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On Reliability Approach for Testing of Many Distributions for Pair of Markov Chains

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Abstract

The problem of three hypotheses logarithmical asymptotically optimal testing for a pair of simple homogeneous stationary Markov chains is examined. It is supposed that M probability distributions are known and each of Markov chains independently of other follows to one of them. The matrix of all error probability exponents (reliabilities) is studied.

1 Introduction

R. Ahlswede and E. Haroutunian [1] formulated and solved a group of problems concerning hypotheses identification and hypotheses testing for many objects, which is based on result of [2] for testing of many hypotheses for one object. In [3]–[8] the problem of multiple hypotheses testing for one Markov chain was considered. In [9] three hypotheses testing problem for two independent objects with independent observations was studied. In [10] hypotheses testing of two distributions for two Markov chains was considered. For solving these problems the method of types and Kullback - Leibler divergence [11] were used. In this paper we consider the case of two Markov chains which independently follow to one from M given probability distributions. We consider in detail the case $M = 3$ to make the presentation simpler.

Let the finite set $\mathcal{X} = \{1, 2, \ldots, S\}$ be the state space of Markov chain, and let $\mathbf{x}^1 = (\mathbf{x}_0^1, \mathbf{x}_1^1, \mathbf{x}_2^1, \ldots, \mathbf{x}_N^1)$, $\mathbf{x}^2 = (\mathbf{x}_0^2, \mathbf{x}_1^2, \mathbf{x}_2^2, \ldots, \mathbf{x}_N^2)$, x_n^1, $x_n^2 \in \mathcal{X}$, $n = \overline{1, N}$, $\mathbf{x}^1$, $\mathbf{x}^2 \in \mathcal{X}^{\mathcal{N}+\infty}$, be vectors of observed states of two simple homogeneous stationary Markov chains. These Markov chains are characteristics of two independent objects. There are three possible transition probabilities of both chains, $P_1 = \{P_1(t|s)\}$, $P_2 = \{P_2(t|s)\}$, $P_3 = \{P_3(t|s)\}$, $t, s = \overline{1, S}$. In three cases for such chains there exist corresponding stationary distributions $Q_1 = \{Q_1(t)\}$, $Q_2 = \{Q_2(t)\}$ and $Q_3 = \{Q_3(t)\}$ not necessarily unique, such that

$$\sum_{t=1}^{S} Q_l(t) P_l(t|s) = Q_l(s), \qquad \sum_{t=1}^{S} Q_l(t) = 1, \quad l = \overline{1,3},\ s = \overline{1,S}.$$

We denote $Q \circ P(s,t)$ the probability $Q(s)P(t|s)$, $\quad s, t \in \mathcal{X}$.

The probability of the vector $\mathbf{x}^i \in \mathcal{X}^{N+1}$ of the i-th Markov chain with transition probabilities P_l and one of stationary distribution Q_l is

$$Q_l \circ P_l^N(\mathbf{x}^i) \triangleq Q_l(x_0^i) \prod_{n=1}^{N} P_l(x_n^i | x_{n-1}^i), \quad l = \overline{1,3}, \quad i = 1,2,$$

$$Q_l \circ P_l^N(\mathcal{A}) \triangleq \bigcup_{\mathbf{x}^i \in \mathcal{A}} (Q_l \circ P_l)^N(\mathbf{x}^i) \quad l = \overline{1,3}, \quad i = 1,2, \qquad \mathcal{A} \subset \mathcal{X}^{N+1}.$$

We have three hypotheses H_1, H_2, H_3 and call the procedure of making decision on the base of $N+1$ observations the test, which we denote by Φ^N. The test Φ^N for this model can be composed by the pair of tests φ_1^N and φ_2^N for the corresponding objects: $\Phi^N = (\varphi_1^N, \varphi_2^N)$. The test φ_i^N is defined by a partition of the space $\mathcal{X}^{N+1}$ on three sets $\mathcal{A}_{l,i}^N = \{\mathbf{x}^i : \varphi_i^N(\mathbf{x}^i) = l\}$, $i = 1,2$, $l = \overline{1,3}$, i.e. the hypotheses l must be accepted for i–th object, when $\mathbf{x}^i \in \mathcal{A}_{l,i}^N$ Let us denote by $\Phi = (\varphi_1^N, \varphi_2^N)$, the infinite sequences of tests.

We denote by $\alpha_{l_1,l_2|k_1,k_2}^N(\Phi^N)$ the probability of the erroneous acceptance by the test Φ^N of the pair of hypotheses (H_{l_1}, H_{l_2}) provided that the pair (H_{k_1}, H_{k_2}) is true,

$$\alpha_{l_1,l_2|k_1,k_2}^N(\Phi^N) = Q_{k_1} \circ P_{k_1}^N(\mathcal{A}_{l_1,1}^N) Q_{k_2} \circ P_{k_2}^N(\mathcal{A}_{l_2,2}^N), \ (k_1,k_2) \neq (l_1,l_2), k_i, l_i = \overline{1,3}, i = 1,2.$$

The probability to reject a true pair of hypotheses (H_{k_1}, H_{k_2}) is the following

$$\alpha_{k_1,k_2|k_1,k_2}^N(\Phi^N) = \sum_{(k_1,k_2)\neq(l_1,l_2)} \alpha_{l_1,l_2|k_1,k_2}^N(\Phi^N). \tag{1}$$

We study error probability exponents of the sequence of tests Φ, which we call "reliabilities":

$$E_{l_1,l_2|k_1,k_2}(\Phi) \triangleq \limsup_{N\to\infty} -\frac{1}{N} \log \alpha_{l_1,l_2|k_1,k_2}^N(\Phi^N), \ k_i, \ l_i = \overline{1,3}, \ i = \overline{1,2}. \tag{2}$$

It is easy to show using (1) and (2) that

$$E_{k_1,k_2|k_1,k_2}(\Phi) = \min_{(k_1,k_2)\neq(l_1,l_2)} E_{l_1,l_2|k_1,k_2}(\Phi). \tag{3}$$

The matrix $E(\Phi) = \{E_{k_1,k_2|k_1,k_2}(\Phi)\}$ is called the reliability matrix of the sequence Φ of tests.

As in [9] we call the test sequence Φ^* logarithmically asymptotically optimal (LAO) for this model if for given values of the elements $E_{1,1|3,1}$, $E_{1,1|1,3}$, $E_{2,2|3,2}$, $E_{2,2|2,3}$, it provides maximal values for all other elements of the matrix $E(\Phi^*)$.

Our goal is to define conditions on $E_{1,1|3,1}$, $E_{1,1|1,3}$, $E_{2,2|3,2}$, $E_{2,2|2,3}$ under which there exists LAO sequence of tests Φ^*, and to show how other elements of $E(\Phi^*)$ can be found as functions of given four ones.

We name the second order type of the vector $\mathbf{x}$ [4] the square matrix of S^2 relative frequencies $\{N(t,s)N^{-1}, t = \overline{1,S}, s = \overline{1,S}\}$ of the simultaneous appearance of the states t and s on the pairs of neighbor places in $\mathbf{x}$. It is clear that $\sum_{(t,s)\in\mathcal{X}^2} N(t,s) = N$. Note that there are other definitions of types for Markov chains, for example in [6].

Denote by $\mathcal{T}_{Q\circ P}^N$ the set of vectors from $\mathcal{X}^{N+1}$, which have the type such that for some joint PD $Q \circ P$

$$N(t,s) = NQ(t)P(t|s), \quad t,s = \overline{1,S}.$$

We denote by $D(Q \circ P||Q_l \circ P_l)$ Kullback - Leibler divergence of the distribution

$$Q \circ P = \{Q(t)P(t|s), \quad t, s = \overline{1, S}\}$$

from the distribution

$$Q_l \circ P_l = \{Q_l(t)P_l(t|s), \quad t, s = \overline{1, S}\}, \quad l = \overline{1, 3},$$

defined as follows

$$D(Q \circ P||Q_l \circ P_l) = \sum_x Q \circ P(x) \log \frac{Q \circ P(x)}{Q_l \circ P_l(x)}.$$

We shall analyze the reliability matrix $E(\Phi^*) = \{E_{k_1,k_2|k_1,k_2}(\Phi^*)\}$ of LAO test for two Markov chains. The main result is formulated in Section 3.

2 LAO Hypotheses Testing for One Markov Chain

The problem for one Markov chain and two hypothetical distributions was investigated in [3]–[5]. The case of M hypotheses for independent observations was studied in [2] and for case of Markov chains in [7] and [8]. Let us recall results and definitions for the case $M = 3$. The statistician must select one among 3 hypotheses $H_l : P_l, l = \overline{1, 3}$. Let $\mathbf{x} = (\mathbf{x_0}, \mathbf{x_1}, ..., \mathbf{x_N})$ be vectors of observed states of a simple homogenous stationary Markov chain with finite number S of states. The procedure of decision making is a non-randomized test φ^N.

Theorem 1 is proved in [8]. We reformulate it for the case $M = 3$. For given positive elements $E_{1|1}$, $E_{2|2}$ let us denote

$$E^*_{1|1}(E_{1|1}) \triangleq E_{1|1}, \qquad E^*_{2|2}(E_{2|2}) \triangleq E_{2|2}, \tag{5.a}$$

$$E^*_{l|k} \triangleq \inf_{Q \circ P: D(Q \circ P||Q \circ P_k) \leq E_{k|k},\ \exists Q_k:\ D(Q||Q_k) < \infty} D(Q \circ P||Q \circ P_l), \quad l = \overline{1, 3}, \quad k = 1, 2,\ k \neq l, \tag{5.b}$$

$$E^*_{l|3} \triangleq \inf_{Q \circ P: D(Q \circ P||Q \circ P_1) > E_{1|1}, D(Q \circ P||Q \circ P_2) > E_{2|2}} D(Q \circ P||Q \circ P_k), \qquad l = 1, 2, \tag{5.c}$$

$$E^*_{3|3} \triangleq \min_{k=1,2} E_{3|k}. \tag{5.d}$$

Theorem 1. If different conditional distributions and positive numbers $E_{1|1}$, $E_{2|2}$ are given and the following inequalities hold

$$E_{1|1} < \min[\inf_{Q_k} D(Q_k \circ P_k||Q_k \circ P_1),\ k = 2, 3], \tag{6.a}$$

$$E_{2|2} < \min[E^*_{2|1}, \inf_{Q_3} D(Q_3 \circ P_3||Q_3 \circ P_2)], \tag{6.b}$$

then

a) there exists a LAO sequence of tests Φ^* such that all elements of the reliability matrix $E(\Phi^*)$ are defined in (5),

b) if one of the inequalities (6) is violated, then at least one element of the matrix $E(\Phi^*)$ is equal to 0.

Remark. Using the definition of $E^*_{1|3}$ and $E^*_{2|3}$ in (5.c) it can be proved as in [9], that

$$E^*_{1|1} = E^*_{1|3}, \qquad E^*_{2|2} = E^*_{2|3}. \tag{7}$$

3 LAO Hypotheses Testing of Three Distributions for Pair of Markov Chains

The following lemma is an analogue of Lemma for independent observations from [9].

Lemma. [9] If positive elements $E_{1|1}(\varphi_i)$, $E_{2|2}(\varphi_i)$, $i = 1, 2$, satisfy the conditions (6), then the following equalities hold true for the test $\Phi = (\varphi_1, \varphi_2)$ for two objects characterized by Markov chains:

$$E_{l_1,l_2|k_1,k_2}(\Phi) = E_{l_1|k_1}(\varphi_1) + E_{l_2|k_2}(\varphi_2), \quad k_1 \neq l_1, \quad k_2 \neq l_2. \tag{8.a}$$

$$E_{l_1,l_2|k_1,k_2}(\Phi) = E_{l_i|k_i}(\varphi_i), \; k_i \neq l_i, \quad k_{3-i} = l_{3-i}, \quad i = 1, 2. \tag{8.b}$$

Proof. The proof of the Lemma is based on the following equalities:

$$\alpha^N_{l_1,l_2|k_1,k_2}(\Phi^N) = \alpha^N_{l_1|k_1}(\varphi_1^N)\alpha^N_{l_2|k_2}(\varphi_2^N), \quad \mathrm{k}_1 \neq \mathrm{l}_1, \mathrm{k}_2 \neq \mathrm{l}_2. \tag{9.a}$$

$$\alpha^N_{l_1,l_2|k_1,k_2}(\Phi^N) = \alpha^N_{l_i|k_i}(\varphi_i^N)\{1 - \alpha^N_{l_{3-i}|k_{3-i}}(\varphi_{3-i}^N)\}, \quad k_i \neq l_i, \quad k_{3-i} = l_{3-i}, \quad i = 1, 2. \tag{9.b}$$

Now let us consider the case of two Markov chains and 3 hypotheses concerning each of them.

For knowing correctly in which set of test the elements $E_{1,1|3,1}$, $E_{1,1|1,3}$, $E_{2,2|3,2}$, $E_{2,2|2,3}$ of the tests for two objects can be positive we divide set of tests $\Phi = (\varphi_1, \varphi_2)$ into following classes:

$\mathcal{A} \triangleq \{\Phi = (\varphi_1, \varphi_2) : E_{m|m}(\varphi_i) > 0, m = 1, 2, \; i = 1, 2\}$,

$\mathcal{B} \triangleq \{\Phi = (\varphi_1, \varphi_2)$: one, or two m' from $[1, 2]$ exist such that $E_{m'|m'}(\varphi_i) = 0$ for one values of i, but $E_{m'|m'}(\varphi_j) > 0, i \neq j$, and for other $m < 3, E_{m|m}(\varphi_i) > 0, \; i, j = \overline{1, 2}\}$,

$\mathcal{C} \triangleq \{\Phi = (\varphi_1, \varphi_2)$: one or two m' from $[1, 2]$ exist such that $E_{m'|m'}(\varphi_i) = 0$, and for other $m < 3$, $E_{m|m}(\varphi_i) > 0, \; i = \overline{1, 2}\}$.

In other words we divide set of tests into classes taking in consideration zero values of elements of reliability matrix of one Markov chain, because if there are one zero element in reliability matrix (of a Markov chain), then the corresponding element of the reliability matrix of two Markov chains equals to zero too.

Let us define the following family of sets for given positive elements $E_{1,1|3,1}$, $E_{1,1|1,3}$, $E_{2,2|3,2}$, $E_{2,2|2,3}$ to determine LAO test Φ^*:

$$R_1^1 \triangleq \{Q \circ P : D(Q \circ P \| Q \circ P_1) \leq E_{1,1|3,1}, \exists Q_1 : D(Q \| Q_1) < \infty\},$$

$$R_1^2 \triangleq \{Q \circ P : D(Q \circ P \| Q \circ P_1) \leq E_{1,1|1,3}, \exists Q_1 : D(Q \| Q_1) < \infty\},$$

$$R_2^1 \triangleq \{Q \circ P : D(Q \circ P \| Q \circ P_2) \leq E_{2,2|3,2}, \exists Q_2 : D(Q \| Q_2) < \infty\},$$

$$R_2^2 \triangleq \{Q \circ P : D(Q \circ P \| Q \circ P_2) \leq E_{2,2|2,3}, \exists Q_2 : D(Q \| Q_2) < \infty\},$$

$$R_3^1 \triangleq \{Q \circ P : D(Q \circ P \| Q \circ P_1) > E_{1,1|3,1}, D(Q \circ P \| Q \circ P_2) > E_{2,2|3,2}\},$$

$$R_3^2 \triangleq \{Q \circ P : D(Q \circ P \| Q \circ P_1) > E_{1,1|1,3}, D(Q \circ P \| Q \circ P_2) > E_{2,2|2,3}\}.$$

The optimal values of the reliabilities of the LAO test sequence will be the following:

$$E^*_{1,1|3,1}(E_{1,1|3,1}) \triangleq E_{1,1|3,1}, \qquad E^*_{1,1|1,3}(E_{1,1|1,3}) \triangleq E_{1,1|1,3}, \tag{10.a.1}$$

$$E^*_{2,2|3,2}(E_{2,2|3,2}) \triangleq E_{2,2|3,2}, \qquad E^*_{2,2|2,3}(E_{2,2|2,3}) \triangleq E_{2,2|2,3}, \tag{10.a.2}$$

$$E^*_{l_1,l_2|k_1,k_2}(\Phi^*) \triangleq \inf_{Q\circ P\in R^i_{l_i}} D(Q\circ P||Q\circ P_{l_i}), \quad l_i \neq k_i, \quad l_{3-i} = k_{3-i}, \quad i=1,2, \tag{10.b}$$

$$E^*_{l_1,l_2|k_1,k_2}(\Phi^*) \triangleq E^*_{l_1,l_2|l_1,k_2}(\Phi^*) + E^*_{l_1,l_2|k_1,l_2}(\Phi^*), \qquad k_i = l_i, \quad i=1,2, \tag{10.c}$$

$$E^*_{k_1,k_2|k_1,k_2}(\Phi^*) \triangleq \min_{(k_1,k_2)\neq(l_1,l_2)} E^*_{l_1,l_2|k_1,k_2}(\Phi^*). \tag{10.d}$$

Theorem 2. Assume that all distributions P_l, $l=\overline{1,3}$ are different and absolutely continuous relative to each other: $0 < D(Q_l\circ P_l||Q_k\circ P_k) < \infty$, $l\neq k$, $k=\overline{1,3}$, if positive elements $E_{1,1|3,1}$, $E_{1,1|1,3}$, $E_{2,2|3,2}$, $E_{2,2|2,3}$ are given and the following inequalities hold

$$E_{1,1|3,1} < \min[\inf_{Q_3} D(Q_3\circ P_3||Q_3\circ P_1), \inf_{Q_2} D(Q_2\circ P_2||Q_2\circ P_1)], \tag{11.a}$$

$$E_{1,1|1,3} < \min[\inf_{Q_3} D(Q_3\circ P_3||Q_3\circ P_1), \inf_{Q_2} D(Q_2\circ P_2||Q_2\circ P_1)], \tag{11.b}$$

$$E_{2,2|2,3} < \min[E^*_{2,2|2,1}, \inf_{Q_3} D(Q_3\circ P_3||Q_3\circ P_2)], \tag{11.c}$$

$$E_{2,2|3,2} < \min[E^*_{2,2|1,2}, \inf_{Q_3} D(Q_3\circ P_3||Q_3\circ P_2)], \tag{11.d}$$

then

a) there exists a LAO test sequence $\Phi^* \in \mathcal{A}$, the reliability matrix of which $E(\Phi^*)$ is defined in (10) and all elements of it are positive,

b) when even one of the inequalities (11) is violated, then there exists at least one element of the matrix $E(\Phi^*)$ equal to 0,

c) the reliability matrix $E(\Phi^*)$ of the tests Φ^* from the families $\mathcal{B}$ and $\mathcal{C}$ necessarily contains elements equal to zero.

Proof. a) Inequalities presented bellow have been obtained from (6) using (7)

$$E^1_{1|3} < \min[\inf_{Q_3} D(Q_3\circ P_3||Q_3\circ P_1), \inf_{Q_2} D(Q_2\circ P_2||Q_2\circ P_1)], \tag{12.a}$$

$$E^2_{1|3} < \min[\inf_{Q_3} D(Q_3\circ P_3||Q_3\circ P_1), \inf_{Q_2} D(Q_2\circ P_2||Q_2\circ P_1)], \tag{12.b}$$

$$E^1_{2|3} < \min[E^{1*}_{2|1}, \inf_{Q_3} D(Q_3\circ P_3||Q_3\circ P_2)]. \tag{12.d}$$

$$E^2_{2|3} < \min[E^{2*}_{2|1}, \inf_{Q_3} D(Q_3\circ P_3||Q_3\circ P_2)], \tag{12.c}$$

The proof will be fulfilled for the case (12.c), which is the consequence of the inequality (11.c). For all other cases the proof should be executed the by same way. Let us consider a test $\Phi = (\varphi_1, \varphi_2)$, where $E_{2,2|2,3}(\Phi) = E_{2,2|2,3}$, $E_{2,2|2,1}(\Phi) = E^*_{2,2|2,1}$. The relevant error probabilities $\alpha_{2,2|2,3}(\Phi)$ and $\alpha_{2,2|2,1}(\Phi)$ have been obtained based on (9.b). According to (2) and (9) we get that

$$E_{2,2|2,1}(\Phi) < E^{2*}_{2|1} + \lim_{N\to\infty} \sup -\frac{1}{N}\log(1-\alpha_{2|2}(\varphi_1^N)), \tag{13.a}$$

$$E_{2,2|2,3}(\Phi) < E^{2*}_{2|3} + \lim_{N\to\infty} \sup -\frac{1}{N}\log(1-\alpha_{2|2}(\varphi_1^N)), \tag{13.b}$$

where $E^{2*}_{2|1} = E^2_{2|1}$, $E^{2*}_{2|3} = E^2_{2|3}$.

When $\min[E^{*}_{2,2|2,1}, \inf_{Q_3} D(Q_3\circ P_3||Q_3\circ P_2)] = E^{*}_{2,2|2,1}$ it is easy to see that $E^{2}_{2|3} < E^{2*}_{2|1}$ from (13) and (11.c)). We must show (using (13) and inequality $E^{*}_{2,2|2,1} < \inf_{Q_3} D(Q_3\circ P_3||Q_3\circ P_2)$) that $E^{2*}_{2|1} < \inf_{Q_3} D(Q_3\circ P_3||Q_3\circ P_2)$. In this case the inequality (12.c) will be proved.

Let us assume now that $\min[E^{*}_{2,2|2,1}, \inf_{Q_3} D(Q_3\circ P_3||Q_3\circ P_2)] = \inf_{Q_3} D(Q_3\circ P_3||Q_3\circ P_2)$. First of all we shall prove that $E^{2}_{2|3} < \inf_{Q_3} D(Q_3\circ P_3||Q_3\circ P_2)$ if the reliability $E_{2,2|2,3}$ satisfies the condition (11.c). Assume that the opposite statement is true: $E^{2}_{2|3} \geq \inf_{Q_3} D(Q_3\circ P_3||Q_3\circ P_2)$. In this case using (13.b) and (11.c) we can derive:

$$\inf_{Q_3} D(Q_3\circ P_3||Q_3\circ P_2) + \lim_{N\to\infty}\sup -\frac{1}{N}\log(1-\alpha_{2|2}(\varphi_1^N)) \leq E^{2}_{2|3} - \lim_{N\to\infty}\sup\frac{1}{N}\log(1-\alpha_{2,2}(\varphi_1^N)) < \inf_{Q_3} D(Q_3\circ P_3||Q_3\circ P_2).$$

Here we have come to contradiction

$$-\lim_{N\to\infty}\sup\frac{1}{N}\log(1-\alpha_{2|2}(\phi_1^N)) < 0.$$

Because the supposition is not correct.

Now we shall prove that $E^{2}_{2|3} < E^{2*}_{2|1}$ as well. From (11.c) it follows that $E_{2,2|2,3} < E^{*}_{2,2|2,1}$. Using (13) we can obtain $E^{2}_{2|3} < E^{2*}_{2|1}$. In this case (12.c) also holds.

It follows from (7) and (12) that conditions (6) of Theorem 1 hold for both objects. According to Theorem 1 there is LAO sequence of tests φ_1^{N*}, φ_2^{N*} for the each object. The elements of the matrices $E(\varphi_1^*)$ and $E(\varphi_2^*)$ are determined in (5). The test $\Phi^* = (\varphi_1^*, \varphi_2^*)$ has been taken as a test for this model and it is shown that it is LAO. The elements of the matrix $E(\Phi^*)$ are determined in (10).

Applying Lemma we can deduce that the reliability matrix $E(\Phi^*)$ can be obtained from matrices $E(\varphi_1^*)$ and $E(\varphi_2^*)$ as in (8).

Thus we obtain that

$$E_{1,1|1,3} = E^{2}_{1|3},\ E_{1,1|3,1} = E^{1}_{1|3},\ E_{2,2|2,3} = E^{2}_{2|3},\ E_{2,2|3,2} = E^{1}_{3|2}.$$

When (11) takes place according to (8.b), (5), (7) and (14) we obtain the elements $E_{l_1,l_2|k_1,k_2}(\varphi)$, $l_i \neq k_i$, $l_{3-i} = k_{3-i}$, $i = 1,2$, of the matrix $E(\Phi^*)$ determined by relations (10.b). From (8.a) and (10.b) we obtain (10.c). The equality in (10.d) is the particular case of (3). Some elements in the matrix $E(\Phi^*)$ must be equal to 0 when one of the inequalities (11) is violated (this is consequence of (9) and (10.b)).

b) Now let us show that the compound test for two objects is LAO which is optimal. Suppose that for given $E_{1,1|3,1}$, $E_{1,1|1,3}$, $E_{2,2|3,2}$, $E_{2,2|2,3}$ there exists a test $\Phi' = (\varphi_1', \varphi_2')$ with matrix $E(\Phi')$ such that it has at least one element exceeding the respective element of the matrix $E(\Phi^*)$. It is contradiction to the fact that LAO tests φ_1^*, φ_2^* have been used for the objects $\mathbf{x}^1$ and $\mathbf{x}^2$.

Remark 2. When the type $Q\circ P$ of vector $\mathbf{x} = (x_0, x_1, x_2, ..., x_N)$ of observed states of simple homogeneous stationary Markov chain, for some $N = 0, 1, 2, \dots$, is not absolutely continuous relative to one of the probabilities, for example, $Q_2\circ P_2$, that is

$$D(Q\circ P||Q_2\circ P_2) = \infty,$$

then naturally test φ^N doesn't accept second hypothesis, and for such test

$$\alpha_{2|1}^N(\varphi^N) = \alpha_{2|2}^N(\varphi^N) = \alpha_{2|3}^N(\varphi^N) = 0,$$

and the reliability matrix $E(\Phi)$ of the sequence Φ of tests will be the following:

$$E = \begin{pmatrix} E_{1|1} & E_{1|2} & E_{1|3} \\ \infty & \infty & \infty \\ E_{3|1} & E_{3|2} & E_{3|3} \end{pmatrix}.$$

In the same way we can consider the situation for pairs of Markov chains.

4 On LAO Testing of Many Distributions for Pair of Markov Chains

In this section we formulate generalization of the result of the previous section for the case of many hypotheses.

Theorem 3. Let all distributions $Q_l \circ P_l$, $l = \overline{1, L}$, be different and absolutely continuous relative to each other: $0 < D(Q_l \circ P_l || Q_k \circ P_k) < \infty$, $l \neq k$. If positive elements $E_{l,l|L,l}$, $E_{l,l|l,L}$, $l = \overline{1, L-1}$ are given and the following inequalities hold

$$E_{1,1|L,1} < \min_{l=\overline{2,L}} [\min_{Q_L} D(Q_L \circ P_L || Q_3 \circ P_1), \min_{Q_l} D(Q_l \circ P_l || Q_l \circ P_1)],$$

$$E_{1,1|1,L} < \min l = \overline{2, L} [\min_{Q_L} D(Q_L \circ P_L || Q_L \circ P_1), \min_{Q_l} D(Q_l \circ P_l || Q_l \circ P_1)],$$

$$E_{l,l|l,L} < \min[\min_{k=\overline{1,l-1}} E^*_{l,l|l,k}, \min_{k=\overline{l+1,L}} \inf_{Q_k} D(Q_k \circ P_k || Q_k \circ P_l)], \qquad l = \overline{2, L-1},$$

$$E_{l,l|L,l} < \min[\min_{k=\overline{1,l-1}} E^*_{l,l|k,l}, \min_{k=\overline{l+1,L}} \inf_{Q_k} D(Q_k \circ P_k || Q_k \circ P_l)], \qquad l = \overline{2, L-1},$$

then

a) there exists a LAO test sequence $\Phi^* \in \mathcal{A}$, the reliability matrix of which $E(\Phi^*)$ is defined as in to (10) and all elements of it are positive,

b) when even one of the inequalities (11), written for multiple hypotheses is violated, then there exists at least one element of the matrix $E(\Phi^*)$ equal to 0,

c) for given positive numbers $E_{l,l|L,l}$, $E_{l,l|l,L}$, $l = \overline{1, L-1}$, the reliability matrix $E(\Phi)$ of the tests Φ from the class $\mathcal{B}$ necessarily contains elements equal to zero.

The proof of this theorem is similar to the case of three hypotheses, so we will not present it.

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Երկու մարկովյան շղթաների նկատմամբ բազմակի վարկածների ստուգման հուսալիության սկզբունքի մասին

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Ամփոփում

Ուսումնասիրվել է երկու անկախ ստացիոնար մարկովյան շղթաներից կազմված մոդելի համար բազմակի վարկածների ստուգման խնդիրը: $M(\geq 2)$ հավանականությունների բաշխումները հայտնի են: Անկախ օբյեկտներից յուրաքանչյուրը բաշխված է ըստ դրանցից մեկի: Ուսումնասիրված է սխալների հավանականությունների ցուցիչների (հուսալիությունների) զույգերի մատրիցը: