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On LAO Testing of Multiple Hypotheses for Pair of Objects

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Abstract

Many hypotheses testing for a model consisting of two independent by functioning objects is considered. It is known that $M(\geq 2)$ probability distributions are given and objects independently of other follows to one of them. The matrix of asymptotic interdependencies (reliability–reliability functions) of all possible pairs of the error probability exponents (reliabilities) in optimal testing for this model is studied.

This problem was introduced (and solved for the case with two given probability distributions) by Ahlswede and Haroutunian. The situation with three hypotheses was examined by Haroutunian and Hakobyan.

1 Introduction

In [1] (see also [2], [3]) Ahlswede and Haroutunian formulated an ensemble of new problems on multiple hypotheses testing for many objects and on identification of hypotheses. Noted problems are extentions of those investigated in the books [4] and [5]. Problems of identification of distribution and of distributions ranking for one object were solved in [2] completely. Also the problem of hypotheses testing for the model consisting of two independent or two strictly dependent objects (when they cannot admit the same distribution) with two possible hypothetical distributions was investigated in [2]. In this paper we study the model consisting of two objects which independently follow to one of given $M(\geq 2)$ probability distributions. The problem is a generalization of those investigated in [6] for testing of many hypotheses concerning one object. The case of two independent objects with three hypotheses was examined in [7]. Recently Tuncel [10] published an interesting consideration of the problem of multiple hypothesis optimal testing, which differs from the approach of [6], [9].

Let $\mathcal{P}(\mathcal{X})$ be the space of all probability distributions (PDs) on finite set $\mathcal{X}$ of cardinality K. There are given M PDs $G_m \in \mathcal{P}(\mathcal{X}),\ m=\overline{1,M}\varphi$.

Let us recall main definitions from [6] for the case of one object. The random variables (RV) X taking values on $\mathcal{X}$ follows to one of the M PDs G_m, $m = \overline{1,M}$. The statistician must accept one of M hypotheses $H_l : G = G_l,\ l = \overline{1,M}$, on the base of a sequence of results of N observations of the object $\mathbf{x} = (x_1, ..., x_n, ..., x_N)$, $x_n \in \mathcal{X}$, $n = \overline{1,N}$. The procedure of decision making is a non-randomized test φ_N, which can be defined by division of the sample space $\mathcal{X}^N$ on M disjoint subsets $\mathcal{A}_l^N = \{\mathbf{x} : \varphi^N(\mathbf{x}) = l\}$, $l = \overline{1, M}$. The set $\mathcal{A}_l^N$ contain all vectors $\mathbf{x}$ for which the hypothesis H_l is adopted. The probability $\alpha_{m|l}(\varphi_N)$ of the erroneous acceptance of hypothesis H_l provided that H_m is true, is equal to $G_m^N(\mathcal{A}_l^N)$, $l \neq m$. The probability to reject H_m, when it is true, is

$$\alpha_{m|m}(\varphi_N) \triangleq \sum_{l \neq m} \alpha_{m|l}(\varphi_N). \tag{1}$$

The error probability exponents, which it is convienient to call "reliabilities" of the sequence of tests φ, are defined as

$$E_{m|l}(\varphi) \triangleq \overline{\lim_{N \to \infty}} - \frac{1}{N} \log \alpha_{m|l}(\varphi_N), \;\; m, l = \overline{1, M}. \tag{2}$$

From (1) it follows that

$$E_{m|m}(\varphi) = \min_{l \neq m} E_{m|l}(\varphi), \;\; m = \overline{1, M}. \tag{3}$$

The matrix $\mathbf{E}(\varphi) = \{E_{m|l}(\varphi)\}$ is the reliability matrix of the sequence φ of tests. It was studied in [6].

Definition 1: We call the sequence of tests φ^* logarithmically asymptotically optimal (LAO) if for given positive values of $M-1$ diagonal elements of the matrix $\mathbf{E}(\varphi^*)$ maximal values to all other elements of it are provided.

The concept of LAO test was introduced by L. Birge [11] and also elaborated in [6], [7] and [9].

Now let us consider the model with two objects. Let X_1 and X_2 be independent RV taking values in the same finite set $\mathcal{X}$ with one of M PDs, they are characteristics of corresponding independent objects. The random vector (X_1, X_2) assumes values $(x^1, x^2) \in \mathcal{X} \times \mathcal{X}$.

Let $(\mathbf{x}_1, \mathbf{x}_2) = ((x_1^1, x_1^2), ..., (x_n^1, x_n^2), ..., (x_N^1, x_N^2))$, $x_n^i \in \mathcal{X}$, $i = \overline{1, 2}$, $n = \overline{1, N}$, be a sequence of results of N independent observations of the vector (X_1, X_2). The statistician must define unknown PDs of the objects on the base of observed data. The selection for each object must be made from the same set of hypotheses: $H_m : G = G_m$, $m = \overline{1, M}$. We call the procedure of making decision on the base of N pairs of observations the test for two objects and denote it by Φ_N. Because of the objects independence test Φ_N may be considered as the pair of the tests φ_N^1 and φ_N^2 for the respective separate objects. We will denote the whole compound test sequence by Φ.

Let $\alpha_{m_1, m_2|l_1, l_2}(\Phi_N)$ be the probability of the erroneous acceptance by the test Φ_N of the hypotheses pair (H_{l_1}, H_{l_2}) provided that the pair (H_{m_1}, H_{m_2}) is true, where $(m_1, m_2) \neq (l_1, l_2)$, $m_i, l_i = \overline{1, M}$, $i = 1, 2$. The probability to reject a true pair of hypotheses (H_{m_1}, H_{m_2}) by analogy with (1) is the following:

$$\alpha_{m_1, m_2|m_1, m_2}(\Phi_N) \triangleq \sum_{(l_1, l_2) \neq (m_1, m_2)} \alpha_{m_1, m_2|l_1, l_2}(\Phi_N). \tag{4}$$

We have to study corresponding limits $E_{m_1, m_2|l_1, l_2}(\Phi)$ of error probability exponents of the sequence of tests Φ, called also reliabilities

$$E_{m_1, m_2|l_1, l_2}(\Phi) \triangleq \overline{\lim_{N \to \infty}} - \frac{1}{N} \log \alpha_{m_1, m_2|l_1, l_2}(\Phi_N), \quad m_i, l_i = \overline{1, M}, \quad i = 1, 2. \tag{5}$$

As in (3) it follows from (5) that

$$E_{m_1,m_2|m_1,m_2}(\Phi) = \min_{(l_1,l_2)\neq(m_1,m_2)} E_{m_1,m_2|l_1,l_2}(\Phi). \tag{6}$$

Definition 2: The test sequence Φ^* we call LAO for the model with two objects if for given positive values of certain $2(M-1)$ elements of the reliability matrix $\mathbf{E}(\Phi^*)$ the procedure provides maximal values for other elements of it.

The paper is devoted to analysis of the reliability matrix $\mathbf{E}(\Phi^*) = \{E_{m_1,m_2|l_1,l_2}(\Phi^*)\}$ of LAO tests for two objects.

In section 3 we formulate and prove the results on two objects testing and in section 4 we present an example of calculation of reliabilities for one and for two objects.

2 LAO Testing of Hypotheses for One Object

We define the divergence (Kullback-Leibler distance) $D(Q||G)$ for PDs $Q, G \in \mathcal{P}(\mathcal{X})$, as usual (see [8]):

$$D(Q||G) = \sum_x Q(x) \log \frac{Q(x)}{G(x)}.$$

For given positive elements $E_{1|1}, E_{2|2}, \dots, E_{M-1|M-1}$ let us divide $\mathcal{P}(\mathcal{X})$ on M subset.

$$\mathcal{R}_l \triangleq \{Q : D(Q||G_l) \leq E_{l|l}\}, \;\; l = \overline{1, M-1}, \tag{7.a}$$

$$\mathcal{R}_M \triangleq \{Q : D(Q||G_l) > E_{l|l}, \;\; l = \overline{1, M-1}\} = \mathcal{P}(\mathcal{X}) - \bigcup_{l=1}^{M-1} \mathcal{R}_l, \tag{7.b}$$

and consider the following values:

$$E^*_{l|l} = E^*_{l|l}(E_{l|l}) \triangleq E_{l|l}, \;\; l = \overline{M-1}, \tag{8.a}$$

