\cdot\mathbf{e}_0(\mathbf{h}_0\times\mathbf{r})\,\zeta_b(\mathbf{r},t),\\ f_d(\mathbf{r},t)&=\bar{\eta}_d(\mathbf{r},t)+\frac{c}{h_0^2}\cdot\mathbf{h}_0(\mathbf{e}_0\times\mathbf{r})\,\zeta_d(\mathbf{r},t).\end{aligned}\tag{12}$$

Related with the stochastic fields, for simplicity we will supposes that they satisfy to following conditions of correlation:

$$\begin{aligned}\langle f_x(\mathbf{r},t)\rangle&=0,\qquad\langle f_x(\mathbf{r},t)\,f_x(\mathbf{r}',t')\rangle=2\epsilon_x\,\delta(\mathbf{r}-\mathbf{r}')\delta(t-t'),\\ \langle\zeta_x(\mathbf{r},t)\rangle&=0,\qquad\langle\zeta_x(\mathbf{r},t)\,\zeta_x(\mathbf{r}',t')\rangle=2\sigma_x\,\delta(\mathbf{r}-\mathbf{r}')\delta(t-t').\end{aligned}\tag{13}$$

The diffusion constants ϵ_x, σ_x where $x=b,d$, characterize the power of fluctuations in the free or nonperturbed vacuum. Specifically for a quantum noise coming from vacuum fluctuations we understand a stationary Wiener-type source with noise intensity proportional to a "vacuum power" which we write as $\mathcal{P}\equiv\frac{1}{4}\hbar\langle\omega^2\rangle$ and with mean energy $\varepsilon\equiv\frac{1}{2}\hbar\sqrt{\langle\omega^2\rangle}$. Recall that $\hbar$ is the Planck's constant and $\langle\omega^2\rangle$ is the variance of the field frequencies averaged over some appropriate distribution (we assume $\langle\omega\rangle=0$ since ω and $-\omega$ must be considered as independent fluctuations). Note that calculation of $\langle\omega^2\rangle$ for quantum fluctuations is not trivial because vacuum energy density diverges as ω^3 with (assumed) uniform probability distribution denying a simple averaging process unless a physical cutoff at high frequencies exist [2].

Now the important problem consists in receiving the equation for function of distribution of joint probability:

$$P(\xi_b,\xi_d;\mathbf{r},t)=\left\langle\delta(\xi_b(t)-\xi_b^0)\,\delta(\xi_d(t)-\xi_d^0)\right\rangle\Big|_{\{\xi_b^0=\xi_b(0);\,\xi_d^0=\xi_d(0)\}}.$$

Using the system of SDEs (12) it is simple to find [11]:

$$\frac{\partial P}{\partial t} = \Big\{\epsilon_b \frac{\partial^2}{\partial \xi_b^2} + \epsilon_d \frac{\partial^2}{\partial \xi_d^2} - A_b(\mathbf{r},t)\frac{\partial}{\partial \xi_b}(\xi_b - \xi_d) - A_d(\mathbf{r},t)\frac{\partial}{\partial \xi_d}(\xi_b - \xi_d)\Big\} P. \tag{14}$$

Later we will use probability of distribution normalized on unit, which supposes:

$$\bar{P}(\xi_b, \xi_d; \mathbf{r}, t) = C(\mathbf{r},t) P(\xi_b, \xi_d; \mathbf{r}, t), \qquad C^{-1}(\mathbf{r},t) = \int\!\!\int P(\xi_b, \xi_d; \mathbf{r}, t) d\xi_b d\xi_d.$$

Now we can calculate the average values of magnetic and electric fields with taking into account the screening effect of quantum vacuum:

$$\mathbf{b}_0(\mathbf{r},t) = \mu_{vac}(\mathbf{r},t)\,\mathbf{h}_0(\mathbf{r},t), \qquad \mathbf{d}_0(\mathbf{r},t) = \varepsilon_{vac}(\mathbf{r},t)\,\mathbf{e}_0(\mathbf{r},t), \tag{15}$$

where the following designations are made:

$$\mathbf{x}_0(\mathbf{r},t) = \int\!\!\int \mathbf{x}(\mathbf{r}, t : \{\xi_b\}) \bar{P}(\xi_b, \xi_d; \mathbf{r}, t) d\xi_b d\xi_d, \qquad \mathbf{x} = \mathbf{b}, \mathbf{d}, \quad \mathbf{x}_0 = \mathbf{b}_0, \mathbf{d}_0,$$

in addition:

$$\begin{aligned} \mu_{vac}(\mathbf{r},t) &= 1 + \int\!\!\int \xi_b \bar{P}(\xi_b, \xi_d; \mathbf{r}, t) d\xi_b d\xi_d, \\ \varepsilon_{vac}(\mathbf{r},t) &= 1 + \int\!\!\int \xi_d \bar{P}(\xi_b, \xi_d; \mathbf{r}, t) d\xi_b d\xi_d. \end{aligned} \tag{16}$$

Thus, we received expressions for the vacuum's refraction indexes $\varepsilon_{vac}(\mathbf{r},t)$ and $\mu_{vac}(\mathbf{r},t)$ at presence of external electromagnetic field.

3. The statistical properties of nonperturbed quantum vacuum

The equation for quantum distribution at the absence of an external fields, when the vacuum is nonperturbed, may be found from the Eq. (14) easily:

$$\frac{\partial P^{(f)}}{\partial t} = \Big\{\epsilon_b \frac{\partial^2}{\partial \xi_b^2} + \epsilon_b \frac{\partial^2}{\partial \xi_d^2}\Big\} P^{(f)}, \tag{17}$$

where $P^{(f)}(\xi_b, \xi_d; t)$ describes the distribution function in the free quantum vacuum. Later we will suppose that along the both coordinates ξ_b and ξ_d, the diffusion constants are equal $\epsilon_b = \epsilon_d = \epsilon$.

The general solution of Eq. (17) may be found easily, it has the following form:

$$P^{(f)}(\xi_b, \xi_d; t) = \frac{1}{4\pi t\sqrt{\epsilon}} \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} \breve{P}(\theta_b, \theta_d) \exp\Big\{-\frac{(\xi_b - \theta_b)^2 + (\xi_d - \theta_d)^2}{4t\sqrt{\epsilon}}\Big\} d\theta_b d\theta_d, \tag{18}$$

where the distribution function $P^{(f)}(\xi_b, \xi_d; t)$ satisfies to the initial condition:

$$P^{(f)}(\xi_b, \xi_d; t)\Big|_{t=0} = \breve{P}(\xi_b, \xi_d), \tag{19}$$

which is defined on a two-dimensional space $\Xi^2(\xi_b, \xi_d)$.

Note that the type of this space may be specified correspondingly after the definition of physical vacuum's state.

Now using (16), and (18) we can calculate vacuum's refraction indexes:

$$\mu_{vac} = 1 + \int\int \theta_b \check{P}(\theta_b, \theta_d) d\theta_b d\theta_d, \quad \varepsilon_{vac} = 1 + \int\int \theta_d \check{P}(\theta_b, \theta_d) d\theta_b d\theta_d, \tag{20}$$

where the probability distribution satisfies to the normalization condition:

$$\int\int \check{P}(\theta_b, \theta_d) d\theta_b d\theta_d = 1.$$

At the case when electromagnetic wave in vacuum propagates without resistance or when external field is absent obviously takes place equality:

$$\int\int \xi_b P^{(f)}(\xi_b, \xi_d; t)\, d\xi_b d\xi_d = \int\int \xi_d P^{(f)}(\xi_b, \xi_d; t)\, d\xi_b d\xi_d = 0, \tag{21}$$

whence follows, that the refraction indexes of free vacuum identically are equal to unit $\varepsilon_{vac} = \mu_{vac} \equiv 1$.

We will discuss two diametrally different situation of nonperturbed vacuum:

a) when the virtual photons are not localized (free), and

b) vice versa when virtual photons are localized.

In the case of a) the distribution function of non-localized virtual photons in can be written in the kind:

$$\check{P}(\xi_b, \xi_d) = \delta(\xi_b - \xi_b^0)\delta(\xi_d - \xi_d^0). \tag{22}$$

For satisfying to a condition (22), it is necessary to put $\xi_b^0 = \xi_d^0 = 0$. However in this case average value of a square of frequency is equal to zero $\langle \omega^2 \rangle = 0$ that contradicts the observing data.

In the case of b) we will assume, that the distribution function is being described by wavefunction of a inverted $2D$ quantum harmonic oscillator:

$$P^{(p)}(\xi_b, \xi_d) = \prod_{x=b,d} \check{P}(\xi_x), \qquad \check{P}(\xi_x) = |\psi(\xi_x)|^2, \tag{23}$$

where wavefunction $\psi(\xi_x)$ satisfy to equation:

$$\Big\{\frac{d^2}{d\xi_x^2} + \frac{1}{\epsilon}\Big(E^{(x)} + \frac{1}{2}\xi_x^2\Big)\Big\}\psi = 0. \tag{24}$$

Note that the equation (24) is being solved exactly:

$$\psi_n(\xi_x) = \Big(\frac{1}{2^n n!}\sqrt{\frac{\epsilon}{\pi}}\Big)^{1/2} \exp\Big(-\frac{1}{2}\epsilon\xi_x^2\Big) H_n(\sqrt{\epsilon}\,\xi_x), \qquad E_n^{(x)} = -\sqrt{\epsilon}(1/2 + n), \tag{25}$$

where $n = 0, 1, 2..,$.

As concerns of the eigenvalues $E_n^{(b)}$ and $E_n^{(d)}$-they may be interpreted as a quantization energies (in the units of $\hbar$) respectively of the virtual "electrical" and "magnetic" photons in nonperturbed quantum vacuum.

Because in considered case, vacuum is quantized, therefore the expression (23) describes the partial distribution. The full quantum distribution function in nonperturbed vacuum may be constructed by the following image:

$$\check{P}^{(f)}(\xi_b, \xi_d) = C_0 \sum_{n,m=0}^{\infty} c_{nm} \check{P}_n(\xi_b) \check{P}_m(\xi_d), \qquad c_{nm} = e^{-(E_n + E_m)/E_{eq}}, \tag{26}$$

where $E_{eq} < 0$ the average energy of virtual photons in equilibrium state of vacuum, coefficients c_{nm} like a microcanonical distribution which describes the occupation of quantum levels in free vacuum. In (26) the constant C_0 may be found from the normalization condition (20):

$$C_0 \sum_{n,m=0}^{\infty} c_{nm} = C_0\Big(\frac{e^{-\gamma}}{1-e^{-2\gamma}}\Big)^2 = 1, \tag{27}$$

where $e^{-\gamma} < 1$ and $\gamma = -\sqrt{\epsilon}/(2E_{eq}) > 0$.

Finally substituting expression (26) into Eq.s (20) we can be sure, that the equations are being satisfied and correspondingly the refraction indexes are equal to unit $\mu_{vac} = \epsilon_{vac} = 1$.

To answer the question, how much is exactly the considered model of a stochastic field it is useful to conduct calculation of average value of the frequency's square of a virtual electromagnetic fields:

$$\langle\omega^2\rangle = \int \breve{P}^{(f)}(\xi_d)\xi_d^2\, d\xi_d = \frac{1}{2}(1-e^{-2\gamma})\big(1+3e^{-2\gamma}+5e^{-4\gamma}+...\big)\epsilon^2. \tag{28}$$

Now if to suppose that $\epsilon = \sqrt{\langle\omega^2\rangle}$, then we can receive the following transcendental equation for definition of unknown parameter γ:

$$1 = \frac{1}{2}(1-e^{-2\gamma})(1+3e^{-2\gamma}+5e^{-4\gamma}+...). \tag{29}$$

Having solved the equation (26) with taking into account (24) we find, that:

$$\gamma \cong 1/2, \tag{30}$$

whence follows, the important equality $E_{eq} \cong -\sqrt{\epsilon} = -\sqrt{\langle\omega^2\rangle}$.

The received result vindicate, that the proposed model of quantum noise is very-well describes the quantum vacuum fluctuations. Let's note that the model of localized photons may be different, however this fact doesn't influence on the importance of the conclusion that the corresponding space $\Xi^2(\xi_b, \xi_d)$ is a compactified $2D$ space.

At last, note that the vacuum has the same statistical properties in compactified space on all directions of real three-dimensional space. It in particular follows from similarity of the equations (10) with the equations (11) when the external field is absent or when it doesn't experience resistance from vacuum.

4. Propagation of electromagnetic waves in vacuum

The solution of Eq. (14) may be represented in the form:

$$P(\xi_b, \xi_d; \mathbf{r}, t) = P^{(p)}(\xi_b, \xi_d; t_0)[1 + \chi(\xi_b, \xi_d; \mathbf{r}, t)], \tag{31}$$

where $\chi(\xi_b, \xi_d; \mathbf{r}, t)$ describes the deformation of initial (nonperturbed) quantum distribution $P^{(p)}(\xi_b, \xi_d; t_0)$. It is necessary to note that the free evolution of quantum vacuum does not change its statistical properties therefore we can put $t_0 = 0$.

Now substituting (31) into Eq. (14) we receive the following equation:

$$\frac{\partial \chi_{nm}}{\partial t} = \Big\{\epsilon\Big(\frac{\partial^2}{\partial \xi_b^2} + \frac{\partial^2}{\partial \xi_d^2}\Big) + 2\epsilon\Big(\frac{\partial \ln \breve{P}_n^{(b)}}{\partial \xi_b}\frac{\partial}{\partial \xi_b} + \frac{\partial \ln \breve{P}_m^{(d)}}{\partial \xi_d}\frac{\partial}{\partial \xi_d}\Big)$$
$$-(\xi_b - \xi_d)\Big(A_b(\mathbf{r},t)\frac{\partial}{\partial \xi_b} + A_d(\mathbf{r},t)\frac{\partial}{\partial \xi_d}\Big)\Big\}\chi_{nm}. \tag{32}$$

For solution of Eq. (32) we will supposes that to take place the following initial and boundary conditions:

$$\chi_{nm}(\xi_b,\xi_d;\mathbf{r},t)|_{t=0}=0, \qquad \chi_{nm}(\xi_b,\xi_d;\mathbf{r},t)|_S=0, \tag{33}$$

where S is a border.

Now we can construct the total distribution of quantum states on the $6D$ space-time:

$$P_{tot}(\xi_b,\xi_d;\mathbf{r},t)=\breve{P}(\xi_b,\xi_d;\mathbf{r},t)=C_0^2\sum_{n,m=0}^{\infty}c_{nm}\breve{P}_{nm}(\xi_b,\xi_d)[1+\chi_{nm}(\xi_b,\xi_d;\mathbf{r},t)]. \tag{34}$$

Using the expressions (16) and (34) for the permittivity and permeability of the quantum vacuum, at presence of external field we receives:

$$\mu_{vac}(\mathbf{r},t)=1+N_0^{-1}(\mathbf{r},t)\sum_{n,m=0}^{\infty}c_{nm}\int\int\xi_b\breve{P}_{nm}(\xi_b,\xi_d)\chi_{nm}(\xi_b,\xi_d|\mathbf{r},t)d\xi_b d\xi_d,$$
$$\varepsilon_{vac}(\mathbf{r},t)=1+N_0^{-1}(\mathbf{r},t)\sum_{n,m=0}^{\infty}c_{nm}\int\int\xi_d\breve{P}_{nm}(\xi_b,\xi_d)\chi_{nm}(\xi_b,\xi_d|\mathbf{r},t)d\xi_b d\xi_d, \tag{35}$$

where $N_0(\mathbf{r},t)=\sum_{n,m=0}^{\infty}c_{nm}\int\int\breve{P}_{nm}(\xi_b,\xi_d)[1+\chi_{nm}(\xi_b,\xi_d;\mathbf{r},t)]d\xi_b d\xi_d$ is the constant of normalization.

Thus, it is obvious that in considered case the vacuum's refraction indexes differ from unit. As shows the analysis of the equations, depending on value and behavior of an external field the specified difference can be essential.

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6. Conclusion

Within the context of quantum field theory the vacuum is the seat of all energetic particle and field fluctuations. In other words vacuum is characterized by physical parameters and structure that constitute an energetic medium which pervades the entire extent of the universe. If the quantum field theory can be accurately described through perturbation, then the properties of the vacuum are analogous to the properties of the of a quantum mechanical harmonic oscillator (or more accurately, the ground state of a QM problem). We considered for the first time this problem within the limits of the stochastic equations of type ML. It has allowed us to construct the regular theory without application of the perturbation methods. It has allowed us to develop the regular theory for quantum distribution in vacuum without application of the perturbation method. Last circumstance has given us the possibility to investigate the structure and statistics of an electromagnetic component of vacuum in detail. In particular, it is shown that the accounting of quantum vacuum in the schema of Maxwell's electrodynamics is described with the two additional measures which are compactified. For quantum distribution of vacuum under the influence of an external field the equation Fokker-Plank type is received and the refraction indexes of vacuum are constructed. It is shown that they can change under the influence of external fields.

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Մաքսվելի էլեկտրոդինամիկական հաշվի առնված քվանտային Մաքսվելիվակումի ֆլուկտուացիաները

Ա. Գևորգյան և Ար. Գևորգյան

Ամփոփում

Քվանտային դաշտի տեսության շրջանակներում վակումը իրենից ներկայացնում է տեղ, որտեղ առկա են էներգետիկ ամեն տեսակ մասնիկներ և դաշտերի քաջատումներ: Այլ խոսքերով, վակումը բնութագրվում է ֆիզիկական չափերով և կառուցվածքով, որը կազմում է բացասական էներգետիկ միջավայրը, որը տեզերքում թափանցում է ամենուրեք: Եթե քվանտային դաշտի տեսությունը խոտորումների միջոցով կարողանար ճշգրիտ նկարագրեր երևույթները, ապա վակումի հատկությունները նման կլինեին քվանտային ներդաշնակ տատանակի հատկություններին: Մենք առաջին անգամ այս խնդիրը դիտարկել ենք Մաքսվել–Լանժևեն տիպի պատահական դիֆերենցիալ հավասարումների շրջանակներում: Դա մեզ հնարավորություն է տվել վակումում՝ քվանտային բաշխման համար զարգացնել կանոնավոր տեսություն առանց օգտագործելու խոտորումների մեթոդները: Վերջին հանգամանքը հնարավորություն է տվել մանրամասնորեն հետազոտել վակումի էլեկտրոմագնիսական բաղադրի վիճակագրությունը և կառուցվածքը: Մասնավորապես ցույց է տրված, որ քվանտային վակումի ներառումը մաքսվելի էլեկտրոդինամիկայի ուրվագծում նկարագրվում է երկու լրացուցիչ չափերով, որոնք կոմպակտֆիկացված են: Արտաքին դաշտում գտնվող վակումի քվանտային բաշխման համար ստացված է Ֆոկկեր–Պլանկի տիպի հավասարում և կառուցված են վակումի բեկման ցուցիչները: Ցույց է տրված, որ նրանք կարող են փոխվել արտաքին դաշտի ազդեցության տակ: