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PRODUCTION OF GREEN POWER IN ARMENIA: THE RISK OF CLIMATIC CONDITIONS

One important feature of green power production, frequently documented by various researchers, is volatility, coming from its dependence on weather and climatic conditions, in general. The question is how the government, on the one side, and power stations and regular consumers, on the other side, should deal with such a volatility. In this regard, some European countries have occasionally made use of strategies, such as demand management, storage of electricity (in pumped-storage plants), double structure systems, buffering strategies and so on[1]. In all these examples there is one thing extremely important for efficient operation of the respective systems, namely, the ability to predict the future "states of nature" that will confront individual production units.

Thus, below we suggest a simple structural framework that allows to better understand the implications of the uncertainty in climatic conditions for the production of hydropower and provides insights into calibration of the probabilities characterizing this uncertainty.

It is well established that the major risk, confronting the operations of hydropower stations, relates to the level of the pressure of water flow fuelling their production systems. For instance, quarterly financial statements of "ARTSAKH HEK" OJSC[2], a medium-sized hydropower cascade situated in the Nagorno-Karabakh Republic, identify only two types of risks. The first one is the currency risk that is typical not only to hydropower plants, but also to almost every organizational unit operating in a developing economy. The

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[1] **Auer, H., and Haas, R.**, "On Integrating Large Shares of Variable Renewables into the Electricity System", *Energy* 115, 2016, pp. 1592–1601.
Bertsch, J., Growitsch C., Lorenczik S., and Nagl S., "Flexibility in Europe's Power Sector — An Additional Requirement or an Automatic Complement?", *Energy Economics* 53, 2016, pp. 118–131.
Edenhofer, O., Hirth L., Knopf B., Pähle M., Schlörner S., Schmid E., Ueckerdt F., "On the Economics of Renewable Energy Sources", *Energy Economics* 40, Supplement 1, 2013, p. S 12 – S 23.
Grand, D., Le Brun Ch., Vidil R., and Wagner F., "Electricity Production by Intermittent Renewable Sources: A Synthesis of French and German Studies", *The European Physical Journal Plus* 131, 2016, pp. 329–340.

[2] http://artsakhhek.am/?page_id=394.

second source of uncertainties is termed a risk of climatic conditions and is characterized as follows: “Climatic conditions have a significant impact on the company. The company’s profit is largely due to the volumes of water flow. Despite the fact that the analysis of indicators of water volumes for the past couple of years makes it possible to make quite accurate predictions, the volumes of the company’s profits and hence the Company’s financial results are due to climatic factors”. In fact, the risk of climatic conditions represents the uncertainty that gives rise to the volatility of the output of hydropower stations.

To understand the consequences of the risk of climatic conditions for the production of hydroelectricity, we consider a simple optimization problem, specified in ex ante. In particular, similar to standard moral hazard models (Grossman and Hart, 1983[1], Holmstrom and Milgrom, 1987[2]) we assume that the relationship between the hydropower station and the distribution operator is governed by a contract which is signed before the station commences production process. The contract specifies electricity production schedule, i.e. the amount of electricity that should be produced by the hydropower station depending on the volume of water flow. In line with the conventional literature on incentives (Myerson, 1979[3], Rogerson, 1985[4]), at the time of contracting the parties are confronted with postcontractual uncertainty, that is, at that time they do not know what the pressure of water flow will be after the contract is signed. This interpretation seemingly resembles the situation of moral hazard with the exception that in the former case there is no private information. However, different from Holmstrom (1982)[5] this interaction represents a decision under risk without any incentive compatibility issue because the interests of parties are highly aligned and they make a decision collaboratively[6].

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1 **Grossman, S., and O. Hart**, “An analysis of the principal-agent problem”, *Econometrica 52*, 1983, pp. 7-45.

2 **Holmstrom, B., and P. Milgrom**, “Aggregation and linearity in the provision of intertemporal incentives”, *Econometrica 55*, 1987, pp. 303-328.

3 **Myerson, R.**, “Incentive compatibility and the bargaining problem”, *Econometrica 47*, 1979, pp. 61-74.

4 **Rogerson, W.**, “Repeated moral hazard”, *Econometrica 53*, 1985, pp. 69-76.

5 **Holmstrom, B.**, “Moral hazard in teams”, *Bell Journal of Economics 13*, 1982, pp. 324-340.

6 In fact, strictly speaking, the optimiation problem, formulated below, approximates the decision of a single agent. the hydropower company. However, in our view, this broader interpretation indicates the possibility of extending the underlying environment (also formally).

The initial notations necessary for the formulation of the problem in continuous distributions are as follows:

- W –the level of water flow in the part of the river near the hydropower station; $W \in [0, \bar{W}]$ with density $f(W)$ and distribution $F(W)$,
- $E(W)$ –production of electricity as a function of the volume of water; an unknown function and the choice "variable" of the model,
- $C(E(W), W)$ –total costs of producing electricity in the hydropower station; a known composite function of W,
- P –the price at which the hydopower station sells electricity to the distribution network operator; a given number which will be interpreted as an average revenue and be made dependent on the state of nature for the purpose of consistent calibration of the discrete version of the model,
- $\bar{E}(W)$ –the optimal size of the production in the given hydropower station, a given function,
- $\bar{\Pi}$ –the minimum necessary level of profit, a given number.

Notice that all of the aforementioned objects are given except for $E(W)$.

Assumption: The average cost of hydroelectricity production is independent of the production level, implying that the total cost function is linear in output, i.e.,

$$C(E(W), W) = c(W) * E(W),$$

where $c(W)$ is a given function of the average cost.

As already emphasized, in this setting the main source of risks is the stochastic and uncertain nature of water supply, which triggers a further inefficiency in the production phase. To reflect this feature in the model, W is assumed to be a given random variable. Since, by definition, electricity production is a function of water consumption, it is random as well. The same is true for the production costs, which turn out to constitute a composite function of W.

The objective function of the hydropower station, that is concerned about minimizing the risk of climating conditions, is defined as follows:

$$R[E(W)] = \int_0^{\bar{W}} [E(W) - \bar{E}(W)]^2 f(W) dW. \qquad (1)$$

Designing the optimal production schedule the company should not forget about its minimum profit requirement. This constraint is formalised in the following way:

$$\Pi[E(W)] = \int_0^{\bar{W}} [P - c(W)]E(W)f(W)dW \geq \bar{\Pi}, \forall W \in [0, \bar{W}]. \tag{2}$$

Notice that, by definition, R and Π are *functionals* depending on $E(W)$. Further, the problem of the hydropower station takes on the following form:

$$\min_{E(W)} R[E(W)] \tag{3}$$

subject to

$$\Pi[E(W)] \geq \bar{\Pi}, \quad \forall W \in [0, \bar{W}]. \tag{4}$$

Since we intend to bring the model (3)-(4) to the data, it is more convenient to solve it in discrete distributions. Let W_i be the i-th realization of the random variable W, and $E_i = E(W_i)$, $c(W_i) = c_i$, $prob(W = W_i) = \pi_i$, $i = 1, \dots, N$. Then, we can rewrite the problem as follows:

$$\max_{(E_i)_{i=1}^N} \sum_{i=1}^N \pi_i (E_i - \bar{E}_i)^2 \tag{5}$$

subject to

$$\sum_{i=1}^N \pi_i (P - c_i) E_i \geq \bar{\Pi}. \tag{6}$$

For ease of mathematical derivations, we assume that at the optimum the constraint (6) holds as an equality. Then the problem (5)-(6) can be solved via Lagrange method.

The Lagrangian of (5)-(6) takes on the following form:

$$\mathcal{L} = \sum_{i=1}^N \pi_i (E_i - \bar{E}_i)^2 + \lambda \left(\sum_{i=1}^N \pi_i (P - c_i) E_i - \bar{\Pi} \right). \tag{7}$$

The First Order Conditions will be:

$$\frac{\partial \mathcal{L}}{\partial E_k} = 2(E_k - \bar{E}_k)\pi_k + \lambda (P - c_k)\pi_k = 0, k = 1, \dots, N. \tag{8}$$

$$\frac{\partial \mathcal{L}}{\partial \lambda} = \sum_{i=1}^N \pi_i (P - c_i) E_i - \bar{\Pi} = 0. \tag{9}$$

Solving the system of equations (8) for $E_k, k = 1, \dots N$, and inserting these values into (9), we get the following equation:

$$\sum_{i=1}^{N} \pi_i (P - c_i)[2\bar{E}_i - \lambda(P - c_i)] = 2\bar{\Pi}. \tag{10}$$

(10) implies that:

$$\lambda = \frac{2\bar{E}_i \sum_{i=1}^{N} \pi_i (P - c_i) - 2\bar{\Pi}}{\sum_{i=1}^{N} \pi_i (P - c_i)^2}. \tag{11}$$

Hence:

$$E_k = \bar{E}_k \frac{\sum_{i \neq k} \pi_i (P - c_i)(c_k - c_i)}{\sum_{i=1}^{N} \pi_i (P - c_i)^2} + \bar{\Pi} \frac{P - c_k}{\sum_{i=1}^{N} \pi_i (P - c_i)^2}, k = \overline{1, N} \tag{12}$$

Then the following result immediately follows from (12):

Proposition: Irrespective of climatic conditions, an increase in the average profit leads to a higher production of electricity, as long as $c_k \leq P$ for every $k = 1, \dots, N$.

This result becomes apparent once we additionally presume that in every state of nature production is below its exogeneosly given optimum. In such a case, in terms of risk an increase in all possible states is definitely preferable to any other configuration. Therefore, the power plant ends up in a situation described in the proposition.

In the next section under some refinements we apply the model to historical data of "ARTSAKH HEK" OJSC. One thing you might worry about is that by construction the model is static in contrast to temporal nature of the data. However, as will become evident below, our calibration strategy abstracts from time dimension in modifying the sequence of elements of the data and thereby addresses this concern.

A methodology for calibration of probabilities: Consider the following information that is available in financial statements of hydropower plants (unfortunately, below the notations of the model are slightly abused):

- $\{E_t\}_{t=1}^{T}$ –the amount of electricity produced in a hydropower station,
- $\{AC_t\}_{t=1}^{T}$ –the average cost of production,
- $\{AR_t\}_{t=1}^{T}$ –the average revenue of production,
- $\{\Pi_t\}_{t=1}^{T}$ –the profit of a hydropower station.

Further, the optimal level of electricity production as a function of t is defined as the trend component of $\{E_t\}_{t=1}^{T}$ and is denoted by $\{\bar{E}_t\}_{t=1}^{T}$.

Consider three states of nature, $k \in K = \{H, L, M\}$, with probabilities $Prob(k = H) = \pi_H \geq 0$, $Prob(k = L) = \pi_L \geq 0$, $Prob(k = M) = \pi_M \geq 0$, $\pi_H + \pi_M + \pi_L = 1$. H (L, M) corresponds to the situation when climatic conditions are favourable (disadvantageous, middling) to the electricity production. Define two cut-off values of the produced electricity, E^1 and E^2 $(E^1 < E^2)$, such that, by convention, $L \coloneqq \{E_t : E_t \leq E^1, t = 1, \dots, T\}$ is produced in state L, $M \coloneqq \{E_t : E^1 < E_t < E^2, t = 1, \dots, T\}$ –in state M, and $H = \{E_t : E_t \leq E^1, t = 1, \dots, T\}$ –in state H. Moreover, one of the criteria for choosing E^1 and E^2 should advicably be that L, M and H have more or less the same number of elements (so that ex ante probabilities do not affect the final outcome). Let $|L| = T_L, |M| = T_M, |H| = T_H$ $(T_L + T_M + T_H = T)$ and let $T^* = min\{T_L, T_M, T_H\}$. Afterwards, arrange the entries of each group in chronological order. Denote by E_{ki} the i-th $(i = 1, \dots, T_k)$ element of the ordered group k $(k = L, M, H)$. Eliminate the last $T_k - T^*$ elements of each ordered group k $(k = L, M, H)$ and, thus, for every i $(i = 1, \dots, T^*)$ match together the three elements E_{Hi}, E_{Mi} and E_{Li}. This procedure is directed at matching each element of every group with, in a certain sense, the closest elements of the two other groups[1]. Then other characteristics, such average cost, average revenue, profit, optimal production, need to be attached to ever element E_{ki} forming the following vector of attributes for every k $(k = H, L, M)$ and i $(i = 1, \dots, T^*)$:

$$(E_{ki}, \bar{E}_{ki}, AC_{ki}, AR_{ki}, \Pi_{ki}).$$

Eventually let $\bar{\Pi} = \frac{1}{3T^*} \sum_{k \in K} \sum_{i=1}^{T^*} \Pi_{ki}$.

Using a sufficient statistic approach and endogenizing the price (that is approximated by the average revenue) with respect to the state of nature, in the context of empirical analysis (12) boils down to the folloing system of equations $(i = 1, \dots, T^*)$:

$$E_{Hi} = \bar{E}_{Hi} \frac{\pi_L(AR_{Li} - AC_{Li})(AC_{Hi} - AC_{Li}) + \pi_M(AR_{Mi} - AC_{Mi})(AC_{Hi} - AC_{Mi})}{\pi_L(AR_{Li} - AC_{Li})^2 + \pi_M(AR_{Mi} - AC_{Mi})^2 + \pi_H(AR_{Hi} - AC_{Hi})^2} +$$

[1] For more on matching strategies, see, for example, **Cameron, C., Trivedi, P.**, "Microeconometrics: Methods and Applications", Cambridge University Press, 2009, pp. 860-896.

$$+\bar{\Pi}\frac{AR_{Hi}-AC_{Hi}}{\pi_L(AR_{Li}-AC_{Li})^2+\pi_M(AR_{Mi}-AC_{Mi})^2+\pi_H(AR_{Hi}-AC_{Hi})^2}, \quad 13)$$

$$E_{Mi}=\bar{E}_{Mi}\frac{\pi_L(AR_{Li}-AC_{Li})(AC_{Mi}-AC_{Li})+\pi_H(AR_{Hi}-AC_{Hi})(AC_{Mi}-AC_{Hi})}{\pi_L(AR_{Li}-AC_{Li})^2+\pi_M(AR_{Mi}-AC_{Mi})^2+\pi_H(AR_{Hi}-AC_{Hi})^2}+$$

$$+\bar{\Pi}\frac{AR_{Mi}-AC_{Mi}}{\pi_L(AR_{Li}-AC_{Li})^2+\pi_M(AR_{Mi}-AC_{Mi})^2+\pi_H(AR_{Hi}-AC_{Hi})^2}, \quad (14)$$

$$E_{Li}=\bar{E}_{Li}\frac{\pi_M(AR_{Mi}-AC_{Mi})(AC_{Li}-AC_{Mi})+\pi_H(AR_{Hi}-AC_{Hi})(AC_{Li}-AC_{Hi})}{\pi_L(AR_{Li}-AC_{Li})^2+\pi_M(AR_{Mi}-AC_{Mi})^2+\pi_H(AR_{Hi}-AC_{Hi})^2}+$$

$$+\bar{\Pi}\frac{AR_{Li}-AC_{Li}}{\pi_L(AR_{Li}-AC_{Li})^2+\pi_M(AR_{Mi}-AC_{Mi})^2+\pi_H(AR_{Hi}-AC_{Hi})^2}. \quad (15)$$

The model closes with the following definition:

Definition: Equilibrium in hydropower production under the risk of climatic conditions is defined as a vector of probabilities (π_H,π_M,π_L) such that optimality conditions (13), (14) and (15) hold true for any given values of $\{E_k,\bar{E}_k,AC_k,AR_k,\bar{\Pi}\}_{k\in K}$.

Taking this definition into account, our objective is to calibrate equilibrium values of π_H,π_M,π_L, given the relationships in (13), (14) and (15). That is, we are interested in probabilities under which the model (5)-(6) reproduces the actual production data as closely as possible[1]. Because the model is based on the assumption of "random" water flow, these probabilities, in fact, would reflect the risk of climatic conditions.

With an aspiration to quantify the equilibrium, we proceed as follows: with a slight abuse of notation, the values, that are determined by (13), (14) and (15), are denoted by $\hat{E}_{Hi}(\pi_H,\pi_M,\pi_L),\hat{E}_{Mi}(\pi_H,\pi_M,\pi_L),\hat{E}_{Li}(\pi_H,\pi_M,\pi_L)$, whereas E_{Hi},E_{Mi},E_{Li} stand for the actual (observed) values. Naturally, we attempt to minimize the difference between the actual data and the data predicted by (13)-(15) taking into account that π_L,π_M and π_H are probabilities. Thus, in the end we arrive at the following problem:

[1] This procedure can also be interpreted in the following way: we ask what values the probabilities would take on, if the hydropower station operated in an optimal manner.

$$\max_{\pi_H,\pi_M,\pi_L} \sum_{i=1}^{T^*} [\left(E_{Hi} - \hat{E}_{Hi}(\pi_H,\pi_M,\pi_L)\right)^2 + (E_{Li} - \hat{E}_{Li}(\pi_H,\pi_M,\pi_L))^2 + \\ +(E_{Mi} - \hat{E}_{Mi}(\pi_H,\pi_M,\pi_L))^2] \quad (16)$$

subject to

$$\pi_H + \pi_M + \pi_L = 1, \quad (17)$$

$$\pi_H \geq 0, \pi_M \geq 0, \pi_L \geq 0 \quad (18)$$

Alternatively, one could additionally weigh the terms $(E_{ki} - \hat{E}_{ki}(\pi_H,\pi_M,\pi_L))^2$ by π_k for $k = H, M, L$, in the objective function (16). However, since this latter approach is not directly backed by the structure of the model, we refrain from implementing it and choose the former method.

At this point, we can already apply the aforementioned methodology to the quarterly data of "ARTSAKH HEK" covering the period from the third quarter of 2009 to the third quarter of 2016. First, it is helpful to look at the density of the produced electicity.

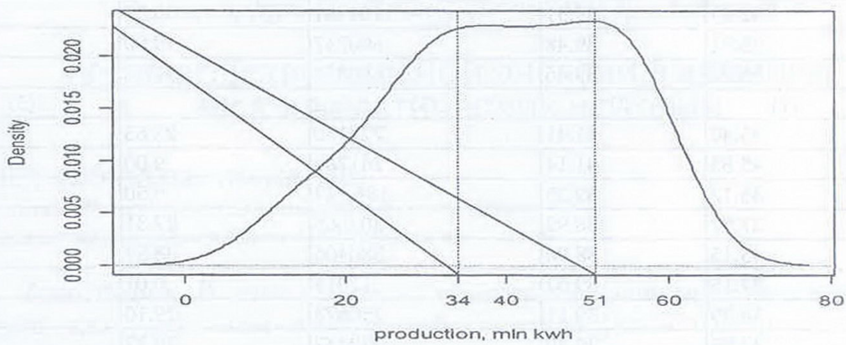

Figure. The density function of the power production in "ARTSAKH HEK" and its two thresholds.

The two cut-off values classifying the production stand out immediately: all the entries below 34 mln kwh are qualified as type L, the entries between 34 and 51- as type M, and the enties above 51- as type H . Thus, groups H and M comprise 10 and group L- 9 elements. This means that the effective number of elements is 9.

The "ordered" states of nature H, M and L are fully characterized in table 2.

We solve the problem (16)-(18) via some version of an evolutionary (genetic) algoritm and obtain the following equilibrium probabilities:

$$\begin{cases} \pi_H^* \approx 0.36, \\ \pi_M^* \approx 0.45, \\ \pi_L^* \approx 0.19. \end{cases}$$

These numbers can be interpreted as follows: given the uncertainty in the water flow, in the subsequent quarter the climatic conditions will be favourable (middling, unfavourable) to the production of electricity in "ARTSAKH HEK" with probability 0.36 (0.45, 0.19). Hence, it is most likely that in the fourth quarter of 2016 the level of the production of electricity in "ARTSAKH HEK" will be either high or low.

Table 2

The characteristics of goups H, M and L in "ARTSAKH HEK"[1].

State H	E, mln kwh (1)	E*, mln kwh (2)	Π, thousand drams (3)	AR, dram/kwh (4)	AC, dram/kwh (5)
2010-Q2	58.61	40.85	719412	16.07	-2.31
2010-Q3	56.69	40.54	1016728	22.55	-3.93
2010-Q4	51.37	40.22	1157427	31.44	-6.03
2013-Q4	52.77	39.29	701181	41.09	-12.01
2014-Q1	54.37	39.37	487230	16.38	-2.96
2014-Q4	56.78	39.49	863419	49.23	-12.15
2015-Q2	52.09	39.51	1101481	33.57	-6.97
2016-Q1	55.51	39.48	690747	19.00	-3.46
2016-Q2	56.92	39.46	1388874	37.53	-6.65
State M	(1)	(2)	(3)	(4)	(5)
2009-Q4	45.40	41.41	727170	23.63	-5.24
2010-Q1	45.83	41.14	241780	9.00	-1.41
2011-Q3	35.17	39.35	184,524	9.50	-2.34
2012-Q3	37.97	38.92	302666	27.31	-0.01
2012-Q4	45.15	38.96	380406	33.59	-0.01
2013-Q1	37.15	39.02	12015	0.01	0.00
2013-Q2	34.39	39.11	250673	29.10	-9.09
2013-Q3	42.95	39.20	229447	30.37	-10.8[illegible]
2014-Q2	42.29	39.42	902385	37.43	-7.71
State L	(1)	(2)	(3)	(4)	(5)
2009-Q3	26.00	41.67	421775	25.55	-6.34
2011-Q1	28.07	39.90	47126	9.33	-2.87
2011-Q2	31.62	39.60	43501	12.11	-5.12
2011-Q4	31.27	39.15	189,229	9.99	-3.19
2012-Q1	21.22	39.01	32151	10.75	-5.12
2012-Q2	28.86	38.94	74,316	20.62	-7.61
2014-Q3	12.98	39.46	814050	138.76	-38.96
2015-Q3	14.07	39.50	991721	143.28	-39.24
2015-Q4	31.42	39.49	1058334	83.17	-24.41

[1] The data are taken from the quarterly financial statements of "ARTSAKH HEK", OJSC.

ԿԱՆԱՉ ԷՆԵՐԳԻԱՅԻ ԱՐՏԱԴՐՈՒԹՅՈՒՆԸ ՀԱՅԱՍՏԱՆՈՒՄ. ԿԼԻՄԱՅԱԿԱՆ ՊԱՅՄԱՆՆԵՐԻ ՌԻՍԿԸ

ԱՇՈՏ ԱՐԱՐԱՏԻ ՆԱՆՅԱՆ
Երևանի պետական համալսարանի տնտեսագիտության
և կառավարման ֆակուլտետի ասպիրանտ

Համառոտագիր: Հոդվածում առաջարկված է տնտեսագիտամաթեմատիկական մոդել՝ ուղղված էներգետիկայի այս ճյուղին բնորոշ կլիմայական պայմանների ռիսկի կառավարմանը: Մոդելը կիրառված է «Արցախ ՀԷԿ» ՓԲԸ-ի եռամսյակային տվյալների նկատմամբ, և գնահատված են հիդրոէլեկտրակայանի «հնարավոր վիճակների» հավասարակշիռ հավանականությունները:

Բանալի բառեր. կանաչ էներգետիկա, կլիմայական պայմանների ռիսկ, հավասարակշիռ հավանականություններ, գենետիկ ալգորիթմ, մաթեմատիկական մոդել, գնահատում:

ПРОИЗВОДСТВО ЗЕЛЕНОЙ ЭНЕРГИИ В АРМЕНИИ: РИСК КЛИМАТИЧЕСКИХ УСЛОВИЙ

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Аннотация: В данной статье предложена экономико-математическая модель для управления риском климатических условий, характерной для данной отрасли энергетики. Модель применена к квартальным данным ЗАО «Арцах ГЭС», и оценены равновесные вероятности «возможных состояний» гидроэлектростанции.

Ключевые слова: *зеленая энергетика, риск климатических условий, равновесные вероятности, генетический алгоритм, математическая модель, оценка.*

PRODUCTION OF GREEN POWER IN ARMENIA: THE RISK OF CLIMATIC CONDITIONS

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Abstract The paper proposes an economic-mathematical model built by the author that will enable to manage the risk associated with climatic and/or environmental conditions typical to this sector of the electricity industry. The model is applied with respect to the quarterly data on "ARTSAKH HEK" OJSC, and equilibrium probabilities of "possible states" of the hydropower station are estimated.

Keywords: *Green energy, risk associated with climatic conditions, equilibrium probabilities, genetic algorithm, mathematical model, estimation.*