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An Algorithm of Test Generation in Some Class of Analog Circuits

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Abstract

A mathematical formalization of thr test design problem for some class of analog circuits is proposed; an algorithm of the test design is given based on this formalisation.

Design of tests for analog circuits is the actual problem of contemporary theory of testing (see for example [1], [2], [3]), however in most cases this problem was investigated from the point of view of the requirements of practical engineering. We shall consider below the same problem in the mathematical interpretation. Of course, this mathematical approach must take into account different technical and physical peculiarities of a situation.

We consider analog circuits which can be characterized by a transfer function $H(p)$ having the form of algebraic fraction (i.e. quotient of two polynomials). We suppose that the considered circuit has a single input and a single output; we suppose also that if the input signal has sinusoidal form then the output signal has a similar form and its frequency is equal to the frequency of the input signal. So the following statement holds:

$$V_{out}(p) = V_{in}(p) \bullet H(p), \tag{1}$$

where $V_{out}(p)$ and $V_{in}(p)$ are corresponding characteristics of the output and input signal. The operational variable p can be replaced here ([4]) by $j\omega$, where j is the imaginary unit and ω is the frequency of the input or output sinusoidal signal $V = V_0 \exp j(\omega t + \varphi)$.

Let us suppose that a circuit S of the mentioned kind (for example, an analog filter) is given and some class W of its faults is fixed. The notion of tests for the given S and W is defined in the natural way. Indeed, let the circuit S_w is obtained by introducing the fault $w \in W$ into the circuit S and let H and H_w are transfer functions correspondingly for S and S_w. Then the fault $w \in W$ is said to be testable if there exists a frequency such that

$$\mid H(j\omega) \mid \neq \mid H_w(j\omega) \mid; \tag{2}$$

(we introduce this more strong condition than $H(j\omega) \neq H_w(j\omega)$, because the last condition must be observed both by measurement of amplitude and by a measurement of initial phase of output signal; but the measurement of initial phase of a signal is essentially more difficult than the measurement of its amplitude and the testing of the condition (2) (connected only with the measurement of amplitude) is more natural from the technical point of view). In this case ω is said to be testing frequency for the fault w. The system $(\omega_1, \omega_2, \ldots, \omega_n)$ is said to be a test for S and Wif this system consists of all testing frequencies for all testable faults $w \in W$.

Of course, this notion does not take into account the behaviour of the circuit in the case of non-sinusoidal input signal and some other physical and technical circumstances. However, additional investigations are need to be made for this cases and we do not consider them here.

Let the circuit S and the class W of its faults are given and we want to find a test for S and W. We shall discuss mathematical methods for the solving of this problem. The inequality $|H(j\omega)| \neq |H_w(j\omega)|$ is equivalent to

$$|H(j\omega)|^2 - |H_w(j\omega)|^2 \neq 0 \tag{3}$$

where in the left hand we have a real function of real variable ω; this function is an algebraic fraction. So the first variant of a mathematical interpretation of the problem is as follows: an algebraic fraction $f(\omega)$ (which is in fact the left side of inequality (3)) is given and a real number ω_0 must be found such that $f(\omega_0) \neq 0$. Of course, the problem in such variant has a trivial solution. However, it is natural to change this mathematical formulation taking into account the following technical circumstances: 1) not all frequencies are available for the technical use and we have to introduce some restrictions on ω, for example $a \leq \omega \leq b$, where a and b are limits of corresponding technical possibilities; 2) the accuracy of technical measurement is limited and it is natural to consider for satisfactory testing the inequality

$$\left||H(j\omega)|^2 - |H_w(j\omega)|^2\right| > \varepsilon \tag{4}$$

instead of (3) (here ε is the number representing the accuracy of our technical measurements). So the another mathematical interpretation of the problem can be formulated as follows: an algebraic fraction $f(\omega)$, a segment $[a, b]$, a number $\varepsilon > 0$ are given (we suppose that $f(\omega)$ has no singularities on $[a, b]$) and a number $\omega_0 \in [a, b]$ must be found such that $|f(\omega_0)| > \varepsilon$. Of course, such a number ω_0 sometimes does not exists and we must add the possibility of negative answer to the proposed question. Besides, we shall consider our problem in more general conditions: a) the solution will be formulated not only for algebraic functions but for larger class of functions, namely, for the functions satisfying the Lipschitz condition [5], i.e. there exist a constant C such that $|f(x) - f(y)| \leq C\,|x - y|$ for all $x, y \in [a, b]$ (it is well known that all functions having a continuous derivative on the segment satisfy this condition as well as all algebraic functions having no singularities on $[a, b]$); b) different constants ε_1 and ε_2 will be fixed for the cases of positive and negative answer to the main question (otherwise when only one number $\varepsilon > 0$ is given, the difficulties arise in the case when the upper bound of $|f(\omega)|$ is equal precisely to ε).

Finally we introduce the following formulation for our mathematical interpretation of the problem mentioned above. We suppose that the following objects are fixed: (1) the segment $[a, b]$; (2) the function $f(x)$ everywhere defined on $[a, b]$; (3) the constant C such that $|f(x) - f(y)| \leq C\,|x - y|$ for every $x, y \in [a, b]$; (4) the rational numbers ε_1 and ε_2 such that $0 < \varepsilon_1 < \varepsilon_2$. The following information must be found: either we must find the number $\omega_0 \in [a, b]$ such that $|f(\omega)| > \varepsilon_1$, or we must establish that $|f(\omega)| < \varepsilon_2$ for all $\omega \in [a, b]$.

Now let us describe an algorithm giving the solution of this problem. We shall denote it as "algorithm A".

Description of algorithm A.

Let all the objects (1), (2), (3), (4), mentioned above are fixed. We compute the value of f in the following points:

on the first step - in the point $(a+b)/2$;
on the second step - in the points $a+(b-a)/4$ and $a+3(b-a)/4$;
on the third step - in the points $a+(b-a)/8$, $a+3(b-a)/8$, $a+5(b-a)/8, \ldots$,
on the n-th step - in the points $a+(2k+1)(b-a)/2^n$ for all integer k such that $0 \le k \le 2^{n-1}-1$.

Every number $|f(a+(2k+1)(b-a)/2^n)|$ (which we denote as $F_{k,n}$) we compute with the accuracy $(\varepsilon_2-\varepsilon_1)/2$, i.e we find rational approximations u and v such that $u < F_{k,n} < v$ and $v-u < (\varepsilon_2-\varepsilon_1)/2$. For every number $F_{k,n}$ we note what a possibility among the following ones takes place: either $F_{k,n} < (\varepsilon_2-\varepsilon_1)/2$ or $F_{k,n} > \varepsilon_1$ (the first possibility takes place if $u \le \varepsilon_1$, the second one - if $u > \varepsilon_1$). The process of working of the algorithm A is interrupted when for some $F_{k,n}$ the inequality $F_{k,n} > \varepsilon_1$ is found; in such a case we have found the number $\omega_0 = a+(2k+1)(b-a)/2^n$ such that $|f(\omega_0)| > \varepsilon_1$ and we say in this case that the algorithm gave the "positive answer" concerning ω_0. Otherwise the process is continued until m step, where m is minimal integer such that $m \ge 1+log_2(c(b-a)/(\varepsilon_2-\varepsilon_1))$. If for all $F_{k,m}$ where $0 \le k \le 2^{m-1}-1$, we obtain the inequality $F_{k,m} < (\varepsilon_2-\varepsilon_1)/2$ then we say that the algorithm gave a "negative answer" concerning ω_0 (we shall prove below that in this case $|f(\omega)| < \varepsilon_2$ for all $\omega \in [a,b]$).

Theorem. Let $f(x)$ is the function everywhere defined on the segment $[a,b]$ and such that $|f(x)-f(y)| \le C\,|x-y|$ for all $x,y \in [a,b]$ and for some constant C. Let rational numbers ε_1 and ε_2 satisfy the inequality $0 < \varepsilon_1 < \varepsilon 2$. Then if the algorithm A applied to $[a,b]$, f, c, ε_1, ε_2 gives the positive answer concerning some $\omega_0 \in [a,b]$, then $|f(\omega_0)| > \varepsilon_1$; if algorithm A applied to the same objects gives the negative answer, then $|f(\omega_0)| < \varepsilon_2$ for all $\omega \in [a,b]$.

Proof. If the algorithm A gives the positive answer, then such $F_{k,n}$ is found that $F_{k,n} > \varepsilon_1$ and hence the number $\omega_0 = a+(2k+1)(b-a)/2^n$ satisfies the condition $|f(\omega_0)| > \varepsilon_1$. Now let us consider the case of the negative answer, Let $m \ge 1+log_2(c(b-a)/(\varepsilon_2-\varepsilon_1))$ and for all $F_{k,m}$ such that $0 \le k \le 2^{m-1}-1$ the inequality $F_{k,m} < (\varepsilon_2-\varepsilon_1)/2$ holds. Then we prove that $|f(\omega)| < \varepsilon_2$ for all $\omega \in [a,b]$. Indeed, if $\omega \in [a,b]$, then there exists an integer k, such that $0 \le k \le 2^{m-1}-1$ and $|\omega-(2k+1)(b-a)/2^m| \le (b-a)/2^m$, hence $||f(\omega)|-|f((2k+1)(b-a)/2^m)|| \le C(b-a)/2^m \le (\varepsilon_2-\varepsilon_1)/2$, so that $||f(\omega)|-F_{k,m}| \le (\varepsilon_2-\varepsilon_1)/2$. But $F_{k,m} < (\varepsilon_2-\varepsilon_1)/2$ and we obtain $|f(\omega)| < \varepsilon_2$. This completes the proof.

The algorithm described above can be used not only as a precise algorithm but also as an approximate one. Sometimes the upper bound $1+log_2(c(b-a)/(\varepsilon_2-\varepsilon_1))$ for m is too great for computational implementation; in this cases we have to consider only feasible steps of the algorithm. However in many cases the required frequency ω_0 can be found quickly. In the approximate form the algorithm is formulated as follows: the process of its working is continued until its steps are implementable by the used computational means; if the required ω_0 such that $|f(\omega_0)| > \varepsilon_1$ is not found within these possibilities, then the algorithm gives the negative answer. Such a form of the considered algorithm, probably can be suitable from the practical point of view.

The algorithm of test generation for the given circuit S and the class of its faults W now can be described as follows. Let H be the transfer function for S. We consider all the faults $w \in W$ and for every w we find the circuit S_w and the transfer function H_w for it. Then we apply the algorithm A to the function $|H|^2-|H_w|^2$ (for example, we use the approximate variant of A). If the algorithm A gives the negative answer, then we adopt that the corresponding fault w is not testable by our means (in this case indeed the difference between H and H_w cannot be great). If the algorithm A gives the positive answer then we include the obtained frequency in the list of testing frequencies. The required test is obtained as the list of all these testing frequencies.

The given mathematical interpretation must be considered of course, only as a first approximation to the mathematical description of the mentioned problem.

References

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