$$E^*_{m|l} = E^*_{m|l}(E_{l|l}) \triangleq \inf_{Q\in\mathcal{R}_l} D(Q||G_m), \;\; m = \overline{1, M}, \;\; m \neq l, \;\; l = \overline{1, M-1}, \tag{8.b}$$

$$E^*_{m|M} = E^*_{m|M}(E_{1|1}, \dots, E_{M-1|M-1}) \triangleq \inf_{Q\in\mathcal{R}_M} D(Q||G_m), \;\; m = \overline{1, M-1}, \tag{8.c}$$

$$E^*_{M|M} = E^*_{M|M}(E_{1|1}, \dots, E_{M-1|M-1}) \triangleq \min_{l=\overline{1,M-1}} E^*_{M|l}. \tag{8.d}$$

The main result of paper [6] is

Theorem 1: If the distributions G_m, $m = \overline{1, M}$, are different, that is all elements of the matrix $\{D(G_l||G_m)\}$, are strictly positive, then two statements hold:

a) when the given numbers $E_{1|1}, E_{2|2}, \dots, E_{M-1|M-1}$ satisfy conditions

$$0 < E_{1|1} < \min_{l=\overline{2,M}} D(G_l||G_1), \tag{9.a}$$

$$0 < E_{m|m} < \min[\min_{l=\overline{1,m-1}} E^*_{m|l}(E_{l|l}), \min_{l=\overline{m+1,M}} D(G_l||G_m)], \;\; m = \overline{2, M-1} \tag{9.b}$$

then there exists a LAO sequence of tests φ^*, the reliability matrix of which $\mathbf{E}(\varphi^*) = \{E^*_{m|l}\}$ is defined in (8) and all elements of it are strictly positive;

b) even if one of conditions (9) is violated, then the reliability matrix of any such test include at least one element equal to zero (that is the corresponding error probability does not tend to zero exponentially).

It will be useful for the sequel to formulate the following corollaries of Theorem 1.

Corollary 1: From definitions (8) and conditions (9) it follows that

$$E^*_{m|m} = E^*_{m|M}, \quad m = \overline{1, M-1}. \tag{10}$$

Proof: Applying theorem of Kuhn-Tucker in (8.b) we can derive that the elements $E^*_{l|l}$, $l = \overline{1, M-1}$ may be determined by elements $E^*_{m|l}$, $m \neq l$, $m = \overline{1, M}$ by the following inverse function

$$E^*_{l|l}(E^*_{m|l}) = \inf_{Q: D(Q||G_m) \leq E^*_{m|l}} D(Q||G_l).$$

From conditions (9) we see that $E^*_{m|m}$ can be equal only to one from $E^*_{m|l}$, $l = \overline{m+1, M}$. Assume that (10) is not true, that is $E^*_{m|m} = E^*_{m|l}$, for $l = \overline{m+1, M-1}$. From (8.b) it follows that

$$E^*_{l|l}(E^*_{m|l}) = \inf_{Q: D(Q||G_m) \leq E^*_{m|l}} D(Q||G_l) = \inf_{Q: D(Q||G_m) \leq E^*_{m|m}} D(Q||G_l) = E^*_{l|m},$$

$$m = \overline{1, M-1}, l = \overline{1, M-1}, m \neq l,$$

but from conditions (9) it follows that $E^*_{l|l} < E^*_{l|m}$ for $m = \overline{1, l-1}$. Our assumption is not true, hence (10) is valid.

Corollary 2: If in contradiction to conditions (9) one element $E_{m|m}$, $m = \overline{1, M-1}$, of the reliability matrix of an object is equal to zero, then the corresponding elements of the matrix determined as functions of $E_{m|m}$, will be given as in the case of Stain's lemma [8]:

$$E'_{l|m}(E_{m|m}) = D(G_m||G_l), \quad l = \overline{1, M}, \ l \neq m, \tag{11.a}$$

and the remaining elements of the matrix $\mathbf{E}(\varphi)^*)$ defined by $E_{l|l} > 0$, $l \neq m$, $l = \overline{1, M-1}$, as follows from Theorem1:

$$E'_{k|l} = \inf_{Q: D(Q||G_l) \leq E_{l|l}} D(Q||G_k), \tag{11.b}$$

$$E'_{k|M} = \inf_{Q: D(Q||G_i) > E_{i|i}, i = \overline{1, M-1}} D(Q||G_k). \tag{11.c}$$

That is there exists LAO sequence of tests φ', the reliability matrix $\mathbf{E}(\varphi')$ of which is defined by (11).

Remark 1: The number of elements $E_{m|m}$ taken equal to zero may be any between 1 and $M-1$, corresponding generalization of Corollary 2 is straightforward.

3 LAO Testing of Hypotheses for Two independent Objects

Let us consider the case of two independent objects and M hypotheses concerning each of them.

Now let us denote by $\mathbf{E}(\varphi^i)$ the reliability matrices of the sequences of tests φ^i, $i = 1, 2$, for each of the objects. The following Lemma was used in [2] and [7].

Lemma: If elements $E_{m|l}(\varphi^i)$, $m, l = \overline{1, M}$, $i = 1, 2$, are strictly positive, then the following equalities hold for $\Phi = (\varphi^1, \varphi^2)$:

$$E_{m_1,m_2|l_1,l_2}(\Phi) = \sum_{i=1}^{2} E_{m_i|l_i}(\varphi^i), \text{ if } m_1 \neq l_1, \ m_2 \neq l_2, \tag{12.a}$$

$$E_{m_1,m_2|l_1,l_2}(\Phi) = E_{m_i|l_i}(\varphi^i), \text{ if } m_{3-i} = l_{3-i}, \ m_i \neq l_i, \ i = 1, 2. \tag{12.b}$$

The equalities (12.a) are void also, if the reliabilities $E_{m|l}(\varphi^i) = 0$, for several m, l and several i.

Proof: It follows from the independence of the objects that

$$\alpha_{m_1,m_2|l_1,l_2}(\Phi_N) = \alpha_{m_1|l_1}(\varphi_N^1)\ \alpha_{m_2|l_2}(\varphi_N^2), \text{ if } m_1 \neq l_1, \ m_2 \neq l_2, \tag{13.a}$$

$$\alpha_{m_1,m_2|l_1,l_2}(\Phi_N) = \alpha_{m_i|l_i}(\varphi_N^i)\ [1 - \alpha_{m_{3-i}|l_{3-i}}(\varphi_N^{3-i})], \text{ if } m_{3-i} = l_{3-i}, \ m_i \neq l_i. \tag{13.b}$$

Remark that in the case of two objects we need to consider also the probabilities of right (not erroneous) decisions. According to the definitions (4) and (5), and we obtain (12) from equalities (13).

Our aim is to find LAO test from the set of the compound tests $\{\Phi = (\varphi^1, \varphi^2)\}$ when strictly positive elements $E_{m,m|m,M}$ and $E_{m,m|M,m}$, $m = \overline{1, M-1}$, of the reliability matrix are given .

The elements $E_{m,m|m,M}$ and $E_{m,m|M,m}$, $m = \overline{1, M-1}$ of the test for two objects can be positive only in the following two subsets of tests $\{\Phi = (\varphi^1, \varphi^2)\}$:

$\mathcal{A} \triangleq \{\Phi = (\varphi^1, \varphi^2) : E_{m|m}(\varphi^1) > 0, E_{m|m}(\varphi^2) > 0, m = \overline{1, M-1}\}$,

$\mathcal{B} \triangleq \{\Phi = (\varphi^1, \varphi^2) :$ one or several m' from $[1, M-1]$ exist such that $E_{m'|m'}(\varphi^1) = 0, E_{m'|m'}(\varphi^2) = 0$, and for other $m < M, E_{m|m}(\varphi^1) > 0, E_{m|m}(\varphi^2) > 0\}$.

Let us define the following subsets of $\mathcal{PX}$ for given positive elements $E_{m,m|M,m}$, $E_{m,m|m,M}$, $m = \overline{1, M-1}$:

$$\mathcal{R}_m^{(1)} \triangleq \{Q : D(Q||G_m) \leq E_{m,m|M,m}\}, \ m = \overline{1, M-1},$$

$$\mathcal{R}_m^{(2)} \triangleq \{Q : D(Q||G_m) \leq E_{m,m|m,M}\}, \ m = \overline{1, M-1},$$

$$\mathcal{R}_M^{(1)} \triangleq \{Q : D(Q||G_m) > E_{m,m|M,m}, \ m = \overline{1, M-1}\},$$

$$\mathcal{R}_M^{(2)} \triangleq \{Q : D(Q||G_m) > E_{m,m|m,M}, \ m = \overline{1, M-1}\}.$$

Let also

$$E^*_{m,m|m,M} \triangleq E_{m,m|m,M}, \ E^*_{m,m|M,m} \triangleq E_{m,m|M,m}, \ m = \overline{1, M-1}, \tag{14.a}$$

$$E^*_{m_1,m_2|l_1,l_2} \triangleq \inf_{Q:Q\in R^{(i)}_{l_i}} D(Q||G_{m_i}), \quad m_i \neq l_i, \ m_{3-i} = l_{3-i}, \ i = 1,2, \tag{14.b}$$

$$E^*_{m_1,m_2|l_1,l_2} \triangleq E^*_{m_1,m_2|m_1,l_2} + E^*_{m_1,m_2|l_1,m_2}, \quad m_i \neq l_i, \ i = 1,2, \tag{14.c}$$

$$E^*_{m_1,m_2|m_1,m_2} \triangleq \min_{(l_1,l_2)\neq(m_1,m_2)} E^*_{m_1,m_2|l_1,l_2}. \tag{14.d}$$

The main result of the present paper is formulated in

Theorem 2: If all distributions G_m, $m = \overline{1,M}$, are different, that is $(D(G_l||G_m) > 0$, $l \neq m$, $l, m = \overline{1,M})$, then the following three statements are valid:

a) when given elements $E_{m,m|m,M}$ and $E_{m,m|M,m}$, $m = \overline{1,M-1}$, meet the following conditions

$$0 < E_{1,1|M,1} < \min_{l=\overline{2,M}} D(G_l||G_1), \tag{15.a}$$

$$0 < E_{1,1|1,M} < \min_{l=\overline{2,M}} D(G_l||G_1), \tag{15.b}$$

$$0 < E_{m,m|M,m} < \min[\min_{l=\overline{1,m-1}} E^*_{m,m|l,m}, \min_{l=\overline{m+1,M}} D(G_l||G_m)], \ m = \overline{2,M-1}, \tag{15.c}$$

$$0 < E_{m,m|m,M} < \min[\min_{l=\overline{1,m-1}} E^*_{m,m|m,l}, \min_{l=\overline{m+1,M}} D(G_l||G_m)], \ m = \overline{2,M-1}, \tag{15.d}$$

then there exists a LAO test sequence $\Phi^* \in \mathcal{A}$, the reliability matrix of which $\mathbf{E}(\Phi^*) = \{E_{m_1,m_2|l_1,l_2}(\Phi^*)\}$ is defined in (14) and all elements of it are positive,

b) when even one of the inequalities (15) is violated, then there exists at least one element of the matrix $\mathbf{E}(\Phi^*)$ equal to 0,

c) for given positive numbers $E_{m,m|m,M}$, $E_{m,m|M,m}$, $m = \overline{1,M-1}$ the reliability matrix $\mathbf{E}(\Phi)$ of the tests $\Phi \in \mathcal{B}$ necessarily contains elements equal to zero.

Proof: a) Conditions (15) imply that inequalities (9) hold simultaneously for the both objects. Really, using equalities (10) we can rewrite inequalities (9) for both objects as follows:

$$0 < E_{1|M}(\varphi^1) < \min_{l=\overline{2,M}} D(G_l||G_1), \tag{16.a}$$

$$0 < E_{1|M}(\varphi^2) < \min_{l=\overline{2,M}} D(G_l||G_1), \tag{16.b}$$

$$0 < E_{m|M}(\varphi^1) < \min[\min_{l=\overline{1,m-1}} E^*_{m|l}(\varphi^1), \min_{l=\overline{m+1,M}} D(G_l||G_m)], \quad m = \overline{2,M-1}, \tag{16.c}$$

$$0 < E_{m|M}(\varphi^2) < \min[\min_{l=\overline{1,m-1}} E^*_{m|l}(\varphi^2), \min_{l=\overline{m+1,M}} D(G_l||G_m)], \ m = \overline{2,M-1}. \tag{16.d}$$

We shall prove, for example, the inequalities (15.d), which are the consequence of the inequalities (15.d). Let us consider the tests $\Phi \in \mathcal{A}$ such that $E_{m,m|m,M}(\Phi) = E_{m,m|m,M}$ and $E_{m,m|m,l}(\Phi) = E^*_{m,m|m,l}$, $l = \overline{1,m-1}$, $m = \overline{1,M-1}$. The corresponding error probabilities $\alpha_{m,m|m,M}(\Phi_N)$ and $\alpha_{m,m|m,l}(\Phi_N)$ are given as products defined by (13.b). Because $\Phi \in \mathcal{A}$, then

$$\overline{\lim_{N\to\infty}} -\frac{1}{N}\log(1-\alpha_{m|m}(\varphi^2_N)) = 0, \ m = \overline{2,M-1}, \tag{17.a}$$

$$\overline{\lim_{N\to\infty}} -\frac{1}{N}\log(1-\alpha_{m|m}(\varphi^1_N)) = 0, \ m = \overline{2,M-1}. \tag{17.b}$$

According to (5), (13) and (17) we obtain that

$$E^*_{m,m|m,M}(\Phi) = E^*_{m|M}(\varphi^2), \quad m = \overline{2, M-1}, \tag{18.a}$$

$$E^*_{m,m|m,l}(\Phi) = E^*_{m|l}(\varphi^2), \quad m = \overline{2, M-1}. \tag{18.b}$$

So (16.d) is consequence of (15.d).

It follows from (10) and (16) that conditions (9) of Theorem 1 take place for both objects. According to Theorem 1 there exist LAO sequences of tests $\varphi^{*,1}$ and $\varphi^{*,2}$ for the first and the second objects such that the elements of the matrices $\mathbf{E}(\varphi^{*,1})$ and $\mathbf{E}(\varphi^{*,2})$ are determined according to (8). We consider the sequence of tests Φ^*, which is composed of the pair of sequences of tests $\varphi^{*,1}$, $\varphi^{*,2}$ and we will show that Φ^* is LAO and other elements of the matrix $\mathbf{E}(\Phi^*)$ are determined according to (14).

It follows from (16), (10) and (9) that the requirements of Lemma are fulfilled. Applying Lemma we can deduce that the reliability matrix $\mathbf{E}(\Phi^*)$ can be obtained from matrices $\mathbf{E}(\varphi^{*,1})$ and $\mathbf{E}(\varphi^{*,2})$ as in (12).

When conditions (15) take place, we obtain according to (12.b), (8), (10) and (18), that the elements $E_{m_1,m_2|l_1,l_2}(\Phi^*)$, $m_i \neq l_i$, $m_{3-i} = l_{3-i}$, $i = 1, 2$, of the matrix $\mathbf{E}(\Phi^*)$ are determined by relations (14.b). From (12.a) and (12.b) we obtain (14.c). The equality in (14.d) is a particular case of (6). From (14.b) it follows that all elements of $\mathbf{E}(\Phi^*)$ are positive.

Now let us show that the compound test Φ^* for two objects is LAO, that is it is optimal. Suppose that for given $E_{m,m|m,M}$, $E_{m,m|M,m}$, $m = \overline{1, M-1}$, there exists a test $\Phi' \in \mathcal{A}$ with matrix $\mathbf{E}(\Phi')$, such that it has at least one element exceeding the respective element of the matrix $\mathbf{E}(\Phi^*)$. This contradicts to the fact, that LAO tests have been used for the objects X_1 and X_2.

b) When one of the inequalities (15) is violated, then from (14.b) we see, that some elements in the matrix $\mathbf{E}(\Phi^*)$ must be equal to zero.

c) When $\Phi \in \mathcal{B}$, then from (10) and (12.a) it follows that the elements $E_{m',m'|M,M} = 0$.

Remark 2: For every $\Phi \in \mathcal{B}$, from independence of two objects, the definition (4) and equalities (10) we have that

$$E_{m',m'|M,m'}(\Phi) = \overline{\lim_{N\to\infty}} -\frac{1}{N}\log(1 - \alpha_{m'|m'}(\varphi^2)) > 0, \tag{19.a}$$

$$E_{m',m'|m',M}(\Phi) = \overline{\lim_{N\to\infty}} -\frac{1}{N}\log(1 - \alpha_{m'|m'}(\varphi^1)) > 0. \tag{19.b}$$

For $\Phi \in \mathcal{B}$ we obtain

$$E_{m_1,m_2|l_1,l_2}(\Phi) = E_{m_1|l_1}(\varphi^1) + E_{m_2|l_2}(\varphi^2), \quad m_1 \neq l_1, \quad m_2 \neq l_2, \tag{20.a}$$

$$E_{m,m_2|m,l_2}(\Phi) = E_{m_2|l_2}(\varphi^2), \quad E_{m_1,m|l_1,m}(\Phi) = E_{m_1|l_1}(\varphi^1), \quad m_i \neq l_i, \ i = 1, 2. \tag{20.b}$$

From (19) we obtain that,

$$E_{m',m_2|m',l_2}(\Phi) = E_{m_2|l_2}(\varphi^2) + E_{m',m'|m',M}, \quad m_2 \neq l_2, \tag{20.c}$$

$$E_{m_1,m'|l_1,m'}(\Phi) = E_{m_1|l_1}(\varphi^1) + E_{m',m'|M,m'}, \quad m_1 \neq l_1. \tag{20.d}$$

But in this case the elements $E_{m',m'|M,M}(\Phi) = 0$.

From (20) we see, that LAO test $\Phi' = (\varphi'^1, \varphi'^2)$, is composed of the tests φ'^1, φ'^2 discussed in Corollary 2.

4 Example

Let us consider for example the set $\mathcal{X} = \{0, 1\}$ of two elements and the following probability distributions given on $\mathcal{X}$: $G_1 = \{0,10; 0,90\}$, $G_2 = \{0,85; 0,14\}$, $G_3 = \{0,23; 0,77\}$. In Fig. 1 and Fig. 2 the results of calculations of functions $E_{2|1}(E_{1|1})$ and $E_{2,1|1,2}(E_{1,1|3,1}, E_{2,2|2,3})$ are presented. For this distributions we have $\min(D(G_2, G_1), D(G_3, G_1)) \approx 2,2$ and $\min(E_{2,2|2,1}, D(G_3, G_2)) \approx 1,4$. We see that when the inequality (15.b) and (15.c) are violated, then $E_{2,1|1,2} = 0$ and, when the first inequality in (9) is violated then $E_{2|1} = 0$.

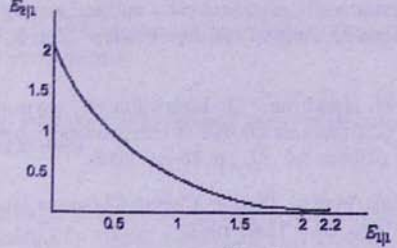

Fig. 1

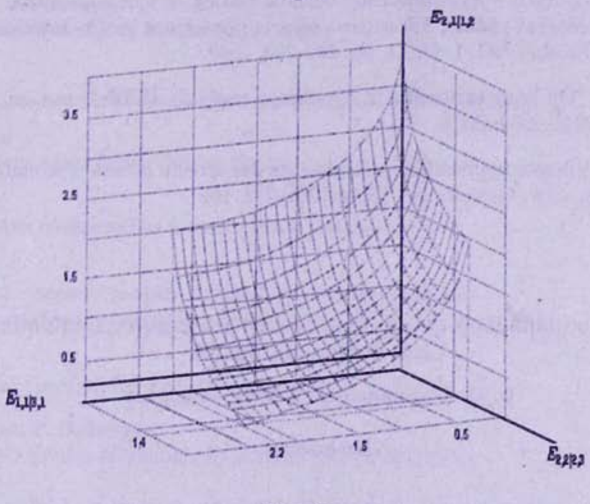

Fig. 2

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Երկու օբյեկտների զույգի նկատմամբ բազմակի վարկածների ԼԱՕ ստուգման մասին

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Ամփոփում

Լուծված է երկու անկախ օբյեկտներից կազմված մոդելի համար բազմակի վարկածների ստուգման խնդիրը: $M(\geq 2)$ հավանականային բաշխումները հայտնի են, և օբյեկտներից յուրաքանչյուրը անկախորեն ընդունում է դրանցից մեկը: Այս մոդելի համար ուսումնասիրվել է բոլոր հնարավոր զույգերի սխալների հավանականությունների ցուցիչների (հուսալիությունների) փոխկախվածությունը: Այս խնդիրը առաջադրել են (և լուծել երկու հավանականային բաշխումների դեպքի համար) Հարությունյանը և Ալսվեդեն: