# Orthogonal S-CDMA System with Non-Coherent MSK Modems

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#### Abstract

Simplified non-coherent Spread Spectrum (SS) MSK receiver's structures is presented. It is shown that optimal non-coherent SS receiver for pure (with rectangular baseband pulses offset QPSK signals can serve as suboptimal (performance loss≈0.9 dB) non-coherent receiver for SS MSK signals. The conditions of orthogonality of SS MSK signals are derived in synchronous and quasi-synchronous Code Division Multiple Access (CDMA) systems.

#### 1 Introduction

Spread Spectrum Multiple Access(SSMA) communication has been considered by many authors, (e.g. [4]). The most common form of such communication is direct sequence spread spectrum: the carrier is phase modulated by the digital data sequence and the code sequence. In recent years there is increased interest in Continuous Phase Modulation (CPM) and in Minimum Shift Keying (MSK) particularly, (e.g. [6]). The advantages of MSK modulation have given the reasons to consider this modulation in SS communication [1], [5], [7]. The performance of asynchronous CDMA system with coherent for each user MSK modem in terms of signal to noise ratio and average bit error probability was evaluated in [3]. In this paper we consider synchronous and quasi-synchronous CDMA systems. As it is well known synchronization essentially increases the capacity of CDMA system. The synchronous conditions between different users can be assumed for downlink communication and quasi-synchronous conditions for uplink communication in radio microcell network.

## 2 SS MSK Signals Tifisansmitted by Single User

There can be several ways to view MSK modulation [8]. We consider two of them in order to obtain 1) antipodal SS MSK signals; 2)orthogonal SS MSK signals

A. MSK as a Type of Continuous Phase Modulation

To simplify notation we consider SS MSK signal for one symbol (data bit) duration  $[0;T_b]$ .

Depending on codeword (vector)  $\underline{C}=(c_0,c_1,...,c_{N-1})$ , components  $c_n$  of which are +1 or -1 and called chips, the transmitted SS MSK signal is

$$x(t,\underline{C},\varphi) = D\cos[w_{\theta}t + p(t,\underline{C}) + \varphi], \tag{1}$$

where information carrying phase during n-th chip time interval  $nT < t \le (n+1)T$  is

$$p(t,\underline{C}) = c_n 90^{\circ} (\frac{t - nT}{T}) + S(n,\underline{C})$$
 (2a)

In above expressions

D - amplitude of signal

φ - initial phase

wo- central frequency

T=Tb/N - chip duration, Tb-bit (or symbol)duration

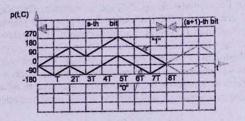
n = 0, N - 1 and the phase states S(n, C) in (2a) are defined by

S(0,C)=0

 $S(n,\underline{C}) = 90^n \sum_{j=0}^{n-1} c_j, \qquad n = \overline{1, N-1}$ 

Antipodal Signals: The first question is: how to choose a codeword  $C_I$  as the representation of the data bit "1" (+1) and a codeword  $\underline{C}_0$  as the representation of the data bit "0" (-1) to have antipodal SS MSK signals  $x(t,\underline{C}_1,\varphi) = -x(t,\underline{C}_0,\varphi)$ 

Because of requirement of phase continuity we assume that  $S(N,\underline{C}(s))=S(0,\underline{C}(s+1))=0$  (2b), i.e. s-th data bit SS MSK signal coincides with initial phase of the next (s+1)th data bit signal. Thus, we are forced to use quasi-antipodal signals which means that signals are antipodal in every chip time excluding first one and last one (see phase trajectories in Fig.1).



C1= 1 1-1 1 1-1-1-1

Co=-1 1-1 1 1-1-1 1

Fig.1 Phase trajectories of quasi antipodal SS MSK 8-chip signals

$$\underline{C}_{I} = c_{0} c_{1} c_{2} \dots c_{N-2} c_{N-1} 
\underline{C}_{0} = -c_{0} c_{1} c_{2} \dots c_{N-2} - c_{N-1}$$
(3)

B. MSK as a Type of Offset QAM.

The expression for SS MSK signal depending on the inphase chip sequence

 $\underline{A}$ =(a<sub>0</sub> a<sub>1</sub> a<sub>2</sub> ... a<sub>N-1</sub>), and the quadrature chip sequence  $\underline{B}$ =(b<sub>0</sub> b<sub>1</sub> b<sub>2</sub> ... b<sub>N-1</sub>), which are constrained by the conditions

$$a_n=a_{n-1}$$
, if n is even;  
 $b_n=b_{n-1}$ , if n is odd (4a)

is as follows (within chip time interval  $nT < t \le (n+1)T$ ,  $n = \overline{0, N-1}$ ):

$$x(t, \underline{A}, \underline{B}, \varphi) = \begin{cases} \frac{D}{\sqrt{2}} (\cos \omega_n t - \varphi) a_n \cos(90^n \frac{t - nT}{T}) + \frac{D}{\sqrt{2}} (\sin \omega_n t - \varphi) b_n \sin(90^n \frac{t - nT}{T}), n - even \\ \frac{D}{\sqrt{2}} (\cos \omega_n t - \varphi) a_n \sin 90^n (\frac{t - nT}{T}) + \frac{D}{\sqrt{2}} (\sin \omega_n t - \varphi) b_n \cos(90^n \frac{t - nT}{T}), n - odd \end{cases}$$

$$(4b)$$

Note, that

$$x(t,\underline{C},\varphi) = x(t,\underline{A},\underline{B},\varphi) \Rightarrow$$

$$a_0 = a_{N-1} = 1, \quad a_n = a_{n-1} = \cos S(n,\underline{C}), \quad n - even$$

$$b_n = b_{n-1} = \sin S(n,\underline{C}), \quad n - odd$$

$$(5)$$

Condition  $a_0=a_{N-1}=1$  in (5) for any signal comes into an agreement with above mentioned necessity instead of perfectly antipodal SS MSK signals  $x(t,\underline{A},\underline{B},\varphi)$  and  $x(t,-\underline{A},-\underline{B},\varphi)$  to deal with quasi-antipodal SS MSK signals (3) which equivalently are

$$x(t,\underline{A},\underline{B},\varphi)$$
 and  $x(t,-\underline{A}',-\underline{B},\varphi)$  (6a)

where  $-\underline{A}'$  is opposite to  $\underline{A}$ , except first component:

$$-\underline{A}' = (a_0 - a_1 - a_2 \dots - a_{N-1})$$
 (6b)

Orthogonal SS MSK signals as representatives of the data 1 and -1

In Section 4 the orthogonality conditions of SS MSK signals will be derived. In particular, it will be shown that two MSK signals  $x(t, \underline{C}, \varphi)$  and  $x(t, \underline{-C}, \varphi)$  or equivalently  $x(t, \underline{A}, \underline{B}, \varphi)$  and  $x(t, \underline{A}, \underline{-B}, \varphi)$  are orthogonal. It will be shown also that these signals with arbitrary initial phases are still orthogonal, i.e.

$$\int_{0}^{T_{p}} x(t, \underline{A}, \underline{B}, \varphi_{1}) \cdot x(t, \underline{A}, -\underline{B}, \varphi_{2}) dt = 0$$
(7a)

for any  $\varphi_1$  and  $\varphi_2$  if and only if

where

$$\langle \underline{A}, \underline{B} \rangle = \sum_{n=0}^{N-1} a_n b_n$$
 (7c)

### 3 Non-Coherent SS MSK Receivers

As it was noticed in [1] conventional FM discriminators used in typical MSK receivers create non-linearity before the SS codeword digital correlators and, thus, CDMA operation with this type of MSK receiver is not possible. The optimum non-coherent SS MSK receiver [1],

simplified non-coherent receiver considered in this section allow CDMA operation because the outputs of the correlators of these receivers are linear.

# A.An Optimum Non-Coherent SS MSK Receiver structure

We consider the model where transmitted orthogonal SS MSK signals  $x(t, d\underline{C}, \varphi)$  d=1 or -1 are distorted by AWGN n(t) and received signal is:

The receiver knows the codeword  $\underline{C}$  and does not know the data bit d nor the initial phase  $\varphi$ , which is equally likely to be any-where between 0° and 360°. As it was derived in [1] maximum likelihood non-coherent receiver must as usually at first correlate the received signal with SS MSK signal in inphase and quadrature coordinates:

$$\widetilde{a}(d) = \int_{0}^{T_{b}} y(t) \cos[\omega_{o}t + p(t, d\underline{C})dt$$
(9a)

$$\widetilde{b}(d) = \int_{0}^{T_{b}} y(t) \sin[\omega_{0}t + p(t, d\underline{C})] dt$$
(9b)

where d=1 and d=-1, and then make the decision by rule:

Choose the data bit to be "1" if and only if

$$\sqrt{\tilde{a}(1)^2 + \tilde{b}(1)^2} > \sqrt{\tilde{a}(-1)^2 + \tilde{b}(-1)^2}$$
 (10)

# B. A Suboptimum Non-Coherent SS MSK receiver

In order to simplify the optimum receiver presented in previous paragraph let's consider the receiver which instead of calculating  $\tilde{a}(d)$  (9a) and  $\tilde{b}(d)$  (9b) calculates the following values:

$$a(d) = \sum_{n=0}^{N-1} \int_{nT}^{(n+1)T} y(t) \cos(\omega_0 t + \frac{S(n, d\underline{C}) + S(n+1, d\underline{C})}{2}) dt$$
(11a)

$$b(d) = \sum_{n=0}^{K-1} \prod_{n=1}^{(n+1)T} y(t) \sin(\omega_n t + \frac{S(n, d\underline{C}) + S(n+1, d\underline{C})}{2}) dt$$
(11b)

This receiver correlates received signal with signal which information carrying phase in every chip time is constant and equal to average value (S(n,dC)+S(n+1,dC))/2 of linear varying phase of MSK signal (see phase trajectory in Fig.1).

In fact, this Averaged Chip Phase (ACP) receiver is an optimum receiver for SS pure offset QPSK signals (with rectangular baseband pulses in inphase and quadrature channels). It can be shown that this receiver allows simple structure implementation (Fig.2)

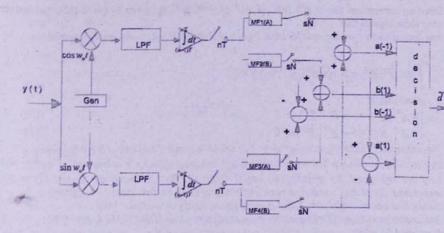


Fig. 2 Block-diagram of SS MSK Non-Coherent ACP receiver
The contents of digital matched filters (MF) in Fig. 2 are the inphase and the quadrature chip sequences  $\underline{M}$  and  $\underline{B}$  cor-responding to the codeword  $\underline{C}$ . The decision rule is defined by (10). To estimate the performance of ACP receiver as non-coherent receiver employing SS MSK signal in AWGN channel one has to compare this receiver with the optimum one (9), (10). Analysis shows that noise at the outputs of both receivers are the same but the ratio of intended signals is:

$$= \frac{\widetilde{b}'(1)}{b(1)} = \frac{\widetilde{a}'(1)}{a(1)} = \frac{\widetilde{a}'(1)}{\overset{nT}{a(1)}} = \frac{\sum_{\substack{nT \\ n=1 \\ nT}}^{(n+1)T} \cos[w_n t + c_n 90^n (\frac{t-nT}{T}) + S(n,\underline{C}) + \varphi] \cos[w_n t + c_n 90^n (\frac{t-nT}{T}) + S(n,\underline{C}) + \varphi] \cos[w_n t + c_n 90^n (\frac{t-nT}{T}) + S(n,\underline{C}) + \varphi] \cos[w_n t + \frac{S(n,\underline{C}) + S(n+1,\underline{C})}{2}] dt}$$

$$= \frac{\frac{T}{2} \cos \varphi + 0(\frac{1}{\omega_n})}{T \frac{\sqrt{2}}{\pi} \cos \varphi + 0(\frac{1}{\omega_n})} \approx 1.11$$

which corresponds to the loss of 0.9 dB in energy.

# 4 Conditions of Orthogonality

In this section we consider the problems:

- How to choose codewords  $\underline{C}(i)$   $i = \overline{1, K}$  (or  $\underline{A}(i)$  and  $\underline{B}(i)$ ) to have a set of mutually orthogonal signals?
- How to provide the orthogonality of users in synchronous and quasi-synchronous CDMA system with suggested in previous section receivers?

A. Conditions of Orthogonality of SS MSK Signals

Inserting expressions for  $x(t,\underline{A}^{(i)},\underline{B}^{(i)},\varphi_i)$  and  $x(t,\underline{A}^{(k)},\underline{B}^{(k)},\varphi_k)$  given by (4) in (12), one derives two non-coherent synchronous SS MSK signals defined by (1),(2) or (4) are orthogonal, i.e.

$$\int_{0}^{T} x(t, \underline{A}^{(t)}, \underline{B}^{(t)}, \varphi_t) \cdot x(t, \underline{A}^{(k)}, \underline{B}^{(k)}, \varphi_k) dt = 0$$
(12)

if and only if 
$$(13a)$$

if and only if (13a) 
$$\left\{ < \underline{A}^{(i)}, \underline{A}^{(k)} > + < \underline{B}^{(i)}, \underline{B}^{(k)} > = 0 \right.$$
 (13b) 
$$\left\{ < A^{(i)}, B^{(k)} > - < \underline{A}^{(k)}, \underline{B}^{(i)} > = 0 \right.$$

The coherent SS MSK signals ( $\phi_k = \phi_i$  in (12) are orthogonal if and only if (13a) is hold. In particular, it follows that signals  $x(t,\underline{A},\underline{B},\varphi_1)$  and  $x(t,\underline{A},-\underline{B},\varphi_2)$  are orthogonal if  $\varphi_1=\varphi_2$  and they are orthogonal for arbitrary  $\varphi_1$ ,  $\varphi_2$  if  $\langle \underline{A}, \underline{B} \rangle = 0$ . We have used this signals (7) as the representatives of data 1 and -1 in non-coherent single user modem (Section 3, fig. 2). Notice that binary codewords  $\underline{A}$  and  $\underline{B}$  in (13) must satisfy to constraint (4a).

B. Conditions of Users' Orthogonality in Quasi-Synchronous (QS)-CDMA System with Simplified Receivers

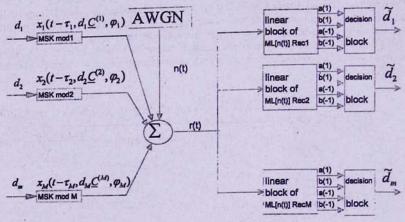


Fig.3 Model of CDMA system with MSK modems

Now we consider the QS-CDMA system model (Fig.3), where time offset not exceeding one chip duration is allowed between SS MSK signals of different users. Analysis shows that

For QS-CDMA (0≤tk≤T, k=1,2...M)system with SS MSK modems, Near Zero Interuser Interference conditions are (14).

It is interesting to note that zero interuser interference conditions for pure O-QPSK SS signals in S-CDMA system coincide with conditions (13) for SS MSK signals, but Near Zero Interuser Interference conditions for pure O-QPSK signals in QS-CDMA system will differ from (14) since analysis for deriving (14) essentially depends on the sine form of baseband MSK signals.

$$\begin{cases} < \underline{A}^{(i)}, \underline{A}^{(k)} >= 0 \\ < \underline{B}^{(i)}, \underline{B}^{(k)} >= 0, & k, i = \overline{1, M}, & k < i \end{cases}$$

$$< \underline{A}^{(i)}, \underline{B}^{(k)}(1) >= 0$$

$$< \underline{A}^{(k)}(1), \underline{B}^{(i)} >= 0$$

$$< \underline{A}^{(k)}(1), \underline{B}^{(k)} >= 0$$

$$< \underline{A}^{(k)}, \underline{B}^{(i)}(1) >= 0, & k, i = \overline{1, M}, & k \le i \end{cases}$$
where
$$< \underline{A}^{(k)}(1), \underline{B}^{(i)} >= \sum_{n=0}^{K-1} a_{n-1}^{(k)} b_n^{(i)}, \text{ etc. (n is taken by modulo N )}$$

## 5 Conclusion

In this work the features and possibilities of communication systems with spread spectrum MSK modulation are investigated. It is shown that non-coherent MSK receiver structure is as simple as a pure offset QPSK receiver structure at the cost of insignificant performance loss (about 1dB). It should be stressed out that the capacity of S- and QS-CDMA system with MSK modems is not less than the capacity of S- and QS-CDMA with QPSK or pure OQPSK modulation in sense of number of orthogonal signals.

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