Mathematical Problems of Computer Science 31, 142–149, 2008.

New Mathematical Approach for Investigation of Statistical Properties of Random Environment of $1D$ Quantum N-Particles System In External Field

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Abstract

The investigation of $1D$ quantum N-particles system (PS) with relaxation in the random environment under the influence of external field is conducted within the limits of the stochastic differential equation (SDE) of Langevin-Schrödinger (L-Sch) type. Using L-Sch equation the $2D$ second order non-stationary partial differential equation is found, which describes the quantum distribution in the environment, depending on energy of nonperturbed $1D$ quantum N-PS and on the external field's parameters. It is shown that the average value of interaction potential between $1D$ disordered quantum N-PS and on the external field, has the *ultraviolet divergence*. This problem is solved by renormalization of equation for the function of quantum distribution. It is shown that it has a sense of dimensional renormalization which is characteristic for the quantum field theory. Critical properties of environment are investigated in detail. The possibility of first-order phase transition in environment depending on amplitude of an external field is shown.

1 Introduction

In this work the problem of $1D$ disordered quantum N-PS under the influence of external field will be discussed. In particular the main attention will be focused on the problems of investigation of the statistic properties of environment by nonperturbed method. Note, that the mentioned task appears in the different complex problems of physics, chemistry, biology and financial mathematics [1]. For investigation of large spectrum of disordered as well as stochastic problems for the first time by author new mathematical approach was proposed [2]. The idea consists of that, that the SDE of type L-Sch is used for receiving the integro-differential equation for joint conditional probability which describes the quantum distribution in environment and occurrence of structures. As was shown the integro-differential equation may be transformed to the partial differential equation of second order if the stochastic processes satisfy the white noise's condition (zero mean value and δ-shaped correlation function).

In the present work using the coordinate along the N-particles chain as a natural parameter (*timing parameter*) the initial random complex Schrödinger equation to L-Sch nonlinear

SDE is reduced. On the basis of this the all necessary investigations for constructing the statistical properties of environment are conducted.

2 Quantum state of an environment of $1D$ N-PC in external field

Let's discuss the problem of relaxation of $1D$ disordered N-particles system in the random environment at interaction with the external field. It is possible to show, that this problem is equivalent to a problem $1D$ N-PS where the interaction between any pair of particles has a random complex value. The quantum state of the $1D$ N-PS can be described within the limits of Schrödinger stochastic differential equation (see for example [3]):

$$\lambda\,\delta U(\varepsilon|x,\mathrm{g}) = \lambda\varepsilon + \Psi^{-1}d_t^2\Psi, \qquad t = x/d_0 \sim N_x, \quad d_t^2 \equiv (d/dt)^2 \equiv d^2/dt^2, \tag{1}$$

where $\mu = m_0/N^{1/(N-1)}$ is an effective mass of D $N >> 1$ particles system, d_0-is an average distance between two particles in the particles-chain, x-is the coordinate along the length of $1D$ particles-chain (PC) and plays a role of the natural parameter (*timing parameter*). ε-is an energy of $1D$ PC, in addition the designation $\lambda = 2\mu d_0^2/\hbar^2$ is made. In the equation (1) the interaction potential $-\delta U(\varepsilon|x,\mathbf{g})$ the random complex function is supposed.

Substituting: $\Psi(t) = \exp\Big(\int_0^t \Xi(t')dt'\Big)$, into (1) it is easy to find the following nonlinear complex stochastic differential equation (SDE) of type Langevin-Schrödinger equation [4]:

$$\Xi_t + \Xi^2 + \lambda(\varepsilon - V) + \lambda f(t) = 0, \qquad \Xi_t \equiv d\Xi/dt, \tag{2}$$

where $\Xi(t)$ describes some complex field: $\Xi(t) = \theta(t) + i\vartheta(t)$, where $\vartheta(t) \geq 0$.

Recall that in (2) the symbol V designates mean value of interaction potential between $1D$ PC and external field and $f(t)$ correspondingly its random part. For definiteness these values in general case we can present in a following kind:

$$V(E) = V^r(E) + iV^i(E).$$

where E amplitude of an external field. As to a random part of function $f(t)$ is supposed, that:

$$f(t) = f^r(t) + if^i(t),$$

where functions $f^r(t)$ and $f^i(t)$ satisfies to the following relations of correlation [5]:

$$\langle f^r(t)f^r(t')\rangle = 2\bar{D}^r\delta(t-t'), \qquad \langle f^r(t)\rangle = 0, \tag{3}$$

$$\langle f^i(t)f^i(t')\rangle = 2\bar{D}^i\delta(t-t') \qquad \langle f^i(t)\rangle = 0. \tag{4}$$

The complex equation (2) for a field is useful, to represent as a system of two real equations:

$$\dot{\theta} + \theta^2 - \vartheta^2 + \lambda(\varepsilon - V^r + f^r(t)) = 0, \tag{5}$$

$$\dot{\vartheta} + 2\vartheta\theta + \lambda(-V^i + f^i(t)) = 0. \tag{6}$$

For the further investigation it is important to receive the evolution equation for the conditional probability (field's distribution) $\{\theta(t),\vartheta(t)\}$:

$$Q(\theta,\vartheta,t|\theta_0,\vartheta_0,t_0) = \Big\langle \delta(\theta(t)-\theta_0)\delta(\vartheta(t)-\vartheta_0)\Big\rangle\Big|_{\{\theta_0=\theta(t_0);\vartheta_0=\vartheta(t_0)\}}.$$

which describes the probability of trajectory $\{\theta(t), \vartheta(t)\}$ starting at the initial point t_0 from the position (θ_0, ϑ_0), at an arbitrary point t comes into the position (θ, ϑ). Using system of SDE (5)-(6) the Fokker-Plank equation is easily to find [5] (see also [4]):

$$\partial_t Q = \hat{L}(\varepsilon, \mathbf{g}|\theta, \vartheta)\, Q, \qquad \partial_t \equiv \partial/\partial t, \tag{7}$$

where $\hat{L}(\varepsilon, \mathbf{g}|\theta, \vartheta) = D^r\partial_\theta^2 + D^i\partial_\vartheta^2 + \partial_\theta[\theta^2 - \vartheta^2 + \lambda(\varepsilon - V^r)] + \partial_\vartheta[2\theta\vartheta - \lambda V^i)]$, $Q \equiv Q(\varepsilon, \mathbf{g}|\theta, \vartheta; t)$, in addition $\partial_x \equiv \partial/\partial x$ and $\partial_x^2 \equiv (\partial/\partial x)^2$, further $D^r = \lambda\bar{D}^r$ and $D^i = \lambda\bar{D}^i$. Note, that the solution of equation (7) satisfies the initial condition:

$$Q(\varepsilon, \mathbf{g}|\theta, \vartheta; t)\Big|_{t=t_0} = \delta(\theta - \theta_0)\delta(\vartheta - \vartheta_0), \tag{8}$$

where the initial phases θ_0 and ϑ_0 are equal to zero. Nevertheless, from the physical point of view the stationary limit of the equation (7) is more interesting. In this case equation (7) is transformed to the following stationary form:

$$\hat{L}(\varepsilon, \mathbf{g}|\theta, \vartheta)\, Q_s = 0, \tag{9}$$

where $Q_s \equiv Q_s(\varepsilon, \mathbf{g}|\theta, \vartheta) = \lim_{t\to t_N} Q(\varepsilon, \mathbf{g}|\theta, \vartheta, t)$ describes the distribution function of conditional probability on the plane of equilibrium field coordinates (θ, ϑ).

For solution of equation (9) it is useful to go over to the system of polar coordinates:

$$\theta = \varrho\cos\varphi, \quad \vartheta = \varrho\sin\varphi, \qquad 0 \le \varrho < +\infty, \quad \varphi \in [0, \pi]. \tag{10}$$

Using the expressions (10) it is possible to make coordinates transformation $(\theta, \vartheta) \to (\varrho, \varphi)$ in equation (9):

$$\tilde{\hat{L}}(\varepsilon, \mathbf{g}|\varrho, \varphi)\, \tilde{Q}_s = 0, \qquad \tilde{Q}_s(\varepsilon, \mathbf{g}|\varrho, \varphi) \equiv Q_s(\varepsilon, \mathbf{g}|\theta, \vartheta), \tag{11}$$

where $\tilde{\hat{L}}(\varepsilon, \mathbf{g}|\varrho, \varphi) = \hat{A}(\varepsilon, \mathbf{g}|\varrho, \varphi)\partial_\varrho^2 + \hat{B}(\varepsilon, \mathbf{g}|\varrho, \varphi)\partial_\varrho + \hat{C}(\varepsilon, \mathbf{g}|\varrho, \varphi)$, in addition:

$$\hat{A}(\varepsilon, \mathbf{g}|\varrho, \varphi) = (D^r\cos^2\varphi + D^i\sin^2\varphi), \quad \hat{B}(\varepsilon, \mathbf{g}|\varrho, \varphi) = -\Big[\varrho^{-1}(D^r\sin^2\varphi + D^i\cos^2\varphi)$$

$$+\varrho^{-1}(D^r - D^i)\sin 2\varphi\partial_\varphi + (\varrho^2\cos 2\varphi + \lambda(\varepsilon - V^r))\cos\varphi + (\varrho^2\sin 2\varphi - \lambda V^i)\sin\varphi\Big],$$

$$\hat{C}(\varepsilon, \mathbf{g}|\varrho, \varphi) = \varrho^{-2}(D^r\sin^2\varphi + D^i\cos^2\varphi)\partial_\varphi^2 + \varrho^{-1}\Big[(2\varrho)^{-1}(D^r - D^i)\sin 2\varphi - (\varrho^2\cos 2\varphi$$

$$+\lambda(\varepsilon - V^r))\sin\varphi + (\varrho^2\sin 2\varphi - \lambda V^i)\cos\varphi\Big]\partial_\varphi + 4\varrho\cos\varphi.$$

The solution of equation (11) is useful to present in the form of decomposition by the polynomials:

$$\tilde{Q}_s(\varepsilon, \mathbf{g}|\varrho, \varphi) = \sum_{\nu=0}^{\infty} G_\nu(\varepsilon, \mathbf{g}|\varrho)P_\nu(\cos\varphi), \tag{12}$$

where $P_\nu(\cos\varphi)$ is a usual Legendre polynomial.

Substituting (12) into the (11) and using the orthogonal behavior of Legendre polynomial is possible to receive the following system of coupled ordinary differential equations:

$$\Big\{A_{\nu\nu'}d_\varrho^2 + B_{\nu\nu'}(\varrho)d_\varrho + C_{\nu\nu'}(\varrho)\Big\}G_{\nu'}(\varepsilon, \mathbf{g}|\varrho) = 0, \tag{13}$$

where matrixes $A_{\nu\nu'}$, $B_{\nu\nu'}(\varrho)$ and $C_{\nu\nu'}(\varrho)$ is being defined by formula: $X_{\nu\nu'}(\varrho) = \int_{-1}^{1} P_{\nu'}(\cos\varphi)\hat{X}(\varepsilon, \mathbf{g}|\varrho,\varphi)P_\nu(\cos\varphi)\,d\cos\varphi$. Now we will pass to definition of conditions necessary for the solution of system of the ordinary differential equations (13). Thereupon. it is natural to assume, that all solutions $G_\nu(\varepsilon,\mathbf{g}|\varrho)$ and their derivatives $\partial G_\nu(\varepsilon,\mathbf{g}|\varrho)/\partial\varrho$ at the limit $\varrho \to \infty$ should aspires to zero. Let's now make the following transformation of coordinate $\varrho \to \xi = \varrho^{-1}$ in the equation (13):

$$\left\{\bar{A}_{\nu\nu'}d_\xi^2 + \bar{B}_{\nu\nu'}(\xi)d_\xi + \bar{C}_{\nu\nu'}(\xi)\right\}\bar{G}_{\nu'}(\varepsilon,\mathbf{g}|\xi) = 0, \tag{14}$$

where $\bar{G}_\nu(\varepsilon,\mathbf{g}|\xi) \equiv G_\nu(\varepsilon,\mathbf{g}|\varrho)$, in addition: $\bar{A}_{\nu\nu'} = A_{\nu\nu'}$, $\bar{B}_{\nu\nu'}(\xi) = 2A_{\nu\nu'}(\varrho)\varrho - B_{\nu\nu'}(\varrho)\varrho^2$, $\bar{C}_{\nu\nu'}(\xi) = C_{\nu\nu'}(\varrho)\varrho^4$. Now we can be convinced, that the equations' system (14) must satisfy of the initial conditions:

$$\bar{G}_\nu(\varepsilon,\mathbf{g},\xi)|_{\xi=0} = 0, \qquad d_\xi\bar{G}_\nu(\varepsilon,\mathbf{g}|\xi)|_{\xi=0} = 0, \tag{15}$$

which are very convenient at computation of problem.

3 Average potential between 1D PC and the external field (problem of renormalization)

Now it is possible to start calculation of mean value of interaction potential between 1D PC with the energy ε and the external field with consideration of its relaxation in the environment. Because the function $\delta U(\varepsilon|x,\mathbf{g})$ subject to x, has a behavior which can be interpreted as a ***deterministic chaos***, it is possible to use Birgoff ergodic hypothesis and change integration over x on the integration over the distribution function of conditional probability $\tilde{Q}_s(\varepsilon,\mathbf{g}|\varrho,\varphi)$:

$$\langle\delta U(\varepsilon|x,\mathbf{g})\rangle_x = (\lambda R)^{-1}\int_0^\infty\int_0^\pi \delta\tilde{U}(\varepsilon|\varrho,\varphi)\tilde{Q}_s(\varepsilon,\mathbf{g}|\varrho,\varphi)\varrho d\varrho d\varphi, \tag{16}$$

where $R = \int_0^\pi\int_0^\infty \tilde{Q}_s(\varepsilon,\mathbf{g}|\varrho,\varphi)\,\varrho d\varrho d\varphi$ is a normalization constant, in addition:

$$\delta U(\varepsilon|x,\mathbf{g})\Big|_{x\to\infty} \to \delta\tilde{U}(\varepsilon|\varrho,\varphi) = \lambda\varepsilon + \varrho^2(\cos 2\varphi + i\sin 2\varphi), \tag{17}$$

describes the interaction potential between the external field and the 1D PC in the limit of stationary processes.

Substituting (12) and (17) in the expression (16), it is possible to find:

$$\langle\delta U(\varepsilon|x,\mathbf{g})\rangle_x = \varepsilon + (\lambda R)^{-1}\left[I_1(\varepsilon,\mathbf{g}) + iI_2(\varepsilon,\mathbf{g})\right], \tag{18}$$

where the following designations are made:

$$I_1(\varepsilon,\mathbf{g}) = \sum_{m=0}^{\infty} A_\nu\int_0^\infty G_\nu(\varepsilon,\mathbf{g}|\varrho)\varrho^3 d\varrho, \qquad A_\nu = \int_0^\pi P_\nu(\cos\varphi)\cos 2\varphi\,d\varphi,$$
$$I_2(\varepsilon,\mathbf{g}) = \sum_{m=0}^{\infty} B_\nu\int_0^\infty G_\nu(\varepsilon,\mathbf{g}|\varrho)\varrho^3 d\varrho, \qquad B_\nu = \int_0^\pi P_\nu(\cos\varphi)\sin 2\varphi\,d\varphi,$$
$$R(\varepsilon,\mathbf{g}) = \sum_{m=0}^{\infty} C_\nu\int_0^\infty G_\nu(\varepsilon,\mathbf{g}|\varrho)\varrho\,d\varrho, \qquad C_\nu = \int_0^\pi P_\nu(\cos\varphi)\,d\varphi. \tag{19}$$

Analysis of integrals in expressions I_1 and I_2 show that they diverging integrals.

Let's consider the behavior of solution of equation (11) near the directions $\varphi = 0$ and $\varphi = \pi$. Substituting the expression (12) into the equation (11) for a both angles the following equation may be found:

$$\Big\{D^r d_\varrho^2 \pm [\varrho^2 + \lambda(\varepsilon - V^r)]d_\varrho \pm 4\varrho\Big\}G(\varepsilon, \mathbf{g}|\varrho) = 0, \tag{20}$$

where $G(\varepsilon, \mathbf{g}|\varrho) = \sum_{\nu=0}^{\infty} G_\nu(\varepsilon, \mathbf{g}|\varrho)$, symbol " + " characterize the equation near the angle $\varphi = 0$ and " − " accordingly the equation near the angle $\varphi = \pi$. Recall that at an equation receiving (20) it was assumed that $D^r \gg D^i$, what in truth nonessential for future discussion. The solution of equation (20) on the big distances has the following behavior:

$$G(\varepsilon, \mathbf{g}|\varrho) \sim \varrho^{-4}. \tag{21}$$

Taking into account (21) it is simple to show that the expression of type $I_{\delta\varphi}(\varepsilon, \mathbf{g}) = \delta\varphi \cdot \int_0^\infty G(\varepsilon, \mathbf{g}|\varrho)\varrho^3 d\varrho$, where $\delta\varphi$ is a finite increment of angle near the angles (0 and π), is diverging logarithmic. The nature of this type of divergence from field theory is well-known and is being named *ultraviolet divergence* (see for example [6]). Recall that the divergence is connected with the problem of infinite energy of vacuum, which includes the considered model of stochasticity. As will be shown below, this problem can be solved by the way of dimensional renormalization.

Let's consider the extended equation:

$$\Big\{D^r \partial_\varrho^2 \pm [\varrho^2 + \lambda(\varepsilon - V^r)]\partial_\varrho \pm (4+\eta)\varrho\Big\}G(\varepsilon, \mathbf{g}, \eta|\varrho) = 0, \tag{22}$$

where $\eta > 0$ auxiliary small parameter. As shows a simple analysis, at the big distances the solution of equation (25) has a behavior of type $\varrho^{-(4+\eta)}$. The last fact mean that the integral:

$$I(\varepsilon, \mathbf{g}, \eta) = \int_0^\infty G(\varepsilon, \mathbf{g}, \eta|\varrho)\varrho^3 d\varrho < \infty, \tag{23}$$

now will be converged. However it is obviously what $\lim_{\eta\to 0} I(\varepsilon, \mathbf{g}, \eta) \to \infty$. Our task to make dimensional renormalization of integral (23).

Let's make differentiation of equation (22) on the variable η:

$$\Big\{D^r \partial_\varrho^2 \pm [\varrho^2 + \lambda(\varepsilon - V^r)]\partial_\varrho \pm 4\varrho\Big\}G_\eta(\varepsilon, \mathbf{g}, \eta|\varrho) \pm \varrho G(\varepsilon, \mathbf{g}, \eta|\varrho) = 0, \tag{24}$$

where $G_\eta(\varepsilon, \mathbf{g}, \eta|\varrho) = \partial G/\partial\eta$. In the equation (24) the form of function $G(\varepsilon, \mathbf{g}, \eta|\varrho)$ isn't fixed. Representing the function in the kind:

$$G(\varepsilon, \mathbf{g}, \eta|\varrho) = \varrho^\eta Y(\varepsilon, \mathbf{g}, |\varrho),$$

from (24) in the limit of $\eta \to 0$ the following equation can be found:

$$\Big\{D^r d_\varrho^2 \pm [\varrho^2 + \lambda(\varepsilon - V^r)]d_\varrho \pm 5\varrho\Big\}Y(\varepsilon, \mathbf{g}, |\varrho) = 0. \tag{25}$$

The solution of this equation on a big distances has the following behavior $Y(\varepsilon, \mathbf{g}|\varrho) \sim \varrho^{-5}$, what mean that the integral $\lim_{\eta\to 0} I(\varepsilon, \mathbf{g}, \eta) = \lim_{\eta\to 0} \int_0^\infty G(\varepsilon, \mathbf{g}, \eta|\varrho)\varrho^3 d\varrho =$

$\int_0^\infty Y(\varepsilon,\mathbf{g}|\varrho)\varrho^3 d\varrho < \infty$ is strongly converged. With the similar manner may be received renormalized form of $2D$ solution:

$$Q_s^r(\varepsilon,\mathbf{g}|\varrho,\varphi) = \sum_{\nu=0}^{\infty} Y_\nu(\varepsilon,\mathbf{g}|\varrho)\, P_\nu(\cos\varphi),$$

where $Q_s^r(\varepsilon,\mathbf{g}|\varrho,\varphi)$ is a renormalized quantum distribution function. Recall that the function of $Y_\nu(\varepsilon,\mathbf{g}|\varrho)$ is a solution of equation (14), where the following replacement only is made $C^*_{\nu\nu'}(\varrho) = C_{\nu\nu'}(\varrho) + \varrho\int_{-1}^{1} P_{\nu'}(x)P_\nu(x)x,dx$, which after a simple computation may be represented by the form: $C^*_{\nu\nu'}(\varrho) = C_{\nu\nu'}(\varrho) + 2\varrho\Big\{\frac{\nu'+1}{(2\nu'-1)^2}\,\delta_{(\nu'+1)\nu} + \frac{\nu'}{(4\nu'^2-1)}\,\delta_{(\nu'-1)\nu}\Big\}$. Thus, for the right calculation of mean value of $\langle\delta(\varepsilon|x,\mathbf{g})\rangle_x$ in the integrals of expressions (19) is necessary in to make replacement $G_\nu(\varepsilon,\mathbf{g}|\varrho) \to Y_\nu(\varepsilon,\mathbf{g}|\varrho)$.

Now it is important to investigate the critical properties of $1D$ PC environment's in the external field. For it, let's introduce Helmholtz's free energy which in the present case can be represented by the following form:

$$F(\varepsilon,\mathbf{g}) = \varepsilon \ln R(\varepsilon,\mathbf{g}), \tag{26}$$

where it is supposed that ε is the most probable energy of $1D$ PC when ensemble of $1D$ PC in equilibrium state, in addition $R(\varepsilon,\mathbf{g})$ describes its partition function (*number of states with the energy in the range from* $-\infty$ *to* $(\varepsilon - V^R)$*, per unit interval*). Note that all thermodynamic properties of the statistical system (environment) in this case may be obtained by means of derivative of free energy by an external field's parameters $\mathbf{g}$. Let's consider the derivation of free energy (26):

$$q(\varepsilon,V^r,\mathbf{g}) = \partial_{V^r}F(\varepsilon,\mathbf{g}) = (\varepsilon/R)\partial_{V^r}R, \tag{27}$$

where value V^r from parameters of the external field's (E_0,Ω) is constructed (see expression (19)) and has a sense of an average energy of external field which falls on $1D$ PC. Analysis of expression (27) in the general case is difficult problem, however in more interesting case when $D^r >> D^i$ quantum probability of processes are concentrated near the directions $\varphi = 0$ and $\varphi = \pi$. In other words, for analysis may be used the $1D$ model. Recall that $1D$ model is well investigated in (see [3]). In particular it was shown that:

$$R_1^{-1}(\varepsilon,V^r,\mathbf{g}) = \int_0^\infty \exp\Big\{-\frac{\varrho^3}{12} - \lambda_0\chi\varrho\Big\}\varrho^{\,1/2}d\varrho, \quad \chi = 1 - \frac{V^r}{\varepsilon}, \tag{28}$$

where R_1 is a partition function of $1D$ model, the parameter $\lambda_0 = \lambda\varepsilon/(D^r)^{2/3} >> 1$. Using (28) it is possible to calculate the derivative of the free energy:

$$q_1(\varepsilon,V^r,\mathbf{g}) = -\lambda_0 R_1 \int_0^\infty \exp\Big\{-\frac{\varrho^3}{12} - \lambda_0\chi\varrho\Big\}\varrho^{1/2}d\varrho, \tag{29}$$

where $q_1(\varepsilon,V^r,\mathbf{g})$ derivative of free energy for the $1D$ model.

Let's calculate the derivative of free energy in two closely located points $V_-^r = \varepsilon - \delta V^r$ and $V_+^r = \varepsilon + \delta V^r$, of scale of energy of external field V^r. If to assume, what takes place the relations $\lambda_0 >> \lambda_0\chi \sim 1$, then the integrals in (29) it is possible to calculate asymptotically by Laplace method (see for example [7]). In particular for the value V_-^r, conducting a simple calculation it is possible to find:

$$q_1^-(\varepsilon,V_-^r,\mathbf{g}) \approx (\lambda_0\chi_-)^{-1} + O(\lambda^{-2}), \qquad \chi_- = \delta V^r/\varepsilon > 0.$$

In a similar way the derivative of free energy for the value V_+^{τ} may be calculated:

$$q_1^{+}(\varepsilon, V_+^{\tau}, \mathbf{g}) \approx \frac{2\pi\lambda_0}{\sqrt{-\lambda_0\chi_+}} \exp\Big\{\frac{4}{3}(\lambda_0\chi_+)^{3/2}\Big\} + O(1), \qquad \chi_+ = -\frac{\delta V^{\tau}}{\varepsilon}.$$

Comparing the values $q_1(\varepsilon, V^{\tau}, \mathbf{g})$ in two close points, it is easy to be convinced, that at $V^{\tau} = \varepsilon$ there is an infinite jump (if to assume $\lambda_0 \to \infty$). In other words in the system occurs phase transition of first-order.

4 Conclusion

In this article a new mathematical method is developed for studying the statistical properties of random environment of $1D$ N-PS in external field. The joint probability distribution which describes the quantum distribution of environment in the limit of thermodynamical equilibrium exactly is constructed. It is shown that the mean value of interaction potential between the $1D$ N-PS and the external field has a *ultraviolet divergence* which is being renormalized by some type of dimensional method. The critical properties of quantum environment is investigated in detail. In particular it is shown, that for energy of an external field $V^{\tau} = \varepsilon$ in the environment the first-order phase transition occurs.

5 Acknowledgment

This work partially was supported by ICTS Grant No. A-1602.

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1D քվանտային N-մասնիկների համակարգի պատահական շրջակայքի վիճակագրական հատկությունները արտաքին դաշտում ուսումնասիրելու նոր մաթեմատիկական պատկերացում

Ա. Գևորգյան և Ար. Գևորգյան

Ամփոփում

1D քվանտային N-մասնիկների համակարգի (ՄՀ) պատահական շրջակայքի ելակսացիան արտաքին դաշտում նկարագրված է Լանժևեն-Շրեդինգերի (Լ-Շր) տիպի պատահական դիֆերենցիալ հավասարման շրջանակներում: Օգտագործելով Լ-Շր ավասարումը՝ ստացված է երկրորդ կարգի 2D ոչ ստացիոնար մասնակի ածանցիալներով իֆերենցիալ հավասարում 1D քվանտային N-ՄՀ շրջակայքի բաշխման՝ կախված ամակարգի չխոտորված էներգիայից և արտաքին դաշտի պարամետրերից: Ցույց է րված, որ N-ՄՀ և արտաքին դաշտի միջև փոխազդեցության պոտենցիալի միջին ծությունը ունի ուլտրամանուշակագույն տարրամիտում: Այս պրոբլեմը լուծվել է վանտային բաշխման հավասարման ռենորմալիզացիայի մեթոդով: Ցույց է տրված, որ ենորմալիզացիան ունի տարածաչափային իմաստ, որը հատուկ է քվանտային դաշտի եսություններին: Մանրամասնորեն ուսումնասիրված է շրջակայքի վիճակագրության րիտիկական հատկություններ"ը և ցույց է տրված, որ այն կախված արտաքին դաշտի մպլիտուդից ունի առաջին կարգի փուլային անցում: