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Intellectual System of Students' Testing

Sergey V. Barkhudaryan

Institute for Informatics and Automation Problems of NAS of RA
e-mail sergeybarkh@mail.ru

Abstract

This paper presents a distance testing system that allows testing of students with dynamically created sequences of tests. The sequence of questions given to students is generated in the process of testing based on his/her answers to the consequent test. Overall performance of developed system is tested by the creation of the corresponding mathematical model.

1 Introduction

The system is aimed at automating the testing processes conducted for graduate and post graduate students as well as representatives of other educational levels. Being rather flexible and dynamic it can serve for testing different subjects. It has been developed with PHP/MySQL/Apache technologies that make it multi-environment [1], [2].

In the beginning of the test the supervisor firstly defines the number of tests offered to students. The system is generating an "intellectual" sequence of tests. Analyzing the answers of a student, the system defines the next test with corresponding complexity. The selected test has a more or less complexity adapted to knowledge of the tester. Let's note that the uniqueness and difference of the system from the similar systems is its intelligence and adaptability.

The system works in the local network as well as via Internet. It is trilingual: Armenian, Russian, and English, and the choice of language is upon the user.

System has three types of users:

- Administrator
- Supervisor
- Student

The Administrator can add users, create questions, add/remove themes, change the system settings and so on.

The Supervisor can create course(s), questions with answers, add students for testing and so on. Through the monitoring module the supervisor can get information about students

taking the test at any time. The Supervisor can also define the days and duration of exam, balls for scoring answers.

The Student has only user status. She/he logs in with his/her username and password, and can view the information about past exams as well as currently available exams and courses.

With the purposes of effective testing it is necessary to build such sequence of tests that provide the most complete test of student knowledge with the given subject.

Let's define the mathematical model of the problem of building the sequence of tests.

2 The task of testing

Let's consider the following task of testing student knowledge in a certain subject. The educational materials consist of educational units, and for testing these units such kind of tests with different complexity are offered.

The task of testing is in drawing up tests with such sequence that meet the following requirements:

1. tests for various educational units are different;
2. the sequence of tests should cover the whole teaching material: if the student has answered the questions included in constructed sequence of tests to an "acceptable degree", she/he will answer tests not belonging to this sequence as well.

Let's assume that $D = \{d_1, d_2, \ldots, d_n\}$ is a set of some educational material, where d_i - corresponds to i-th educational unit (we will call d_i as a educational unit). For d_i different (one or more) tests are developed, based on answers to which the degree of mastering of educational unit is estimated.

For different educational units the tests vary in pairs. Thus, for each educational unit d_i a set of tests $T(d_i)$ is given, where $|T(d_i)| \geq 1, \quad T(d_i) \cap T(d_j) = \emptyset, \quad i \neq j$.

T presents the set $\bigcup_{i=1}^{n} T(d_i)$, which we will call base of tests.

N presents the power of the set T. Notice, that $N \geq n$.

The set of tests T is characterized by a matrix of connection of tests $K = \{k_{ij}\}, \quad i, j = 1, \ldots, N$, where $k_{ij}, \quad 0 \leq k_{ij} < 1$ shows the volume of knowledge checked by the test t_j, which is included in the test t_i. From here follows, that for matrix K the following conditions are satisfied:

$$1) 0 \leq k_{ij} < 1, \quad 2) \sum_{j=1}^{N} k_{ij} = 1, \quad 3) 0 < k_{ii} \leq 1.$$

Let's assume that number k_{ii} is the complexity of the test t_i and designate it as $c(t_i)$.

For checking knowledge of each test $t \in T$ different s variants of answers are defined, which are characterized by numbers $a_1(t), a_2(t), \ldots, a_s(t), \quad s \geq 1$, where $a_i(t)$ shows the degree of knowledge of the test t in percentage. There is a unique $i, \quad 1 \leq i \leq s$, for which $a_i(t) = 100$.

The answer to the test t is estimated by expression $x(t) = (c(t) * a(t))/100$, where complexity $c(t)$ of the test t and selection of the $a_i(t)$ answer to the test t is taken into account. If $a_i(t) = 100$ is selected, then $x(t) = c(t)$, hence, the maximal estimation on the test t is defining as $x_{max}(t) = c(t)$. The task of testing is in finding tests in such sequence

of tests $H = (t_{i_1}, t_{i_2}, \ldots, t_{i_k})$, that answers $x(t_{i_1}), x(t_{i_2}), \ldots, x(t_{i_k})$ allow to more adequately estimate the knowledge of a student.

Notice, that the sequence of tests is characterizing with complexity $C(H) = \sum_{t \in H} c(t)$.

Let's bring the above mentioned task to a task of finding of the strongly connected components in the graph G, corresponding to set of tests T [3], [6].

Graph $G(T, E)$ is built as follows. The vertex set of the graph G is a set of test T. Arc of the graph G is the ordered pair (t_i, t_j) , for which $k_{ij} > 0$. In the graph G the vertexes and arcs are assigned weights: vertex t_i has weight $c(t_i)$, and arc (t_i, t_j) has weight k_{ij}. The top t_j is achievable from top t_i, if from t_i to t_j there is a oriented path.

For graph G the matrix of achievements to $R = \{r_{ij}\}$, $i, j = 1, \ldots, N$ is defined as follows: $r_{ij} = 1$, if top t_j is achievable from the vertex t_i, else $r_{ij} = 0$.

It is clear that in the matrix R all diagonal elements are equal to one, because each vertex is achievable from it's own.

Let's define the set $R(t_i)$, which consists of such vertex t_j for graph G, for which $r_{ij} = 1$. $R(t_i) = \{t_i\} \cup E(t_i) \cup E^2(t_i) \cup \ldots E^p(t_i)$, where $E^p(t_i)$ is a vertex set which are achievable from t_i through the path with length p.

$Q = \{q_{ij}\}$, $i, j = 1, \ldots, N$- is a matrix of counter accessibility, where $q_{ij} = 1$, if the vertex t_i is achievable from the vertex t_j, else $q_{ij} = 0$ [6].

From the definitions above, it follows, that $Q = R^t$, where R^t is the result of transposing of a matrix R.

The strong component of the connectivity of graph G is called maximal (on inclusion) subgraph of graph G, where vertex are mutually achievable.

It is obvious, that strongly connected components break all vertex set of the graph G into mutually not crossed subsets $T_1, T_2, \ldots T_\tau$, which cover all set of tests $\bigcup_{i=1}^{\tau} T_i = T$.

The strongly connected components of the graph G can be defined with the matrix R and Q [6].

Let's designate $R \otimes Q$ as a result of the multiplying the matrixes R and Q element by element.

Notice, that i-th row of matrix $R \otimes Q$ contains units in that j column, from which t_i and t_j are mutually achievable; the rest elements i-th row are equal to 0.

Thus, the two vertices of the graph are in one strongly connected component, if lines corresponding to them in a matrix $R \otimes Q$ are equal. The matrix $R \otimes Q$ through emergence of rows and columns is brought to block-diagonal form.

Each block consists of units, and corresponding lines or columns of matrix $R \otimes Q$ define strongly connected component of graph G. Thus, graph G is broken to strongly connected components. All strongly connected components are covering the set of tests T of the educational material D. It is offered to include tests from strongly connected components into the sequence of tests H, $H \subset T$, and a test with a given complexity is chosen from each connected component.

During the testing, tests constructed from sequence H are offered to a student. The first test is randomly selected from H. Let's assume, that the selected test corresponds to i-th component.

Every other test is defined on the basis of the answers to the previous test. If the answer of the student $x(t) < \frac{1}{2}$, then the next test will be selected from the same component, that has smaller complexity. If $x(t) \geq \frac{1}{2}$, then $(i+1)$-the test from H is given. In case of impossibility of a choice of the next test from H, a test with greater complexity is selected from the same component. The process of definition of tests and testing of the student comes

to the end when quantity of the given tests are equal to the number beforehand determined by the teacher as parameter of the system.

Construction of sequence of tests taken from strongly connected component makes possible inclusion into itself tests of the various complexities covering all teaching material D and being most essential.

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Ուսանողների տեստավորման ինտելեկտուալ համակարգ

Ս. Վ. Բարխուդարյան

Ամփոփում

Առաջարկվող հոդվածում նկարագրված է համակարգ, որը հնարավորություն է տալիս կազմակերպել հեռահար տեստավորում դինամիկ ստեղծվող հարցաշարերի միջոցով: Ուսանողին տրվող հարցերի հաջորդականությունը գեներացվում է տեստավորման ընթացքում պատասխանի հիման վրա: Մշակված համակարգի գործունեության արդյունավետությունը ստուգվում է համապատասխան մաթեմատիկական մոդելի ստեղծմամբ: