

# Modeling and Optimization of the Control Processes in Regional Agriculture Water Distribution under Restricted Resources

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## Abstract

In the paper the dynamical problem of optimal irrigation regimes of plants in a given region under restricted water resources is formulated and solved. The problem is transformed to a mathematical problem of optimal control and solved in the cases of both partial and global restrictions on resources.

Limited nature of useful arable grounds and exploitation stocks of water resources in the droughty regions in the modern conditions is presenting two fundamental requirements for irrigational agriculture: reception of maximum agricultural production from the unit of watering area and from unit of water.

Concerning to the irrigation development problems of arid zones of the earth demanding their urgent solution, the task of irrigation water distribution optimization under the water resources deficit has a special actuality and practical significance. These problems are reduced to the problem of determination of the optimal watering regimes of the sown areas under the condition of the gross output production maximization in terms of money.

Formally, the task of determination of the cultures watering optimal regimes for a separately taken farm, when  $M < \Omega$ , is the following: maximize the degenerative (objective) function (1) by limitations and connections (2-7) on the managing parameters  $m_i(t)$  by conditions of their non negativeness  $m_i(t) \geq 0$ ,  $i \in I_n$  (the notations will be given below):

$$S = \sum_{i=1}^n c_i R_i y_i(t_c) \rightarrow \max, \quad (1)$$

$$\bar{E}_i = \begin{cases} E_i[t, m_i(t)], & \text{if } t_{pi} \leq t \leq t_{ci} \\ 0, & \text{if } t < t_{pi}, t > t_{ci} \end{cases} = -a_i m_i^2(t) + b_i m_i(t) + p_i, \quad a_i > 0, \quad i \in I_n, \quad (2)$$

$$y_i = \int_{t_p}^{t_c} \bar{E}_i[t, m_i(t)] dt, \quad t_p = \min_{I_n} \{t_{pi}\}, \quad t_c = \max_{I_n} \{t_{ci}\}, \quad i \in I_n, \quad (3)$$

$$z_i = \int_{t_p}^{t_c} m_i(t) dt, \quad \tilde{z}_i = \int_{t_p}^{t_c} \omega_i(t) dt, \quad i \in I_n, \quad (4)$$

$$M(t) = \sum_{i=1}^n R_i m_i(t), \quad \Omega(t) = \sum_{i=1}^n R_i \omega_i(t), \quad (5)$$

$$M = \sum_{i=1}^n R_i z_i = \int_{t_p}^{t_c} M(t) dt, \quad \Omega = \sum_{i=1}^n R_i \bar{z}_i = \int_{t_p}^{t_c} \Omega(t) dt, \quad (6)$$

$$R = \sum_{i=1}^n R_i, \quad R_i y_i \geq k_i, \quad i \in I_n. \quad (7)$$

Productivity of culture is examined only in dependence of irrigation norm and that dependency is approximated in the form of parabola of the second order, that is

$$y_i = \bar{y}_i - d_i (z_i - \bar{z}_i)^2, \quad d_i = \frac{\bar{y}_i - y_i^0}{\bar{z}_i^2}, \quad i \in I_n. \quad (8)$$

The following notations for the  $i$ -th culture are introduced:

In these relationships  $I_n$  is the set of the cultures,

$y_i$ —productivity,

$R_i$ —watering area,

$z_i$ —irrigation norm,

$c_i$ —price of purchase,

$k_i$ —volume of planned harvest,

$m_i(t)$ —irrigation regime,

$\bar{z}$ —maximal irrigation norm,

$E_i$ —industrial function,

$\bar{y}_i$  and  $y_i^0$ —maximum and minimum productivity, correspondingly,

$t_p$  and  $t_c$ —common dates of the planting and harvesting, correspondingly,

$M$  and  $\Omega$ —annual and maximum annual irrigation fund of farms water, correspondingly,

$\omega_i(t)$ —maximum irrigation regime.

The formulated task is a complicated variable optimizational problem of objective function (1) with bilateral limitations on managing parameters. It is solved by stages, independently on accepted limitation  $M < \Omega$ , which is disintegrated into three partial limitations:

$$1. \quad z_i \leq \bar{z}_i, \quad i \in I_n.$$

$$2. \quad M(t) \leq \Omega(t), \quad \forall t \in [t_p, t_c].$$

$$3. \quad 0 \leq m_i(t) \leq \omega_i(t), \quad \forall t \in [t_p, t_c], \quad i \in I_n.$$

According to the first limitation, the initial task is solved as a simple problem of optimal control (Mayer's task). Further, according to the limitation 2, the problem is solved by Pontryagin's principle of maximum, as a task of optimal control with fastened left and the free right ends: find the maximum element  $\bar{m}(t) = (\bar{m}_1, \bar{m}_2, \dots, \bar{m}_n)$  of the singular function  $F = \sum_{i=1}^n c_i R_i y_i(t_c)$ , which satisfies below—following equations of connections, limitations and border conditions:

$$\frac{dy_i}{dt} = E_i[t, m_i(t)], \quad i \in I_n, \quad (9)$$

$$0 \leq m_i(t) \leq \omega_i(t), \quad i \in I_n, \quad (10)$$



$$M(t) = \sum_{i=1}^n R_i m_i(t), \quad (11)$$

$$\Omega(t) = \sum_{i=1}^n R_i \omega_i(t), \quad (12)$$

$$y_i(t_p) = 0, \quad i \in I_n. \quad (13)$$

Taking into consideration the planned gross product, the initial task is solved formally by an analogous manner, we take into account the limitation  $y_i(t_c) = \frac{k_i}{R_i}$ ,  $i \in I_H \subset I_n$ , where  $I_H$  is the list of unprofitable cultures, by which the farm fulfils only the planned gross product for guaranteeing the necessary volume of agriculture production.

The problem of optimal water distribution on the level of the regional agriculture is solved in completely analogous way if will be taken  $c_i = R_i = 1$ ,  $i \in I_n$ , i.e. if we consider farms instead of the irrigated cultures and the corresponding character of effective water consumption of farms instead of yield curves of crop.

The solutions of these problems allow to compare the possibilities of the present irrigation canals and nets with the optimal parameters of regional agriculture water distribution control processes. If they provide the necessary blotting capacity and all kinds of re-distributions of irrigating water between the farms of the region in the frame of receipted optimal solution, then quite naturally there is no need of any reconstruction. Otherwise, the solutions of the mentioned problems clearly show and dictate concrete directions, scales and parameters of required regional irrigating net reconstruction.

## References

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