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Modelling of the Relaxation of Auto-Oscillatory Processes in Non-Linear RLC Circuits

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Abstract

A dynamic model describing auto-oscillatory processes of parametric type is analysed. It has been made possible to analyse a family of dynamic equations describing relaxation auto-oscillations and to increase the set of values of amplitudes and periods of auto-oscillations due to the introduction of non linear parameter l. It becomes possible to describe some types of non linear RLC circuits and relaxation of auto-oscillatory processes taking place in these circuits more precisely and comprehensively with the help of varying the coefficients of nonlinerity.

A dynamic model adequately describing relaxation of auto-oscillatory processes in some types of non linear RLC circuits is proposed. Within the framework of the proposed model, particularly at large values of the bifurcation parameter ($\mu \gg 1$), the algorithms for calculating the amplitudes and periods of relaxation of auto-oscillatory processes for the mentioned RLC circuits are found.

Investigation of the nature of processes taking place in non linear systems is of great interest in such areas as physics, engineering, biology, economics and others. The auto-oscillatory processes going on in non linear RLC circuits are of interest. A great number of works has been carried out in this field, in particular in [1,2,3]. A full investigation of those processes could be fulfilled with the help of constructing of the dynamic models endowed with the most important properties of the processes going on in real RLC circuits.

A sample of constructing such dynamic model is brought in this paper to analyse relaxation oscillations in some types of non-linear RLC circuits.

Circuit Description. The following types of RLC circuits are considered (see Fig. 1 and Fig. 2).

a) Ferromagnetic-cored coil, differential capacitor with closed p-n junction and S-type two-pole R_SC_dL connected with them in series.

b) Ferromagnetic-cored coil, differential capacitor with closed p-n junction and S-type two-pole R_SC_dL connected with them in series.

c) Coil without a core, differential capacitor with closed p-n junction and S-type two-pole R_SCL_0 connected with them in series.

d) Coil without a core, capacitor and S-type two-pole R_SCL_0 connected with them in series

e) Coil without a core, capacitor and N-type two-pole R_NCL_0 connected with them in parallel.

The following designations are used:

L is the inductance of the ferromagnetic-cored coil,
L_0 is the inductance of the coil without any core,
C_d is the differential capacity of the closed p-n junction,

C is the capacity of the capacitor,
i_S, is the current in the S-type two-pole,
i_N is the current in the N-type two-pole.

Volt-ampere characteristic of the S-type two-pole is described as following:

$$u_s = \Phi(i_s, l, \mu) = \Phi(x, l, \mu) = -x + \frac{x^{2l+1}}{\mu}, \qquad l = 1,2,3,...$$

Volt-ampere characteristic of the N-type two-pole is described as following:

$$i_N = \Phi(u_N, l, \mu) = \Phi(x, l, \mu)$$

The phase plane for the S-type two-pole with series connection to the LC-circuit is:

$$(x, y) = (i_s [-\frac{C}{L}]^{1/2} u_c)$$

where $x = i_s$, $y = [-\frac{C}{L}]^{1/2} u_c$

And the phase plane for the N-type two-pole with parallel connection to the LC-circuit is determined as following:

$$(x, y) = (u_N, [\frac{L}{c}]^{1/2} i_L)$$

where $x = u_N$, $y = [\frac{L}{C}]^{1/2} i_L$

1 is the coefficient of nonlinearity, m is the coefficient of nonlinearity determined from the following relationships:

$$i_c - m i_c^{2l+1} = C\frac{du_c}{dt} \qquad l = 1,2,3... \qquad (1)$$

In the case of the two-pole connected with an LC circuit in series (see Fig.l), using Kirchhof's laws, we get the following form of the dynamic model

$$\ddot{x} + \left[(2l+1)x^{2l} - \mu\right]\dot{x} + x - m x^{2l+1} = 0, \quad l = 1,2,3... \qquad (2)$$

Note 1. In equations (2) and (3) the differentiation is carried out by scale time that relates to the real time in the following way

$$t = (LC)^{1/2}\tau.$$

where $\mu = \sqrt{\frac{C_d}{L}}$ for the R_SC_dL circuit and $\mu = \sqrt{\frac{C}{L}}$ for the R_SCL circuit.

When m = 0, the model (2) is modified to the following model

$$\ddot{x} + \left[(2l+1)x^{2l} - \mu\right]\dot{x} + x = 0, \quad l = 1,2,3... \qquad (3)$$

The model (3) characterises auto-oscillations in the $R_SC_dL_0$, R_SCL_0, R_NCL_0 circuits, where $\mu=\sqrt{\frac{C_d}{L_0}}$ for $R_SC_dL_0$, $\mu=\sqrt{\frac{C}{L_0}}$ for R_SCL_0, $\mu=\sqrt{\frac{L_0}{C}}$ for R_NCL_0 circuits.

Note 2. In the case of m = 0, l = 1, the equation (2) reduces to the well-known Van der Pol equation having the following form

$$\ddot{x}+\left(3x^2-\mu\right)\dot{x}+x=0 \qquad (4)$$

Let now give the theorems what evidence in analogous to the proof of theorems in work [1].

Theorem 1. Equations (2) and (3) have the only limiting cycles – $\Gamma(\mu,1)$, $l=1,2,3\ldots$ which are orbitally stable.

Theorem 2. When $\mu\gg1$, $\Gamma(\mu,1)$, $l=1,2,3\ldots$, the limiting cycles are described with closed and piecewisely analytic Jordan curves –J, which are schematically presented in Fig.3.

Statement 1. When $\mu\gg1$ the equations (2) and (3) have the solutions $x(\mu,l,t)$ which accept the form of almost discontinuous damping relaxation auto-oscillations. The solutions are schematically presented in Fig. 4.

Statement 2. Let $\mu\gg1$. Limiting amplitudes of equations (2) and (3) represent the only positive real roots of the following algebraic equations.

$$x^{2l+1}-\mu x-\frac{2l}{(2l+1)^{\frac{2l+1}{2l}}}\mu^{\frac{2l+1}{2l}}=0, \quad l=1,2,3\ldots \qquad (5)$$

Statement 3. Let $\mu\gg1$, limiting periods of equations (2) are expressed by the following relationships:

$$T(\mu,l,m)=\frac{\mu(2l+1)}{lm}\ln\frac{2l+1-m\mu}{(2l+1)\left(1-ma^{2l}\right)}+2\mu^2\ln\left[\frac{1}{a}\sqrt[2l]{\frac{\mu}{2l+1}}\right]+\frac{\mu^2}{l}\ln\frac{(2l+1)\left(1-ma^{2l}\right)}{2l+1-m\mu}$$

$$l=1,2,3\ldots \qquad (6)$$

Statement 4. Let $\mu\gg1$. Limiting amplitudes of equations (3) are defined from the following relationships:

$$T_1(\mu,l)=2\mu^2\ln\left[\frac{1}{a}\sqrt[2l]{\frac{\mu}{2l+1}}\right]+\frac{\mu(2l+1)}{l}\left(a^{2l}-\frac{\mu}{2l+1}\right), \qquad l=1,2,3\ldots \qquad (7)$$

It must be noted that for the solution of algebraic equations (5) as well as for the calculation of formulae – $T(\mu, l, m)$ and $T_1((\mu, l)$ it is worked out the algorithm using Fortran.

The values of the functions $T(l,\mu,1)$ and $T_1(\mu,l)$ are presented in Table 1 for some values of quantities l and μ.

Note 3. All the quantities presented in Table 1 are dimensionless so as the following relationships can be used in the deriving equations (2) and (3):

$$\frac{u_S}{u_{bas}}+\frac{u_L}{u_{bas}}=\frac{u_c}{u_{bas}};\quad \frac{i_S}{i_{bas}}=\frac{i_L}{i_{bas}}=-\frac{i_c}{i_{bas}}=x;\qquad ;$$

where the index "bas" designates the basic value of the respective quantity.

Table 1. Some Values of Periods and Amplitudes of Relaxation Auto-Oscillations Described by Equations (2) and (3)

l		μ	T(μ, l, 1)	T(μ, l)	a(l, μ)
1		8.2	18.7714	108.506	3.30656
		8.8	-19.8665	124.965	3.42540
		9.4	-20.9696	142.587	3.65148
		10.0	-22.0789	161.371	3.65148
7		8.2	-58.2277	209.628	1.20905
		8.8	-63.7069	241.490	1.21516
	C_d, C	9.4	-69.1177	275.543	1.22090
		10.0	-74.4189	311.842	1.22631
11		8.2	-56.6227	225.216	1.13037
		8.8	-62.4110	259.381	1.13401
		9.4	-68.2629	295.957	1.13741
		10.0	-74.1634	334.944	1.14062

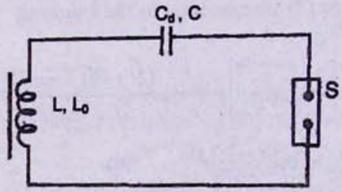

Fig.1. Nonlinear oscillating contour in series connected with a S-type two-pole

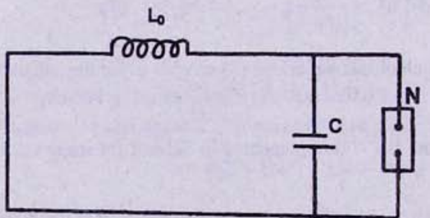

Fig.2. Nonlinear oscillating contour in parallel connected with a N-type two-pole

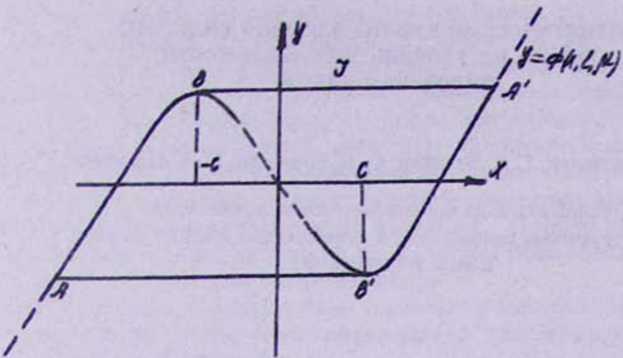

Fig.3. The schematic form of the closed, piecewise analitic Jordan curves - J (ABA'B'), l=1, 2, 3, ...

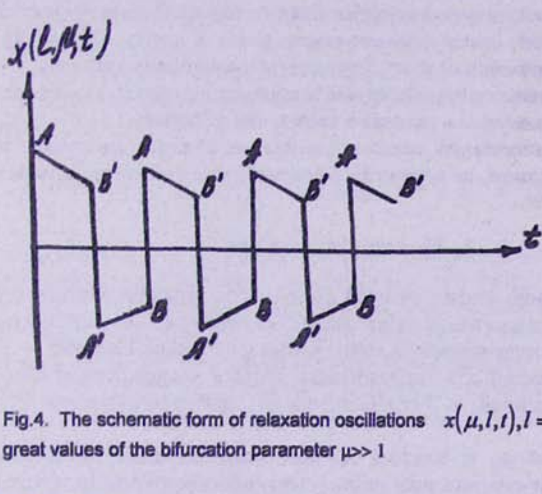

Fig.4. The schematic form of relaxation oscillations $x(\mu,l,t), l = 1,2,3,...$ at great values of the bifurcation parameter $\mu >> 1$

References

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