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On Logarithmically Asymptotically Optimal Hypothesis Testing of Distributions for Pair of Statistically Dependent Objects

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Abstract

The problem of hypotheses testing for a model consisting of two statistically dependent objects is considered. It is supposed that two probability distributions are known for the first object and the second object dependent on the first can be distributed according to one of two given conditional distributions. The matrix of asymptotical optimal interdependencies (reliability–reliability functions) of all possible pairs of the error probability exponents (reliabilities) is studied. The case with two objects which can't have the same probability distribution from two given was discussed by Ahlswede and Haroutunian and for three hypotheses by Haroutunian and Yessayan.

1. Problem Statement and Preliminary Results.

In this paper we consider a generalisation of the problem of many hypotheses testing conserning one object [1]. In paper [2] Ahlswede and Haroutunian and in [3] Haroutunian formulated a number of problems on multiple hypotheses testing and indentification. Haroutunian and Hakobyan in [4] examined the models of two independent objects with three and in [5] with M hypotheses. Haroutunian and Yessayan in [6] studied the model of two objects having different distributions from possible three.

Let X_1 and X_2 be random variables taking values in the finite set $\mathcal{X}$. Let $\mathcal{P}(\mathcal{X})$ be the space of all possible distributions on $\mathcal{X}$. There are given two probability distrubutions (PD) $G_{m_1} = \{G_{m_1}(x^1), x^1 \in \mathcal{X}\}$, $m_1 = 1,2$ from $\mathcal{P}(\mathcal{X})$. The object characterised by X_1 can have one of these two distributions and X_2 can have one of two conditional PDs $G_{m_2/m_1} = \{G_{m_2/m_1}(x^2), x^2 \in \mathcal{X}\}$, $m_1, m_2 = 1,2$ depending on the index m_1 of the PD of the object X_1. Let $(\mathbf{x}_1, \mathbf{x}_2) = ((x_1^1, x_1^2), (x_2^1, x_2^2), ...(x_N^1, x_N^2))$ be a sequence of results of N independent observations of the vector (X_1, X_2). The test, which we denote by Φ^N, is a procedure of making decision on the base of these N observations of both objects. For this model the vector (X_1, X_2) can have one of four probability distributions $G_{m_1,m_2}(\mathbf{x}_1, \mathbf{x}_2)$, $m_1, m_2 = 1,2$, where $G_{m_1,m_2}(\mathbf{x}_1, \mathbf{x}_2) = G_{m_1}(\mathbf{x}_1)G_{m_2/m_1}(\mathbf{x}_2)$. The test Φ^N for this model may be composed by a pair of tests φ_1^N and φ_2^N for the separate objects: $\Phi^N = (\varphi_1^N, \varphi_2^N)$. For object X_1 the non-randomized test $\varphi_1^N(\mathbf{x}_1)$ can be given by division of the sample space $\mathcal{X}^N$ on 2 disjoint subsets $\mathcal{A}_{l_1}^N = \{\mathbf{x}_1 : \varphi_1^N(\mathbf{x}_1) = l_1\}$, $l_1 = 1,2$. The set $\mathcal{A}_{l_1}^N$ consists of all vectors $\mathbf{x}_1$ for which the PD G_{l_1} is adopted. The probability $\alpha_{l_1|m_1}^N(\varphi_1^N)$ of the erroneous acceptance of PD G_{l_1} provided that G_{m_1} is true, $l_1, m_1 = 1,2,\ m_1 \neq l_1$, is defined by the set $\mathcal{A}^N_{l_1}$

$$\alpha^N_{l_1|m_1}(\varphi^N_1) = G_{m_1}(\mathcal{A}^N_{l_1}). \tag{1}$$

The probability to reject G_{m_1}, when it is true, is

$$\alpha^N_{m_1|m_1}(\varphi^N_1) = \alpha^N_{l_1|m_1}(\varphi^N_1) = G_{m_1}(\overline{\mathcal{A}^N_{m_1}}). \tag{2}$$

We denote by φ_1, φ_2 and Φ the infinite sequences of tests. Corresponding error probability exponents for the first object, called "reliabilities", are defined as

$$E_{l_1|m_1}(\varphi_1) = \overline{\lim_{N\to\infty}} - \frac{1}{N} \log \alpha^N_{l_1|m_1}(\varphi^N_1),\ m_1, l_1 = 1,2. \tag{3}$$

It follows from (3) that

$$E_{m_1|m_1}(\varphi_1) = E_{l_1|m_1}(\varphi_1),\ l_1, m_1 = 1,2,\ l_1 \neq m_1. \tag{4}$$

For the object characterised by X_2 the non-randomized test $\varphi^N_2(\mathbf{x}_2, l_1)$, depending on the number of the hypothesis adopted for X_1, can be also given by division of the sample space $\mathcal{X}^N$ on 2 disjoint subsets $\mathcal{A}^N_{l_2/l_1} = \{\mathbf{x}_2 : \varphi^N_2(\mathbf{x}_2, l_1) = l_2\}$, $l_1, l_2 = 1,2$. The set $\mathcal{A}^N_{l_2/l_1}$ consists of all vectors $\mathbf{x}_2$ for which the PD G_{l_2/l_1} is adopted. The probabilities of the erroneous acceptance of PD G_{l_2/l_1} provided that G_{m_2/m_1} is true for all pairs $l_i, m_i = 1,2,\ i = 1,2,\ m_2 \neq l_2$, are the following

$$\alpha^N_{l_2|l_1,m_1,m_2}(\varphi^N_2) = G_{m_2/m_1}(\mathcal{A}^N_{l_2/l_1}),\ m_i, l_i = 1,2,\ i = 1,2,\ m_2 \neq l_2. \tag{5}$$

The corresponding reliabilities, are defined as

$$E_{l_2|l_1,m_1,m_2}(\varphi_2) = \overline{\lim_{N\to\infty}} - \frac{1}{N} \log \alpha^N_{l_2|l_1,m_1,m_2}(\varphi^N_2),\ m_i, l_i = 1,2,\ i = 1,2,\ m_2 \neq l_2. \tag{6}$$

It is clear that

$$E_{m_2|l_1,m_1,m_2}(\varphi_2) = E_{l_2|l_1,m_1,m_2}(\varphi_2),\ m_i, l_i = 1,2,\ i = 1,2. \tag{7}$$

For two objects we study the probability $\alpha_{l_1,l_2|m_1,m_2}(\Phi^N)$ of the erroneous acceptance by the sequence of tests Φ of the pair of PDs $(G_{l_1}, G_{l_2/l_1})$ provided that the pair $(G_{m_1}, G_{m_2/m_1})$ is true, where $(m_1, m_2) \neq (l_1, l_2)$, $m_i, l_i = 1,2,\ i = 1,2$. The probability to reject a true pair of PDs $(G_{m_1}, G_{m_2/m_1})$, is defined as follows

$$\alpha^N_{m_1,m_2|m_1,m_2}(\Phi^N) = \sum_{(l_1,l_2)\neq(m_1,m_2)} \alpha^N_{l_1,l_2|m_1,m_2}(\Phi^N),\ m_i, l_i = 1,2,\quad i = 1,2. \tag{8}$$

The reliabilities of the sequence of tests Φ are the following

$$E_{l_1,l_2|m_1,m_2}(\Phi) \triangleq \overline{\lim_{N\to\infty}} - \frac{1}{N} \log \alpha^N_{l_1,l_2|m_1,m_2}(\Phi^N),\quad m_i, l_i = 1,2,\quad i = 1,2. \tag{9}$$

From (8) we have

$$E_{m_1,m_2|m_1,m_2}(\Phi) = \min_{(l_1,l_2)\neq(m_1,m_2)} E_{l_1,l_2|m_1,m_2}(\Phi),\ m_i, l_i = 1,2,\quad i = 1,2. \tag{10}$$

The matrix $\mathbf{E}(\Phi) = \{E_{l_1,l_2|m_1,m_2}(\Phi),\ m_i, l_i = 1,2,\quad i = 1,2\}$ we call the reliability matrix of the sequence of tests Φ.

Lemma: If the elements $E_{l_1|m_1}(\varphi_1)$ and $E_{l_2|l_1,m_1,m_2}(\varphi_2)$ are positive, then

$$E_{l_1,l_2|m_1,m_2}(\Phi) = E_{l_1|m_1}(\varphi_1) + E_{l_2|l_1,m_1,m_2}(\varphi_2),\ m_1 \neq l_1,\ m_2 \neq l_2, \quad (11.a)$$

$$E_{l_1,l_2|m_1,m_2}(\Phi) = E_{l_1|m_1}(\varphi_1),\ m_1 \neq l_1,\ m_2 = l_2, \quad (11.b)$$

$$E_{l_1,l_2|m_1,m_2}(\Phi) = E_{l_2|l_1,m_1,m_2}(\varphi_2),\ m_1 = l_1,\ m_2 \neq l_2. \quad (11.c)$$

Proof: The following relations hold for error probabilities

$$\alpha^N_{l_1,l_2|m_1,m_2}(\Phi^N) = \alpha^N_{l_1|m_1}(\varphi_1^N)\alpha^N_{l_2|l_1,m_1,m_2}(\varphi_2^N),\ m_1 \neq l_1, m_2 \neq l_2, \quad (12.a)$$

$$\alpha^N_{l_1,l_2|m_1,m_2}(\Phi^N) = \alpha^N_{l_1|m_1}(\varphi_1^N)(1 - \alpha^N_{l_2|l_1,m_1,m_2}(\varphi_2^N)),\ m_1 \neq l_1,\ m_2 = l_2, \quad (12.b)$$

$$\alpha^N_{l_1,l_2|m_1,m_2}(\Phi^N) = (1 - \alpha^N_{l_1|m_1}(\varphi_1^N))\alpha^N_{l_2|l_1,m_1,m_2}(\varphi_2^N),\ m_1 = l_1,\ m_2 \neq l_2. \quad (12.c)$$

Thus, in light of (3) and (6), we obtain (11).

Now we shall reformulate the Theorem from [1] for one object for the case of 2 hypotheses. It is known from [7]–[9] that the type Q of a vector $\mathbf{x}$ is a PD $Q = \{Q_{\mathbf{x}}(x) = \frac{1}{N}N(x|\mathbf{x}), x \in \mathcal{X}\}$, where $N(x|\mathbf{x})$ is the number of repetitions of the symbol x in vector $\mathbf{x}$, and the following estimates for the set $\mathcal{T}^N_Q(X)$ of all vectors of type Q hold

$$(N+1)^{-|\mathcal{X}|}\exp\{NH_Q(X)\} \leq |\mathcal{T}^N_Q(X)| \leq \exp\{NH_Q(X)\}.$$

For entropy $H_Q(X)$ and the informational divergence $D(Q||G_l)$, $l = 1,2$ we use the following notations:

$$H_Q(X) \triangleq -\sum_{x\in\mathcal{X}} Q(x)\log Q(x),$$

$$D(Q||G_l) \triangleq \sum_{x\in\mathcal{X}} Q(x)\log\frac{Q(x)}{G_l(x)}$$

For a given positive number $E_{1|1}(\varphi_1)$ let us define

$$E_{1|1}(\varphi_1^*) \triangleq E_{1|1}(\varphi_1), \quad (13.a)$$

$$E_{1|2}(\varphi_1^*) \triangleq \inf_{Q_{\mathbf{x}_1}:\ D(Q_{\mathbf{x}_1}||G_1)\leq E_{1|1}(\varphi_1)} D(Q_{\mathbf{x}_1}||G_2), \quad (13.b)$$

$$E_{2|1}(\varphi_1^*) \triangleq E_{1|1}(\varphi_1^*) \quad (13.c)$$

$$E_{2|2}(\varphi_1^*) \triangleq E_{2|1}(\varphi_1^*). \quad (13.d)$$

The sequence of tests (φ_1^*) is called logarithmically asymptotically optimal (LAO) if for given $E_{1|1}(\varphi_1^*)$ it provides maximal values for other elements of the matrix $\mathbf{E}(\varphi_1^*)$.

Theorem 1: *If the distributions G_1 and G_2 are such that $0 < D(G_1||G_2) < \infty$, $0 < D(G_2||G_1) < \infty$ and the following inequality holds*

$$0 < E_{1|1}(\varphi_1) < D(G_2||G_1), \quad (14)$$

then:

a) there exists a LAO sequence of tests φ_1^*, *the reliability matrix of which* $\mathbf{E}(\varphi_1^*) = \{E_{l_1|m_1}(\varphi_1^*)|m_1, l_1 = 1,2\}$ *is defined in* (13) *and all elements of which are positive,*

b) if one of inequalities (14) *is violated, then there exists at least one element of the matrix* $\mathbf{E}(\varphi_1^*)$ *equal to* 0.

2. LAO Testing of Two Hypotheses for the Dependent Object

For a given positive number $E_{1|l_1,m_1,1}(\varphi_2)$ for each pair $l_1, m_1 = 1,2$ let us define

$$E_{1|l_1,m_1,1}(\varphi_2^*) \triangleq E_{1|l_1,m_1,1}(\varphi_2), \tag{15.a}$$

$$E_{1|l_1,m_1,2}(\varphi_2^*) \triangleq \inf_{Q_{\mathbf{x}_2}:\ D(Q_{\mathbf{x}_2}||G_{1/l_1}) \le E_{1|l_1,m_1,1}(\varphi_2)} D(Q_{\mathbf{x}_2}||G_{2/l_1}), \tag{15.b}$$

$$E_{2|l_1,m_1,1}(\varphi_2^*) \triangleq E_{1|l_1,m_1,2}(\varphi_2^*), \tag{15.c}$$

$$E_{2|l_1,m_1,2}(\varphi_2^*) \triangleq E_{2|l_1,m_1,1}(\varphi_2^*). \tag{15.d}$$

The sequence of tests φ_2^* we shall call LAO if for given $E_{1|l_1,m_1,1}(\varphi_2^*)$ it provides maximal values for other elements of $\mathbf{E}(\varphi_2^*)$.

Theorem 2 : *If the distributions* G_{1/l_1} *and* G_{2/l_1} *are such that* $0 < D(G_{1/l_1}||G_{2/l_1}) < \infty$, $0 < D(G_{2/l_1}||G_{1/l_1}) < \infty$ *and the following inequality holds*

$$0 < E_{1|l_1,m_1,1}(\varphi_2) < D(G_{2/l_1}||G_{1/l_1}) \tag{16}$$

then:

a) there exists a LAO sequence of tests φ_2^*, *the reliability matrix of which* $\mathbf{E}(\varphi_2^*) = \{E_{l_2|l_1,m_1,m_2}(\varphi_2^*)|\, m_2, l_2 = 1,2\}$ *is defined in* (15) *and all elements of which are positive,*

b) if one of inequalities (16) *is violated, then there exists at least one element of the matrix* $\mathbf{E}(\varphi_2^*)$ *equal to* 0 .

Proof : For $\mathbf{x}_2 \in \mathcal{T}_Q^N(X)$ the conditional probability of $\mathbf{x}_2$ can be represented as follows:

$$G^N_{m_2/m_1}(\mathbf{x}_2) = \prod_{n=1}^{N} G_{m_2/m_1}(x_n^2) = \prod_{x^2} G_{m_2/m_1}(x^2)^{N(x^2|\mathbf{x}_2)} =$$

$$= \exp\{-N[D(Q_{\mathbf{x}_2} \parallel G_{m_2/m_1}) + H_{Q_{\mathbf{x}_2}}(X)]\}.$$

Let us consider the following sequence of tests φ_2 given by the sets

$$\mathcal{B}^N_{1/l_1} = \bigcup_{Q_{\mathbf{x}_2}:\ D(Q_{\mathbf{x}_2}||G_{1/l_1}) \le E_{1|l_1,m_1,1}(\varphi_2)} \mathcal{T}^N_{Q_{\mathbf{x}_2}}(X),$$

and

$$\overline{\mathcal{B}}^N_{1/l_1} \cup \mathcal{B}^N_{1/l_1} = \mathcal{X}^N.$$

We can estimate by the following inequalities

$$\alpha^{(N)}_{1|l_1,m_1,1}(\varphi_2^{*N}) = G^N_{1/m_1}(\overline{\mathcal{B}^N_{1/l_1}}) = G^N_{1/m_1}\left(\bigcup_{Q_{\mathbf{x}_2}: D(Q_{\mathbf{x}_2}||G_{1/l_1}) > E_{1|l_1,m_1,1}(\varphi_2)} \mathcal{T}^N_{Q_{\mathbf{x}_2}}(X)\right) \le$$

$$\le (N+1)^{|\mathcal{X}|} \sup_{Q_{\mathbf{x}_2}: D(Q_{\mathbf{x}_2}||G_{1/l_1}) > E_{1|l_1,m_1,1}(\varphi_2)} G_{1/m_1}(\mathcal{T}^N_{Q_{\mathbf{x}_2}}(X)) \le$$

$$\le (N+1)^{|\mathcal{X}|} \sup_{Q_{\mathbf{x}_2}: D(Q_{\mathbf{x}_2}||G_{1/l_1}) > E_{1|l_1,m_1,1}(\varphi_2)} \exp\{-ND(Q_{\mathbf{x}_2}||G_{1/l_1})\} \le$$

$$\leq \exp\{-N[\inf_{Q_{x_2}:D(Q_{x_2}||G_{1/l_1})>E_{1|l_1,m_1,1}(\varphi_2)} D(Q_{x_2}||G_{1/m_1}) - o_N(1)]\} \leq$$

$$\leq \exp\{-N[E_{1|l_1,1,m_1}(\varphi_2) - o_N(1)]\}. \quad (17)$$

where $o_N(1) \to 0$ with $N \to \infty$.

We evaluate by analogy

$$\alpha^{(N)}_{1|l_1,m_1,2}(\varphi_2^{*N}) = G^N_{2/m_1}(B^N_{1/l_1}) = G^N_{2/m_1}\left(\bigcup_{Q_{x_2}:D(Q_{x_2}||G_{1/l_1})\leq E_{1|l_1,m_1,1}(\varphi_2)} \mathcal{T}^N_{Q_{x_2}}(X)\right) \leq$$

$$\leq (N+1)^{|\mathcal{X}|} \sup_{Q_{x_2}:D(Q_{x_2}||G_{1/l_1})\leq E_{1|l_1,m_1,1}(\varphi_2)} G_{2/m_1}(\mathcal{T}^N_{Q_{x_2}}(X)) \leq$$

$$\leq (N+1)^{|\mathcal{X}|} \sup_{Q_{x_2}:D(Q_{x_2}||G_{1/l_1})\leq E_{1|l_1,m_1,1}(\varphi_2)} \exp\{-ND(Q_{x_2}||G_{2/m_1})\} \leq$$

$$\leq \exp\{-N[\inf_{Q_{x_2}:D(Q_{x_2}||G_{1/l_1})\leq E_{1|l_1,m_1,1}(\varphi_2)} D(Q_{x_2}||G_{2/m_1}) - o_N(1)]\}. \quad (18)$$

Now let us prove the inverse inequality

$$\alpha^{N}_{1|l_1,m_1,2}(\varphi_2^{*N}) = G^N_{2/m_1}(B^N_{1/l_1}) = G^N_{2/m_1}\left(\bigcup_{Q_{x_2}:D(Q_{x_2}||G_{1/l_1})\leq E_{1|l_1,m_1,1}(\varphi_2)} \mathcal{T}^N_{Q_{x_2}}(X)\right) \geq$$

$$\geq \sup_{Q_{x_2}:D(Q_{x_2}||G_{1/l_1})\leq E_{1|l_1,m_1,1}(\varphi_2)} G_{2/m_1}(\mathcal{T}^N_{Q_{x_2}}(X)) \geq$$

$$\geq (N+1)^{-|\mathcal{X}|} \sup_{Q_{x_2}:D(Q_{x_2}||G_{1/l_1})\leq E_{1|l_1,m_1,1}(\varphi_2)} \exp\{-ND(Q_{x_2}||G_{2/m_1})\} \geq$$

$$\geq \exp\{-N[\inf_{Q_{x_2}:D(Q_{x_2}||G_{1/l_1})\leq E_{1|l_1,m_1,1}(\varphi_2)} D(Q_{x_2}||G_{2/m_1}) + o_N(1)]\}. \quad (19)$$

According to the definition (6) the reliability $E_{1|l_1,m_1,2}(\varphi_2^*)$ of the test sequence (φ_2^*) is the following upper limit: $\overline{\lim_{N\to\infty}} -\frac{1}{N}\log\alpha^N_{1|l_1,m_1,2}(\varphi_2^{*N})$. Taking into account (18), (19) and the continuity of the functional $D(Q_{x_2}||G_{1/l_1})$ we obtain that $\overline{\lim_{N\to\infty}} -\frac{1}{N}\log\alpha^N_{1|l_1,m_1,2}(\varphi_2^{*N})$ exists and in correspondence with (16b) equals to $E^*_{1|l_1,m_1,2}$. Thus $E_{1|l_1,m_1,2}(\varphi_2^*) = E^*_{1|l_1,m_1,2}$. From here, in light of (15), we get the reliabilities $E_{2|l_1,m_1,1}(\varphi_2^*) = E^*_{1|l_1,m_1,1}$, $E_{2|l_1,m_1,2}(\varphi_2^*) = E^*_{1|l_1,m_1,2}$.

Thus

$$E_{l_2|l_1,m_1,m_2}(\varphi_2^*) = \overline{\lim_{N\to\infty}} -\frac{1}{N}\log\alpha^N_{l_2|l_1,m_1,m_2}(\varphi_2^{*N}) = E^*_{l_2|l_1,m_1,m_2},\ m_i, l_i = 1,2,\ i = 1,2.$$

The proof of the optimality of φ_2^{*N} and of the part *b)* is similar to the case of one object with two hypotheses [1].

3. LAO Testing of Two Hypotheses for Two Statistically Dependent Objects

Now the sequence of tests Φ^* is called LAO if for given values of the elements $E_{2,1|1,1}(\Phi)$ and $E_{1,2|1,1}(\Phi)$ it provides maximal values for other elements of the matrix $\mathbf{E}(\Phi^*)$.

Theorem 3: *If the distributions G_1 and G_2 are different, $G_{2/1}$ and $G_{1/1}$ are also different, and for given positive elements $E_{2,1|1,1}(\Phi)$ and $E_{1,2|1,1}(\Phi)$ the following inequalities hold*

$E_{1,2|1,1}(\Phi) < D(G_{2/1}||G_{1/1})$, $E_{2,1|1,1}(\Phi) < D(G_2||G_1)$ *then other elements of matrix are defined as follows:*

$$E_{2,2|2,1}(\Phi^*) = E_{1,2|1,1}(\Phi),\ E_{2,2|1,2}(\Phi^*) = E_{2,1|1,1}(\Phi),$$

$$E_{1,1|1,2}(\Phi^*) = E_{2,1|2,2}(\Phi^*) = \inf_{Q_{\mathbf{x}_2}: D(Q_{\mathbf{x}_2}||G_{1/1}) \leq E_{1,2|1,1}(\Phi)} D(Q_{\mathbf{x}_2}||G_{2/1}),$$

$$E_{1,1|2,1}(\Phi^*) = E_{1,2|2,2}(\Phi^*) = \inf_{Q_{\mathbf{x}_1}: D(Q_{\mathbf{x}_1}||G_1) \leq E_{2,1|1,1}(\Phi)} D(Q_{\mathbf{x}_1}||G_2),$$

$$E_{1,1|2,2}(\Phi^*) = E_{1,1|1,2}(\Phi^*) + E_{1,1|2,1}(\Phi^*),\ E_{1,2|2,1}(\Phi^*) = E_{1,1|2,1}(\Phi^*) + E_{2,2|2,1}(\Phi^*),$$

$$E_{2,1|1,2}(\Phi^*) = E_{1,1|1,2}(\Phi^*) + E_{2,2|1,2}(\Phi^*),\ E_{2,2|1,1}(\Phi^*) = E_{1,2|1,1}(\Phi^*) + E_{2,1|1,1}(\Phi^*),$$

$$E_{m_1,m_2|m_1,m_2}(\Phi^*) = \min_{(l_1,l_2)\neq(m_1,m_2)} E_{l_1,l_2|m_1,m_2}(\Phi^*),\ m_i, l_i = 1,2, \quad i = 1,2.$$

The Lemma and the optimality of tests φ_1^* and φ_2^* are bases of the proof similar to the case of independent objects [2].

Corollary: When the objects can accept only different PDs from two given, then $G_{m_2/m_1}(\mathbf{x}_2) = G_{m_2}(\mathbf{x}_2)$, if $m_2 \neq m_1$ and $G_{m_2/m_1}(\mathbf{x}_2) = 0$, if $m_2 = m_1$. In this case we obtain that $G_{m_2}(\mathcal{A}_{l_2}) = 1$, $m_2 \neq m_1$, $l_2 \neq l_1$, $m_2 \neq l_2$. Thus in light of (5) and Lemma, we have the situation equivalent to the case of one object with two hypotheses [1], [2].

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Վիճակագրորեն կախյալ երկու օբյեկտների նկատմամբ վարկածների ստուգման մասին

Ե. Ա. Հարությունյան և Ա. Օ. Եսայան

Ամփոփում

Դիտարկված է վիճակագրորեն կախյալ երկու օբյեկտների նկատմամբ վարկածների լոգարիթմորեն ասիմպտոտորեն օպտիմալ ստուգման խնդիրը: Առաջին օբյեկտը կարող է բաշխված լինել տրված երկու հավանականային բաշխումներից մեկով, իսկ երկրորդը՝ կախված առաջինի բաշխումից, տրված երկու պայմանական հավանականային բաշխումներից մեկով: Ուսումնասիրվել է վարկածների օպտիմալ տեստավորման դեպքում երկու օբյեկտների նկատմամբ վարկածների սխալների հավանականությունների ցուցիչների (հուսալիությունների) փոխկախվածությունը: