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Guessing Subject to Distortion and Reliability Criteria

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Abstract

A problem of guessing messages of a discrete memoryless source $\{X\}$ within given distortion level Δ is considered. It is also demanded that for a given guessing list, distortion level $\Delta \geq 0$ and reliability $E > 0$ the probability that distortions between blocklength N source messages and each of first $L(N) > 0$ guessing vectors are larger than Δ, do not exceed 2^{-NE}.

The minimal (over all guessing lists) $L(N)$ and expected number of required guesses with respect to distortion and reliability criteria are found.

1 Introduction

We investigate the guessing problem with respect to fidelity and reliability criteria. The messages of a discrete memoryless stationary source must be guessed in the framework of given distortion and reliability. The problem is to determine the minimal expected number of sequential guesses until the satisfactory message will be found. The guesser have a testing mechanism by which he can know what estimate is successful, is within given distortion level.

The guessing problem was originally considered by Massey [15] and later was investigated in [2]-[6], [8], [16]. The guessing problem subject to fidelity criterion was considered by Arikan and Merhav in [4]-[6]. We study the problem with addition of the reliability criterion.

The source $\{X\}$ is defined as a sequence $\{X_i\}_{i=1}^{\infty}$ of discrete, independent, identically distributed random variables taking values in the finite set $\mathcal{X}$, which is the alphabet of messages of the source. Let $\mathbf{X} = (X_1, X_2, \ldots, X_N)$ be a sequence of messages. Denote by $\hat{X}$ the reconstruction of the source message, with values in finite set $\hat{\mathcal{X}}$, which in general is different from $\mathcal{X}$ and is called reproduction alphabet. Let $\mathcal{X}^N$ and $\hat{\mathcal{X}}^N$ denote the N-th order Cartesian powers of the sets $\mathcal{X}$ and $\hat{\mathcal{X}}$, respectively. We consider a single-letter distortion measure between source and reproduction alphabets:

$$d : \mathcal{X} \times \hat{\mathcal{X}} \to [0; \infty).$$

The distortion measure between a source vector $\mathbf{x} = (x_1, x_2, \ldots, x_N) \in \mathcal{X}^N$ and a reproduction vector $\hat{\mathbf{x}} = (\hat{x}_1, \hat{x}_2, \ldots, \hat{x}_N) \in \hat{\mathcal{X}}^N$ is defined as the average of the components' distortions:

$$d(\mathbf{x}, \hat{\mathbf{x}}) = \frac{1}{N} \sum_{n=1}^{N} d(x_n, \hat{x}_n).$$

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Let

$$P^* = \{P^*(x), x \in \mathcal{X}\}$$

be the given probability distribution (PD) of the source messages criation. Since we study the memoryless source:

$$P^{*N}(\mathbf{x}) = \prod_{n=1}^{N} P^{*N}(x_n).$$

The ordered list of sequential guesses $\mathcal{G}_N = \{\hat{\mathbf{x}}(1), \hat{\mathbf{x}}(2), \dots\}$ (which are vectors in $\hat{\mathcal{X}}^N$) is called the guessing strategy. For the given guessing strategy $\mathcal{G}_N$ we name a guessing function and note G_N the function that maps each vector $\mathbf{x} \in \mathcal{X}^N$ into a positive integer:

$$G_N : \mathcal{X}^N \to \{1, 2, 3, \dots\},$$

which is the index i of the first such guessing vector $\hat{\mathbf{x}}(i) \in \mathcal{G}_N$ that $d(\mathbf{x}, \hat{\mathbf{x}}(i)) \leq \Delta$. i is the number of sequential guesses for a source vector $\mathbf{x} \in \mathcal{X}^N$ until the successful estimate $\hat{\mathbf{x}}(i) \in \mathcal{G}_N$ will be found.

For a given distortion level $\Delta \geq 0$, positive number $L(N)$, and a guessing strategy $\mathcal{G}_N$ we consider the following set:

$$\mathcal{A}(L_N, \mathcal{G}_N, \Delta) = \{\mathbf{x} \in \mathcal{X}^N : d(\mathbf{x}, \hat{\mathbf{x}}(i)) \leq \Delta, \ i \leq L(N)\},$$

and probability for i to exceed $L(N)$:

$$e(L_N, \mathcal{G}_N, \Delta) = 1 - P^{*N}(\mathcal{A}(L_N, \mathcal{G}_N, \Delta)).$$

A pair of "guessing rates" (R, R') is called (E, Δ)-achievable for $E > 0$, $\Delta \geq 0$, if for every $\varepsilon > 0$ and sufficiently large N there exists a guessing strategy $\mathcal{G}_N$ with function G_N such that (log-s and exp-s are taken to the base 2)

$$\frac{1}{N} \log L(N) \leq R + \varepsilon,$$

$$\frac{1}{N} \log \mathrm{E}\{G_N(\mathbf{X})\} \leq R' + \varepsilon,$$

and

$$e(L_N, \mathcal{G}_N, \Delta) \leq \exp\{-NE\}.$$

Remark that such guessing function G_N may be considered as a "good" encoding function in rate-reliability-distortion coding problem, in which achievable coding rate R ensures demands of receiver both to distortion level and to reliability. This problem (a generalization of the rate-distortion problem) originally was formulated and solved for one-way system by Haroutunian and Mekoush [13] and later was investigated for various multiterminal systems (see for example [12], [14], [17]).

Denote by $\mathcal{R}_G(P^*, E, \Delta)$ and call it the distortion-reliability guessing rate region the set of all (E, Δ)-achievable guessing rates. When $E \to \infty$, $R = \log|\mathcal{X}|$, $R_G(P^*, E, \Delta)$ becomes the distortion guessing rate function $R_G(P^*, \Delta)$, found by Arikan and Merhav [5].

In the next section we specify the distortion-reliability guessing rates region. The proof is presented in the section 3.

2 Formulation of Result

Let $P = \{P(x), x \in \mathcal{X}\}$ be a PD on $\mathcal{X}$ and

$$Q = \{Q(\hat{x} \mid x), x \in \mathcal{X}, \hat{x} \in \hat{\mathcal{X}}\},$$

be a conditional PD on $\hat{\mathcal{X}}$ for given x.

Consider for given $E > 0$ the following set of PD P:

$$\alpha(E) = \{P : D(P \parallel P^*) \leq E\}.$$

Denote by $\Phi(P, \Delta) = Q_P(\hat{x} \mid x) = Q_P$ the function, which puts into the correspondence to PD P some such conditional PD Q_P that for given Δ, the following condition is fulfilled:

$$\mathbf{E}_{P,Q_P} d(X, \hat{X}) = \sum_x P(x) Q_P(\hat{x} \mid x) d(x, \hat{x}) \leq \Delta.$$

Denote by $\mathcal{M}(P, \Delta)$ the set of all such functions $\Phi(P, \Delta)$ for given Δ and P. Below for brevity we shall just write $\Phi(P)$. Let us introduce the following region:

$$\mathcal{R}_G^*(P^*, E, \Delta) = \{(R, R') : \ R \geq R(P^*, E, \Delta), \ R' \geq \max_{P \in \alpha(E)} (R(P, \Delta) - D(P \parallel P^*))\},$$

where $R(P, \Delta)$ is the rate-distortion function for PD P (see [7], [9] and [10]):

$$R(P, \Delta) = \min_{\Phi(P) \in \mathcal{M}(P, \Delta)} I_{P, \Phi(P)}(X \wedge \hat{X}),$$

$R(P^*, E, \Delta)$ is the rate-reliability-distortion function for PD P^* (see [13]):

$$R(P^*, E, \Delta) = \max_{P \in \alpha(E)} \min_{\Phi(P) \in \mathcal{M}(P, \Delta)} I_{P, \Phi(P)}(X \wedge \hat{X}).$$

Theorem: For given PD P^* and every $E > 0$, $\Delta \geq 0$,

$$\mathcal{R}_G(P^*, E, \Delta) = \mathcal{R}_G^*(P^*, E, \Delta).$$

Corollary: When $E \to \infty$, $R = \log |\mathcal{X}|$, we arrive at the result of Arikan and Merhav [5]:

$$R_G(P^*, \Delta) = \max_P (R(P, \Delta) - D(P \parallel P^*)).$$

3 Proof of the theorem

We apply the typical sequences technique (see [9]-[11]). The proof of the inequality

$$R_G^*(P^*, E, \Delta) \subseteq R_G(P^*, E, \Delta) \tag{1}$$

is based on the following random coding lemma about covering of types of vectors, which is a modification of the covering lemmas from [1], [10], [14]:

Lemma: Let for $\varepsilon > 0$,

$$L(P, Q) = \exp\{N(I_{P,Q}(X \wedge \hat{X}) + \varepsilon)\}.$$

Then for every type P and conditional distribution Q for N large enough there exist the collections of vectors

$$\{\hat{\mathbf{x}}_l \in \mathcal{T}_{P,Q}(\hat{X}),\ l = \overline{1, L(P,Q)}\},$$

such that the sets

$$\{\mathcal{T}_{P,Q}(X \mid \hat{\mathbf{x}}_l),\ l = \overline{1, L(P,Q)}\},$$

cover $\mathcal{T}_P(X)$.

The proof of lemma is similar to the proof of lemmas in [1], [10], [12], [14].

Denote by $\mathcal{P}(X,N)$ the set of all distributions P, which for given N are types. Let us represent $\mathcal{X}^N$ as a family of disjoint types

$$\mathcal{X}^N = \bigcup_{P \in \mathcal{P}(X,N)} \mathcal{T}_P(X).$$

Let some $\delta > 0$ be given. Then for N large enough the probability of appearance of the source sequences of types beyond $\alpha(E+\delta)$ can be estimated as follows:

$$P^{*N}\Big(\bigcup_{P \notin \alpha(E+\delta) \cap \mathcal{P}(X,N)} \mathcal{T}_P(X)\Big) = \sum_{P \notin \alpha(E+\delta) \cap \mathcal{P}(X,N)} P^{*N}(\mathcal{T}_P(X)) \le$$

$$\le (N+1)^{|\mathcal{X}|} \exp\{-N \min_{P \notin \alpha(E+\delta) \cap \mathcal{P}(X,N)} D(P \parallel P^*)\} \le$$

$$\le \exp\{-NE - N\delta + |\mathcal{X}| \log(N+1)\} \le \exp\{-N(E+\delta/2\}.$$

Consequently, in order to obtain the desired levels of probabilities $e(L_N, \mathcal{G}_N, \Delta)$, it is sufficient to construct the guessing strategy only for the vectors with types P from $\alpha(E+\delta) \cap \mathcal{P}(X,N)$.

Let us order the types $P \in \alpha(E+\delta) \cap \mathcal{P}(X,N)$ as $\{P_1, P_2, \ldots\}$ according to increasing value of corresponding rate-distortion functions $R(P,\Delta)$:

$$R(P_i,\Delta) \le R(P_{i+1},\Delta),\ 1 \le i \le |\alpha(E+\delta) \cap \mathcal{P}(X,N)|.$$

For fixed i let the set $\{\hat{\mathbf{x}}_{P_i,l} \in \mathcal{T}_{P_i,Q^{\min}_{P_i}}(\hat{X}),\ l = \overline{1, L(P_i, Q^{\min}_{P_i})}\}$, with

$$L(P_i, Q^{\min}_{P_i}) = \exp\{N(\min_{\Phi(P_i) \in \mathcal{M}(P_i,\Delta)} I_{P_i,\Phi(P_i)}(X \wedge \hat{X}) + \varepsilon)\} = \exp\{N(R(P_i,\Delta) + \varepsilon)\}$$

be a collection of vectors for which according to the lemma for N large enough the sets

$$\{\mathcal{T}_{P_i,Q^{\min}_{P_i}}(X \mid \hat{\mathbf{x}}_l),\ l = \overline{1, L(P_i, Q^{\min}_{P_i})}\},$$

cover $\mathcal{T}_{P_i}(X)$.

We construct a guessing strategy as follows:

$$\mathcal{G}^*_N = \{\{\hat{\mathbf{x}}_{P_1,l},\ l = \overline{1, L(P_1, Q^{\min}_{P_1})}\}, \{\hat{\mathbf{x}}_{P_2,l},\ l = \overline{1, L(P_2, Q^{\min}_{P_2})}\}, \ldots\}.$$

The number of required guesses $G_N(\mathbf{x})$ for $\mathbf{x} \in \mathcal{T}_{P_i}(X)$, $P_i \in \alpha(E+\delta) \cap \mathcal{P}(X,N)$, is upper-bounded by

$$G_N(\mathbf{x}) \le \sum_{P_j,\, j \le i} L(P_j, Q^{\min}_{P_j}) \le L(P_i, Q^{\min}_{P_i}) \exp\{N\varepsilon\} = \exp\{N(R(P_i,\Delta) + 2\varepsilon)\}.$$

Hence

$$L(N) \geq \exp\{N(R(P^*, E+\delta, \Delta) + 2\varepsilon)\}.$$

Using upper estimate [9], [10] for probability of the set $\mathcal{T}_P(X)$ we have

$$\mathrm{E}\{G_N^*(\mathbf{X})\} = \sum_{\substack{\mathbf{x} \in \mathcal{T}_P(X), \\ P \in \alpha(E+\delta) \cap \mathcal{P}(X,N)}} P^{*N}(\mathbf{x}) G_N^*(\mathbf{x}) =$$

$$= \sum_i \sum_{\substack{\mathbf{x} \in \mathcal{T}_{P_i}(X), \\ P_i \in \alpha(E+\delta) \cap \mathcal{P}(X,N)}} P^{*N}(\mathbf{x}) G_N^*(\mathbf{x}) \leq$$

$$\leq \sum_i \sum_{\substack{\mathbf{x} \in \mathcal{T}_{P_i}(X), \\ P_i \in \alpha(E+\delta) \cap \mathcal{P}(X,N)}} P^{*N}(\mathbf{x}) \exp\{N(R(P_i, \Delta) + 2\varepsilon)\} \leq$$

$$\leq \sum_{P \in \alpha(E+\delta) \cap \mathcal{P}(X,N)} \exp\{N(-D(P \parallel P^*) + R(P, \Delta) + 2\varepsilon)\} \leq$$

$$\leq \max_{P \in \alpha(E+\delta) \cap \mathcal{P}(X,N)} \exp\{N(-D(P \parallel P^*) + R(P, \Delta) + 3\varepsilon)\} =$$

$$= \exp\{N(\max_{P \in \alpha(E+\delta) \cap \mathcal{P}(X,N)} (-D(P \parallel P^*) + R(P, \Delta) + 3\varepsilon))\} \leq$$

$$\leq \exp\{N(\max_{P \in \alpha(E+\delta)} (-D(P \parallel P^*) + R(P, \Delta) + 3\varepsilon))\}.$$

Therefore a pair of guessing rates (R, R') such that

$$R \geq \frac{1}{N} \log L(N) - \varepsilon \geq R(P^*, E+\delta, \Delta) + \varepsilon,$$

$$R' \geq \max_{P \in \alpha(E+\delta)} (-D(P \parallel P^*) + R(P, \Delta) + 3\varepsilon) + \varepsilon$$

is (E, Δ)-achievable.

Taking into account arbitrariness of ε and δ, continuity of all functions with respect to E, we obtain (1).

Now we shall prove the inclusion

$$R_G(P^*, E, \Delta) \subseteq R_G^*(E, \Delta). \tag{2}$$

Let $\varepsilon > 0$ be fixed and $\mathcal{G}_N$ be an arbitrary guessing strategy for which $e(L_N, \mathcal{G}_N, \Delta) \leq \exp\{-NE\}$ for some $L(N)$. We can write:

$$\left|\mathcal{A}(L_N, \mathcal{G}_N, \Delta) \bigcap \mathcal{T}_P(X)\right| = |\mathcal{T}_P(X)| - \left|\overline{\mathcal{A}(L_N, \mathcal{G}_N, \Delta)} \bigcap \mathcal{T}_P(X)\right|.$$

For $P \in \alpha(E-\varepsilon) \cap \mathcal{P}(X,N)$ for N large enough the following estimates take place:

$$\left|\overline{\mathcal{A}(L_N, \mathcal{G}_N, \Delta)} \bigcap \mathcal{T}_P(X)\right| = \frac{P^{*N}(\overline{\mathcal{A}(L_N, \mathcal{G}_N, \Delta)} \cap \mathcal{T}_P(X))}{P^{*N}(\mathbf{x})} \leq$$

$$\leq \exp\{N(H_P(X) + D(P \parallel P^*))\} \exp\{-NE\} \leq$$

$$\leq \exp\{N(H_P(X) + E - \varepsilon)\} \exp\{-NE\} = \exp\{N(H_P(X) - \varepsilon)\}.$$

Hence for N large enough

$$|\mathcal{A}(L_N,\mathcal{G}_N,\Delta)\bigcap\mathcal{T}_P(X)|\geq(N+1)^{-|\mathcal{X}|}\exp\{NH_P(X)\}-\exp\{N(H_P(X)-\varepsilon)\}=$$
$$=\exp\{N(H_P(X)-\varepsilon)\}\left(\exp\{N\varepsilon\}(N+1)^{|-\mathcal{X}|}-1\right)\geq\exp\{N(H_P(X)-\varepsilon)\}. \quad (3)$$

To each $\mathbf{x}\in\mathcal{A}(L_N,\mathcal{G}_N,\Delta)\bigcap\mathcal{T}_P(X)$ an unique guessing vector $\hat{\mathbf{x}}(i)\in\mathcal{G}_N$ corresponds, such that $G_N(\mathbf{x})=i$. This vector determines a conditional type Q, for which $\hat{\mathbf{x}}(i)\in\mathcal{T}_{P,Q}(\hat{X}\mid\mathbf{x})$. Since $\mathbf{x}\in\mathcal{A}(L_N,\mathcal{G}_N,\Delta)$, then $\mathbf{E}_{P,Q}d(X,\hat{X})=d(\mathbf{x},\hat{\mathbf{x}}(i))\leq\Delta$. So, $Q\in\mathcal{M}(P,\Delta)$. The set of all vectors $\mathbf{x}\in\mathcal{A}(L_N,\mathcal{G}_N,\Delta)\bigcap\mathcal{T}_P(X)$ is divided into classes corresponding these conditional types Q and is the union of all such Q - shells. Let us select the Q_P - shell ($Q_P=\Phi(P)$) having maximal cardinality for given P and denote it by $(\mathcal{A}(L_N,\mathcal{G}_N,\Delta)\bigcap\mathcal{T}_P(X))\,(\Phi(P))$. Using polynomial upper estimate [9], [10] for the number of conditional types Q, we have for N large enough

$$|\mathcal{A}(L_N,\mathcal{G}_N,\Delta)\bigcap\mathcal{T}_P(X)|\leq(N+1)^{|\mathcal{X}||\hat{\mathcal{X}}|}\,|(\mathcal{A}(L_N,\mathcal{G}_N,\Delta)\bigcap\mathcal{T}_P(X))\,(\Phi(P))|\leq$$
$$\leq\exp\{N\varepsilon\}\,|(\mathcal{A}(L_N,\mathcal{G}_N,\Delta)\bigcap\mathcal{T}_P(X))\,(\Phi(P))|. \quad (4)$$

Let $\mathcal{B}(P,\Phi(P),\mathcal{A}(L_N,\mathcal{G}_N,\Delta))$ be the set of all guessing vectors $\hat{\mathbf{x}}(i)\in\mathcal{G}_N$, which satisfy $G_N(\mathbf{x})=i$ for some $\mathbf{x}\in\mathcal{A}(L_N,\mathcal{G}_N,\Delta)\bigcap\mathcal{T}_P(X)$, $\mathbf{x}\in\mathcal{T}_{P,\Phi(P)}(X\mid\hat{\mathbf{x}}(i))$. In accordance with the definition of the guessing strategy

$$|\mathcal{B}(P,\Phi(P),\mathcal{A}(L_N,\mathcal{G}_N,\Delta))|\leq\max_{\mathbf{x}\in\mathcal{A}(L_N,\mathcal{G}_N,\Delta)\bigcap\mathcal{T}_P(X)}G_N(\mathbf{x}).$$

Then

$$\left|\left((\mathcal{A}(L_N,\mathcal{G}_N,\Delta)\bigcap\mathcal{T}_P(X)\right)(\Phi(P))\right|\leq\sum_{\hat{\mathbf{x}}\in\mathcal{B}(P,\Phi(P),\mathcal{A}(L_N,\mathcal{G}_N,\Delta))}\left|\mathcal{T}_{P,\Phi(P)}(X\mid\hat{\mathbf{x}})\right|\leq$$

$$\leq\exp\{NH_{P,\Phi(P)}(X\mid\hat{X})\}\cdot\max_{\mathbf{x}\in\mathcal{A}(L_N,\mathcal{G}_N,\Delta)\bigcap\mathcal{T}_P(X)}G_N(\mathbf{x}).$$

From the last inequality, (3) and (4) we obtain that for $P\in\alpha(E-\varepsilon)\cap\mathcal{P}(X,N)$

$$\max_{\mathbf{x}\in\mathcal{A}(L_N,\mathcal{G}_N,\Delta)\bigcap\mathcal{T}_P(X)}G_N(\mathbf{x})\geq\exp\{N(I_{P,\Phi(P)}(X\wedge\hat{X})-2\varepsilon)\}.$$

Hence

$$L(N)\geq\exp\{N(R(P^*,E-\varepsilon,\Delta)-2\varepsilon)\}.$$

Using inner estimate [9], [10] for probability of the set $T_P(X)$ we obtain

$$\mathrm{E}\{G_N(\mathbf{X})\}=\sum_{\mathbf{x}\in\mathcal{X}^N}P^{*N}(\mathbf{x})G_N(\mathbf{x})\geq\sum_{P\in\alpha(E-\varepsilon)\cap\mathcal{P}(X,N)}\sum_{\mathbf{x}\in\mathcal{T}_P(X)}P^{*N}(\mathbf{x})G_N(\mathbf{x})\geq$$

$$\geq(N+1)^{-|\mathcal{X}|}\max_{P\in\alpha(E-\varepsilon)\cap\mathcal{P}(X,N)}\max_{\mathbf{x}\in\mathcal{T}_P(X)}\{\exp\{-ND(P\parallel P^*)\}G_N(\mathbf{x})\}\geq$$

$$\geq\max_{P\in\alpha(E-\varepsilon)\cap\mathcal{P}(X,N)}\max_{\mathbf{x}\in\mathcal{A}(L_N,\mathcal{G}_N,\Delta)\bigcap\mathcal{T}_P(X)}\{\exp\{-N(D(P\parallel P^*)-\varepsilon)\}G_N(\mathbf{x})\}=$$

$$=\max_{P\in\alpha(E-\varepsilon)\cap\mathcal{P}(X,N)}\{\exp\{-N(D(P\parallel P^*)-\varepsilon)\}\max_{\mathbf{x}\in\mathcal{A}(L_N,\mathcal{G}_N,\Delta)\bigcap\mathcal{T}_P(X)}G_N(\mathbf{x})\}\geq$$

$$\geq \max_{P\in\alpha(E-\varepsilon)\cap\mathcal{P}(X,N)} \exp\{N(-D(P \parallel P^*) + I_{P,\Phi(P)}(X \wedge \hat{X}) - 3\varepsilon)\} \geq$$

$$\geq \max_{P\in\alpha(E-\varepsilon)\cap\mathcal{P}(X,N)} \exp\{N(-D(P \parallel P^*) + R(P,\Delta) - 3\varepsilon)\}.$$

Hence for N large enough

$$R \geq \frac{1}{N}\log L(N) - \varepsilon \geq R(P^*, E-\varepsilon, \Delta) - 3\varepsilon,$$

$$R' \geq \frac{1}{N}\log \mathrm{E}\{G_N(\mathbf{X})\} - \varepsilon \geq \max_{P\in\alpha(E-\varepsilon)\cap\mathcal{P}(X,N)} (-D(P \parallel P^*) + R(P,\Delta) - 3\varepsilon) - \varepsilon.$$

Taking into account arbitrariness of ε, continuity by E of all functions in above expressions, we complete the proof of the inclusion (2).

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