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About 2-Cyclic Orgraph

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Let G be an orgraph of order p with minimum half degree $\delta(G)$. In this paper we prove that:

1) if $p \geq 9$ and $\delta(G) \geq (p + 3(k - 2))/4$, where the integer $k \geq 2$, then G is k–connected or 2-cyclic.

2) if $p \geq 10$ and $\delta(G) \geq p/3$, then G is 2-cyclic.

3) if $p \geq 12$ and $\delta(G) \geq (p - 3)/2$, then G is 2-linked.

1 Introduction and Notaions

In this paper we investigate orgraphs in which any two vertices are on a common cycle (such orgraphs are called 2-cyclic). The class of 2-syclic digraphs is not characterized completely, for example, it is not known whether there exists a natural number k such that every k-connected digraph is 2-cyclic (k must be at least six). Jackson conjectured that every 3-connected orgraph is 2-cyclic [2] .In [3] it was shown some sufficient conditons for a digraph to be 2-cyclic.

All terms not defined in this paper can be find in Harary's book [1] . Without other specifications, G denotes a digraph of order p with vertex set $V(G)$ and arc set $E(G)$. All paths and cycles considered here are oriented and elementary. A digraph G is a strong, if for any two vertices x and y, G contains a path from x to y and a path from y to x. A digraph G is k–connected if the deletion of less than k vertices always gives a strong digraph. The arc from x to y is denoted by xy, and if such an are exists then we say that x dominates y and y is dominated by x. We denote by $od(x)$ and $id(x)$ respectively the outdegree and the indegree of the vertex x and by $\delta(G)$ the minimum outdegree and indegree of a vertex in the digraph G. If $x, y \in V(G)$ then by $d(x, y)$ we denote the length of the shortest path from x to y. An oriented graph (orgraph) is a digraph with no cycle of length two. The connected number of a digraph G , denoted by $k(G)$, is the maximum value of k for which G is k-connected. For any real number x, $[x]$ denotes the integer part of x.

For any $A, B \subset V(G)$ and $x \in V(G)$, we define

$$E(A \to B) = \{zy \in E(G)/z \in A, y \in B\},$$

$$I(x) = \{y \in V(G)/yx \in E(G)\}, \qquad O(x) = \{y \in V(G)/xy \in E(G)\},$$

$$I(x, A) = \{y \in A / yx \in E(G)\}, \qquad O(x, A) = \{y \in A / xy \in E(G)\},$$
$$id(x, A) = |I(x, A)|, \qquad od(x, A) = |O(x, A)|,$$
$$od^*(x) = |V(G)| - od(x) - 1, \qquad id^*(x) = |V(G)| - id(x) - 1.$$

If $A = \{x\}$, then we write x instead of $\{x\}$. The induced subgraph with the set of vertexes A is denoted by $\langle A \rangle$. We write $A \to B$ if $xy \in E(G)$ for each $x \in A$ and for each $y \in B$. If $C \subset V(G), A \to B$ and $B \to C$, then we write $A \to B \to C$.

2 2-Cyclic Orgraphs

We omit the proof of the following simple lemma:

Lemma 1 *Let G be an orgraph of order p. Then G contains vertices x and y with $od(x) \leq (p-1)/2$ and $od^*(y) \geq (p-1)/2$.*

Lemma 2 *Let G be an orgraph of order p $(p \geq 2)$ with $\delta(G) \geq k$. Then G is a $m-$connected, where $m \geq (4k - p + 2)/3$.*

Proof. Immediate from Lemma 1.

Theorem 2.1 *Let G be an orgraph of order p $(p \geq 9)$ with $\delta(G) \geq (p + 3(k-2))/4$, where the integer $k \geq 2$. Then G is $k-$connected or $2-$cyclic.*

Proof. Suppose that G is not $k-$connected and show that G is $2-$cyclic. From Lemma 2 it follows that $k(G) = k - 1$ and $\delta = \delta(G) = (P + 3(k-2) + 1)/4$ or $(P + 3(k-2))/4$. So we have the partition

$$G = A \cup B \cup \{x_1, x_2, ..., x_{k-1}\},$$

where $E(A \to B) = \emptyset$.

Consider the following cases.

Case 1. $\delta = (p + 3(k-2) + 1)/4$.

Then $p = 4\delta - 3(k-2) - 1$ and $a + b = 4\delta - 4k + 6$, where $a = |A|$ and $b = |B|$. We can assume, without loss of generality, that $a \leq 2\delta - 2k + 3$. Hence, by Lemma 1, we have $a = 2\delta - 2k + 3$ and $b = 2\delta - 2k + 3$. By Lemma 1 we have that induced subgraphs $\langle A \rangle$ and $\langle B \rangle$ are regular tournaments and

$$A \to \{x_1, x_2, ..., x_{k-1}\} \to B.$$

Therefore, $E(B \to A) \neq \emptyset$ and G is $2-$cyclic.

Case 2. $\delta = (p + 3(k-2))/4$.

Then $p = 4\delta - 3(k-2)$ and $a + b = 4\delta - 4k + 7$. Without loss of generality, we can assume that $a \leq 2\delta - 2k + 3$. By Lemma 1 $a = 2\delta - 2k + 3, b = 2\delta - 2k + 4$.

$$A \to \{x_1, x_2, ..., x_{k-1}\} \tag{1}$$

and induced subgraph $\langle A \rangle$ is a regular tournament. It is easy to see that for every vertex $z \in A$ there are at least $k - 1$ vertices from B which dominate z and for each vertex $z_1 \in B$

$$E(z_1 \quad A) \quad k \quad 2, \tag{2}$$

$$id(z_1, B) \geq \delta - k + 1. \qquad (3)$$

In order to prove Theorem 1, we need the following remarks:

Remark 1. If the subgraph $\langle B \rangle$ is not strong, then $B = B_1 \cup B_2, B_1 \cap B_2 = E(B_2 \to B_1) = \emptyset, |B_1| = 2\delta - 2k + 3, |B_2| = 1$,the subgraph $\langle B_1 \rangle$ is a regular tournament ,$\{x_1, x_2, ..., x_{k-1}\} \to B_1$ and $E(B_2 \to A) \neq \emptyset$.

Remark 2. If the subgraph $\langle B \rangle$ is exsactly 1−connected, then $B = B_1 \cup B_2 \cup \{z\}, B_1 \cap B_2 = E(B_1 \to B_2) = \emptyset$, $z \notin B_1 \cup B_2$ and $|B_1| \leq 2$, $|B_2| \geq 2\delta - 2k + 1$.

We assume that G is not 2−cyclic and we will show that this leads to a contradicton. Let the vertices u and v are not on a common cycle. But as G is $(k-1)$−connected, we have $d(u,v) \geq k, d(v,u) \geq k$ and if $k \geq 3$, then

$$O(u) \cap I(v) = I(u) \cap O(v) = \emptyset.$$

Consider the following subcases:

Subcase 2.1. $u \in A$ and $v \in B$.

Then for some $i, 1 \leq i \leq k-1$, there is a path from x_i to v whith does not contain the vertices from A. If $E(v \to A) \neq \emptyset$, then there exists a cycle containing u and v. Now we can assume that $E(v \to A) = \emptyset$. Then, by (2), $k = 2$, $a = 2\delta - 1$, $b = 2\delta$ and the vertex v is adjacent to all vertices of $A \cup \{x_1\}$. If $x_1 v \in E(G)$, then it is easy to find a path from v to u not containing x_1. So we can assume that $x_1 v \notin E(G)$. We have $v x_1 \in E(G)$. Therefore, from $E(v \to A) = \emptyset$ and from Remark 1 follows, that subgraph $\langle B \rangle$ is strong. Since $id(v, B) \geq \delta$ and $od(x_1, B) \geq \delta$, then the set B contains a vertex z for which $x_1 z, zv \in E(G)$.

Assume that $E(B - \{v, z\} \to A) \neq \emptyset$. Let $E(w \to A) \neq \emptyset$ and $w \in B - \{v, z\}$. In $\langle B \rangle$ each path from v to w contains the vertex z. Hence, by Remark 2, $od(v) \leq 2$, we obtain a contradiction.

Assume now that $E(B - \{v, z\} \to A) = \emptyset$. Then, by $E(v \to A) = \emptyset$, we have $z \to A$. As in case $E(B - \{v, z\} \to A) \neq \emptyset$ it follows that $x_1 \to O(v, B)$. Since

$$E(O(v, B) \to I(v, B) - \{z\}) \neq \emptyset,$$

then $E(O(v, B) \to z) = \emptyset$. Hence $I(v, B) - \{z\} \to z$ and for each $y \in O(v, B)$

$$|E(y \to I(v, B) - \{z\})| \geq 2.$$

Let $y_1 y_3, y_2 y_4 \in E(G)$, where $y_1, y_2 \in O(v, B)$ and $y_3, y_4 \in I(v, B) - \{z\}$.Therefore the cycle $u x_1 y_1 y_3 v y_2 y_4 z u$ contains the vertices u and v, which gives a contradiction.

Subcase 2.2. $u, v \in B$.

If the subgrapf $\langle B \rangle$ is not strong, then by Remark 1, the vertices u and v are on a common cycle. So we can assume now that the subgraph $\langle B \rangle$ is strong. From $E(u, v) = \emptyset$ and from $k \geq 2$, it follows that $E(u \to A) \neq \emptyset$ and $E(v \to A) \neq \emptyset$. Since $\langle B \rangle$ is strong, then $u, v \notin O(x_i)$, for each $i, 1 \leq i \leq k-1$. By Lemma 1 there is a vertex $x_j, 1 \leq j \leq k-1$, with $od(x_j, B) \geq \delta - (k-2)/2$ and let $j = 1$. Thus there is a $w \in O(x_1, B)$ that $wv \in E(G)$. If in $\langle B \rangle$ there is a path from v to u which does not contain the vertex w, then the vertices u and v are on a common cycle. So we can assume, that in $\langle B \rangle$ each path from v to u contains the vertex w. Therefore, the vertex w is a cut vertex for the vertices u and v. By Remark 2, it is clear that $u \in O(x_1)$, which gives a contradiction.

Subcase 2.3. $u \in \{x_1, x_2, ..., x_{k-1}\}$ and $v \in B$.

Since $E(u, v) = \emptyset$, then there is a vertex z, such that $uz, zv \in E(G)$. Hence $k = 2$ and $E(v \quad u \quad A) =$. Therefore the vertices u and v are on a common cycle.

Subcase 2.4. $u, v \in \{x_1, x_2, ..., x_{k-1}\}$.
Then $k \geq 3$ and

$$|O(u,B) \cap O(v,B)| \geq 2.$$

Let $y, z \in O(u,B) \cap O(v,B)$ and $y \neq z$. By (2) there exist vertices $y_1, z_1 \in A$, such that $yy_1, zz_1 \in E(G)$. By (1) we can assume that $y_1 = z_1$. Therefore, by (2), $k = 3$ and

$$|E(y \to A)| = |E(z \to A)| = 1.$$

There exist two vertices $z_2 \in A - \{z_1\}$ and $w \in B - \{y, z\}$ such that $wz_2 \in E(G)$. It is clear that $E(\{u, v, y, z\} \to w) = \emptyset$ and $id(w, B - \{y, z\}) \geq \delta$. Since $od(y, B) \geq \delta - 1$, then there is a vertex $y_2 \in B - \{z\}$ such that $yy_2, y_2w \in E(G)$. So we have a cycle $uyy_2wz_2vzz_1u$ containing u and v.The proof of the Theorem 1 is completed.

Notice that for $k = 2$ there is an orgraph of order 8 , which is not 2-connected and is not 2-cyclic.

Theorem 2.2 *Let G be an orgraph of order p $(p \geq 10)$ with $\delta(G) \geq (p-5)/2$. Then G is 2-cyclic.*

Proof. Suppose that G is not 2-cyclic. Let the vertices x and y are not on a common cycle. From Lemma 1 and 2 we have that G is 2-connected. Therefore $E(x, y) = \emptyset$ and

$$O(x) \cap I(y) = O(y) \cap I(x) = \emptyset. \tag{4}$$

Consider the following cases:

Case 1. $E(O(x) \to I(y)) \neq \emptyset$.

Let $x_1y_1 \in E(G)$, where $x_1 \in O(x)$ and $y_1 \in I(y)$. Therefore, each path from y to x contains the vertices x_1 and y_1. Thus the set $\{x_1, y_1\}$ is a cut-set and we have the partition

$$V(G) = A \cup B \cup \{x_1, y_1\},$$

where $E(A \to B) = \emptyset$, $y \in A$ and $x \in B$. Hence from Lemma 2 we have that $p \leq 13$ and $\delta(G) = [(p-4)/2]$. As $a + b = p - 2$, where $a = |A|$ and $b = |B|$, then without loss of generality we can assume that $a \leq (p-2)/2$. Therefore, by Lemma 1, there is a vertex $z \in A$, such that $od(z) \leq (a+3)/2$ and $od(z, A) \leq (a-1)/2$.

Assume that $p = 12$ or 13. Then $\delta(G) = 4, a \leq 5$ and $od(z, A) = 2$. Therefore the subgraph $\langle A \rangle$ is a regular tournament and $A \to \{x_1, y_1\}$, which contradicts to $y_1y \in E(G)$.

The proof in the case $p = 10$ or 11 is left to the reader.

Case 2. $E(O(x) \to I(y)) = \emptyset$.

By Lemma 1, there is a vertex $z \in O(x)$ for which

$$2 + |I(y)| + [|O(x)|/2] \leq od^*(z) \leq n + 1 + i, \tag{5}$$

where $p = 2n + i, i = 0$ or $i = 1$.

We divide this case into the following subcases.

Subcase 2.1. $p = 2n$.

From (5) it follows that

$$n + [(n \quad 2)/2] \quad od^*(z) \quad n + 1.$$

Hence $n = 5$, $id(y) = od(x) = 3$ and the subgraphs $\langle A \rangle$ and $\langle B \rangle$ are regular tournaments.
Let

$$D = V(G) - (O(x) \cup I(y) \cup \{x, y\}).$$

Then $|D| = 2$, $O(x) \to D \to I(y)$ and $yu_1, v_1x \in E(G)$, where $u_1 \in O(x)$ and $v_1 \in I(y)$. Therefore the cycle $xu_2z_1v_2yu_1z_2v_1x$, where $z_1, z_2 \in D, u_2 \in O(x)$ and $v_2 \in I(y)$, contains the vertices x and y,which gives a contradiction.

Subcase 2.2. $p = 2n + 1$.

If $n \geq 7$ we obtain a contradiction by using Lemma 1, so assume $n \leq 6$. We shall consider the cases $n = 6$ and $n = 5$ separately. Suppose first that $n = 6$. Then $p = 13$ and, by Lemma 1 $od(x) = id(y) = 4$. We have $|D| = 3$. Let $O(x) = \{x_1, x_2, x_{3,}, x_4\}$, $I(y) = \{y_1, y_2, y_3, y_4\}$ and $D = \{z_1, z_2, z_3\}$. It is easy to see that two vertices from $O(x)$ (resp. $I(y)$) dominate (resp. dominated by) the all vertices of D . Let

$$\{x_1, x_2\} \to D \to \{y_1, y_2\}.$$

It is easy to see that $yz_i \in E(G)$ for some $i, 1 \leq i \leq 3$. Then $E(\{y_1, y_2\} \to x) = \emptyset$. Hence, using case 1, we have $|O(y) \cap D| \leq 1$. Analogously, $|I(x) \cap D| \leq 1$. Therefore, we have

$$|O(x) \cap O(y)| \geq 3 \text{ and } |I(y) \cap I(x)| \geq 3.$$

Let $yx_1, y_1x \in E(G)$. Then the cycle $xx_2z_1y_2yx_1z_2y_1x$ contains the vertices x and y, but this contradicts to the assumption that the vertices x and y are not on a common cycle.

The case when $n = 5$ we leave to the reader. The proof of Theorem 2 is completed.

We will use the following.

Lemma 3 *Let G be an orgraph of order p ($p \geq 7$) with $\delta(G) \geq (p-3)/3$. Then*
1) if $p \neq 12$ and $p \neq 18$, then for every two vertices x and y $d(x, y) \leq 4$.
2) if $p = 12$ or $p = 18$, then for every two vertices x and y $d(x, y) \leq 4$ or $d(y, x) \leq 3$.

The proof of the Lemma 3 is left to the reader.

Theorem 2.3 *Let G be an orgraph of order p ($p \geq 10$), with $\delta(G) \geq p/3$. Then G is $2-$cyclic.*

Proof. If $p \leq 15$, then the Theorem 3 follows from Theorem 2, so assume $p \geq 16$. If G is 4−connected, then the Theorem 3 follows from Lemma 3, so we can assume that G is not 4−connected. Hence from Lemma 2 it follows that $p = 17, 18$ or 21 and G is 3−connected. Therefore we have the partition

$$V(G) = A \cup B \cup \{x, y, z\},$$

where $E(A \to B) = \emptyset$.

If $p = 17$ or $p = 21$, then the subgraphs $\langle A \rangle$ and $\langle B \rangle$ are regular tournaments and $A \to \{x, y, z\} \to B$. Since $E(B \to A) \neq \emptyset$, then it is not difficult to see that G is 2−cyclic. Now assume that $p = 18$. Without loss of generality, we can assume that $|A| = 7$. Then the subgraph $\langle A \rangle$ is regular tournament and $A \to \{x, y, z\}$. Now we note that the rest of the proof of the Theorem 3 follows by similar arguments, as in the case 2 of the Theorem 1. These details are left for the reader. The completes the proof.

3 Other Cyclic Properties in Orgraphs

In this section we consider other properties which imply that the considered digraph is 2–cyclic. The digraph G is pancyclic if it has cycles of every lenght n, $3 \leq n \leq |V(G)|$.

We say that a digraph G has property (T) [3] if, for any three vertices x, y, z in G there exists a path from x to y containing z.

We say that a digrapf G is k-linked if for every family of $2k$ (not necessarily distinct) vertices $x_1, x_2, ..., x_k, y_1, y_2, ..., y_k$ there exist k internal vertex disjoint paths from x_i to y_i, $1 \leq i \leq k$.

Problems connected with the k-linked digraphs and digraphs with property (T), in particular, are considered in [3].

Points (1) and (2) of the following theorem are proved in [4] and [5], the point (3) is proved below.

Theorem 3.4 *Let G be an orgraph of order p with $\delta(G) \geq (p-3)/2$. Then*
1) if $p \geq 10$, then G is pancyclic ([4]).
2) if $p \geq 8$, then G has proprety $[T]$ ([5]).
3) if $p \geq 12$, then G is 2–linked.

Proof of Theorem 4.3. Suppose the contrary. Then there are vertices a, b, c and d for which there are no internally disjoint paths from a to b and from c to d. According to Lemma 2 and 1 we have $k(G) \geq 4$.

Let us define

$$A = O(a) - \{c, d\}, \qquad B = I(b) - \{c, d\},$$
$$C = O(c) - \{a, b\}, \qquad D = I(d) - \{a, b\}.$$

Consider the following cases.

Case 1. $E(A \to B) \neq \emptyset$.

Let $uv \in E(A \to B)$. It is easy to see that $k(G) = 4$ and $p \leq 15$. Therefore the set $\{a, b, c, d\}$ is a cut-set for the vertices c and d. We have the partition

$$V(G) = X \cup Y \cup \{a, b, u, v\},$$

where $E(X \to Y) = \emptyset$ and $a \in X, b \in Y$. Note that $|X| \leq 6$ and $|Y| \leq 6$. Without loss of generality we may suppose that $|X| \leq |Y|$. Hence, if

$$14 \leq p \leq 15, \quad \text{then} \quad |X| = 5, \tag{6}$$

and if

$$12 \leq p \leq 13, \quad \text{then} \quad 3 \leq |X| \leq 4. \tag{7}$$

We distinguish two subcases.

Subcase 1.1. The subgraph $\langle X \rangle$ is a regular tournament.

Then $X \to \{a, b, u, v\}$. By (6) and (7) we have $E(\{u, v\} \to d) = \emptyset$ and $od(a, Y - \{y_2\}) \geq 3$. Consequently, there are vertices $x \in X - \{c\}$ and $y \in Y - \{d\}$ such that $ay, yx \in E(G)$. Since $E(u \to X \cup \{a, b, d\}) = \emptyset$, then $od(u, Y - \{d, y\}) \geq 3$. Hence, by $E(X \cup \{u, v\} \to d) = \emptyset$, there is a vertex $y_1 \in Y - \{y, d\}$ for which $uy_1, y_1 d \in E(G)$. So we have two vertex disjoint paths $ayxb$ and $cuy_1 d$, but this contradicts to our assumption.

Subcase 1.2. The subgraph X is not a regular tournament.

Then, by (6) and (7) $|X| = 4$ and $p = 12$ or 13. By Lemma 1, there are at least two vertices from X which dominates the vertices a, b, u and v. Let $x \in X - \{c\}$ and $x \rightarrow \{a, u, v, b\}$. From this and from $E(a \rightarrow \{v, b\}) = \emptyset$ follows that $od(a, Y - \{d\}) \geq 1$. It is not difficult to see that if $ay \in E(G)$ and $y \in Y - \{d\}$, then in the subgraph $\langle X \cup \{a, b, y\} - \{c\} \rangle$ there is a path P from a to b.

We assume first that $E(c \rightarrow \{u, v\}) \neq \emptyset$.Let $cw \in E(G)$, where $w \in \{u, v\}$. We have $E(w \rightarrow F) = \emptyset$, where $F \subseteq \{c, b, d, x, z, x_1\}, x_1 \in X - \{x, c\}, z \in \{a, u\}$ and $|F| \geq 5$. Therefore there is a vertex $y_1 \in Y - \{y, d\}$for which $wy_1 \in E(G)$. It is not difficult to see that $y_1 d \notin E(G)$ and $y_2 d \in E(G)$ for some $y_2 \in Y - \{y, y_1, d\}$. Therefore

$$E(X \cup \{w, y_1, d\} \rightarrow y_2) = \emptyset$$

and $ay_2, y_3 y_2 \in E(G)$, where $y_3 \in Y - \{y, y_1, y_2, d\}$. Since $E(X \cup \{w, y_1, y_2\} \rightarrow y_3) = \emptyset$, then $wy, dy_3 \in E(G)$ and $yd \in E(G)$. So we have the path $cwyd$. This path and the path P from a to b in the subgraph $\langle X \cup \{a, b, y_2\} - \{c\} \rangle$ are vertex disjoint, this gives a contradiction.

The proof in the case when $E(c \rightarrow \{u, v\}) = \emptyset$ can be given similarly. We leave it to the reader.

Case 2. $E(A \rightarrow B) = \emptyset$.

We show first that $|B| = n - 3$, where $p = 2n + i$ and $i = 1$ or 0. Assume that $|B| \geq n - 2$. Then, by Lemma 1, there is a vertex $a \in A$, for which

$$n + i \geq od^*(a) \geq |B| + 2 + [|A|/2].$$

From this and from $|A| \geq n - 3$ it follows that $|B| = n - 2$, $|A| = n - 3 = 3$ and $p = 13$. Therefore $a \rightarrow \{c, d\}$, $\langle A \rangle$ is a regular tournament and $A \rightarrow H \cup \{c, d\}$, where $|H| = 2$ and

$$H = V(G) - (A \cup B \cup \{a, b, c, d\}).$$

It is easy to see that $E(c \rightarrow B) \neq \emptyset$ and for each $u \in B$ $E(u \rightarrow A) \neq \emptyset$. Let $cu, uv \in E(G)$, where $u \in B$ and $v \in A$. Hence, it is not difficult to see that $E(H \rightarrow B - \{u\}) = \emptyset$. Therefore

$$E(H \rightarrow A \cup B \cup \{b\} - \{u\}) = \emptyset$$

which gives a contradiction. This contradiction proves that $|B| = n - 3$.

Analogously we have

$$|A| = |C| = |D| = n - 3.$$

From $|A| = |B| = n - 3$ it follows that $a \rightarrow \{c, d\} \rightarrow b$, which contradicts to $|C| = |D| = n - 3$. The proof of Theorem 4.3 is completed.

Let us note that there is an orgraph G of order 8 with $\delta(G) = 3$ and there is an orgraph G of order 12 with $\delta(G) = 4$,which is not 2-linked.

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