АКАДЕМИЯ НАУК АРМЯНСКОЙ ССР

АСТРОФИЗИКА

ТОМ 7 АВГУСТ, 1971 ВЫПУСК 3

AN ANGULAR-DIAMETER—RED-SHIFT RELATION FOR RICH CLUSTERS OF GALAXIES

G. PAAL

Received 30 October 1970

A simple method of determining a characteristic size of clusters of galaxies has been developed and applied to all rich clusters with known red-shifts to establish a new observational relation for cosmology: an apparent-size—red-shift relation. Empirical evidence has been obtained suggesting the presence of large-scale inhomogeneities in the distribution of clusters of galaxies, the evolution of the clusters, the reality of the expansion of the Universe and the possibility of correcting the value of the Hubble "constant" as well as the deceleration parameter of the universal expansion.

1. *Introduction.* Cosmology, cosmogony and study of individual clusters is in great need of a practical definition of the diameter of clusters of galaxies. Although numerous attempts have been undertaken to define one [1—5], it is generally known that the definitions so far advanced fail to give reliable characteristic sizes for a good many clusters which have their redshifts measured, i. e. which might profitably be used for the purposes of observational cosmology and cosmogony. Photographs covering larger areas of the sky around the clusters and reaching fainter limiting magnitudes would be needed for those definitions to be useful in comparing near-by and far away clusters [3, 4, 6, 7], even if we restrict ourselves to well-separated clusters with smooth backgrounds. The present paper aims at providing a definition of wider applicability for a characteristic size in clusters of galaxies and making use of it for constructing an angular-diameter—red-shift relation for all rich clusters with known red-shifts.

2. *An operational definition of cluster diameter.* There are some natural requirements that a suitable definition of cluster diameter has to meet: 1) accuracy, 2) simplicity, 3) quality of producing commensur-

able results for clusters at different distances (no errors systematically increasing with distance), 4) applicability to the existing photographic plate material. In our specified program the last requirement expresses the necessity of using mainly the National Geographic Society Palomar Sky Survey (hereafter called P. S. S.) plates or they being unavailable the corresponding prints. Other existing plates cover only the region of a few clusters. This in turn places restrictions on the part of the luminosity function which may be compared in the study.

Selecting the galaxies for investigation. Owing to the fact, that the members of the most distant clusters can be recognised on P. S. S, only in a rather small magnitude range, the magnitude difference, Δm, between the brightest and faintest members to be examined in near clusters must also be small by virtue of the 3rd requirement (cp. the last point of this Section). The value of Δm may at best amount to 2, just like in the case of Abell's work [8]*. To make the commensurability of the results quite exact the strict identity of the magnitude intervals studied ought to be insisted upon, a condition which would necessitate the use of reliable magnitude standards. Lack of the latter caused us to replace the selection of galaxies of equal brightness-categories (relative to the brightest members) by that of galaxies of equal size-categories (relative to the largest members) when defining the new kind of cluster diameter for the purposes of the present study. No doubt these two categories are in fact closely related, but apart from the existence or otherwise of some connection between them, the procedure here suggested provides a proper substitute for the conventional one by distinguishing the members of another comparable subsystem within the clusters instead of the system constituting the bright end of the luminosity function of the clusters. Obviously the isophotos defining the size of the galaxies depend for their position upon several factors: optics of the telescope used, atmospheric seeing condition, quality of the plates, exposure time, red-shift etc, nevertheless the ratio of the isophotal diameters which will be specified below, remains practically unchanged for any given pair of galaxies whatever the influence of the factors mentioned may be, provided these galaxies are elliptical. The reason for this lies in the fact, that the elliptical galaxies have a common luminosity distribution law well represented by Hubble's interpolation formula [9]

$$B/B_0 = [(r/a) + 1]^{-2},$$

* For the same reason as in Abell's work the red P. S. S charts were used throughout this study.—cp. [8] p. 213.

where B/B_0 is the ratio of the observed surface brightness to that at the centre of the nucleus, r is the angular distance from the centre and a is a parameter angle fixing the scale. In the case of measuring faint isophotal contours on P. S. S. this formula reduces to

$$B/B_0 = (r/a)^{-2},$$

which makes our statement obvious. Rich spherical clusters, most useful in deriving reliable cluster diameters, are predominantly populated by ellipticals so that—by suitably fixing the ratio of the diameters of the largest cluster members to those of the smallest ones to be studied—comparable subsystems of the clusters can be defined in a way almost independent of the circumstances of observation and without reference to any standards. This holds true at least for regular clusters, while at the same time the procedure of selecting the galaxies in irregular clusters remains no doubt slightly indefinite. For the sake of completeness we note that some minor factors affecting the position of isophotals, such as diffraction, irradiation and neighbourhood effect [10] lead to a combined error less than the uncertainty of the isophotal diameters estimated on P. S. S. prints in the case of our relative measurements of galaxies of similar size in each cluster.

In what follows, the ratio of the isophotal diameters of the largest galaxies to those of the smallest ones in the subsystems selected for examination is taken to be 2, a number roughly equivalent to a magnitude interval somewhat less than $\Delta m = 2$. The diameter of the "largest" members of a cluster is determined by averaging the diameters measured for the 3rd and the 5th largest galaxy on a field chosen by mere inspection as being the area of the "main condensation" of the cluster. After the cluster diameter is derived by the procedure which we shall come to presently, the 3rd and 5th largest galaxies within the corresponding (newly defined) area are again identified. If they differ from the galaxies picked out previously within the earlier estimated "main condensation", the procedure is repeated using the new "standard" galaxies until a self-consistent system of data is reached (cp. [6]). The writer has found from experience that—instead of a single isophotal diameter—the square root of the product of the major and minor isophotal diameters of every galaxy may profitably be used in the study.

In spite of all caution exercised in selecting the galaxies to fulfil the 3rd requirement, the possibility of selecting not strictly corresponding parts of the clusters with errors slowly growing as the distance increases will be admitted and a method of measuring cluster diameters developed which is insensitive to moderate alterations of the limiting isophotal diameter (and also to the other source of systematic error: the increasing interference of the field). On the other hand the best possible fulfilment of the last requirement appears to be assured by our selection alone (cp. also the last point of the present Section).

Method of measuring the cluster diameter. Having selected the cluster members with which to work we proceed to count them as usual in circular rings around the apparent centre of the cluster [3, 4, 11] and to construct the function $N(\vartheta)$, which represents the total number of galaxies found within the circle defined by radius ϑ. For a considerable part of rich clusters the diagram $N(\vartheta)$ shows one major *change of slope* in the range $20 < N < 50$. (The possibility of random occurence of these features will be discussed later). In order to illustrate this phenomenon we present the corresponding diagrams for a few near-by regular clusters on Fig. 1 (upper part). Since the "deflection"

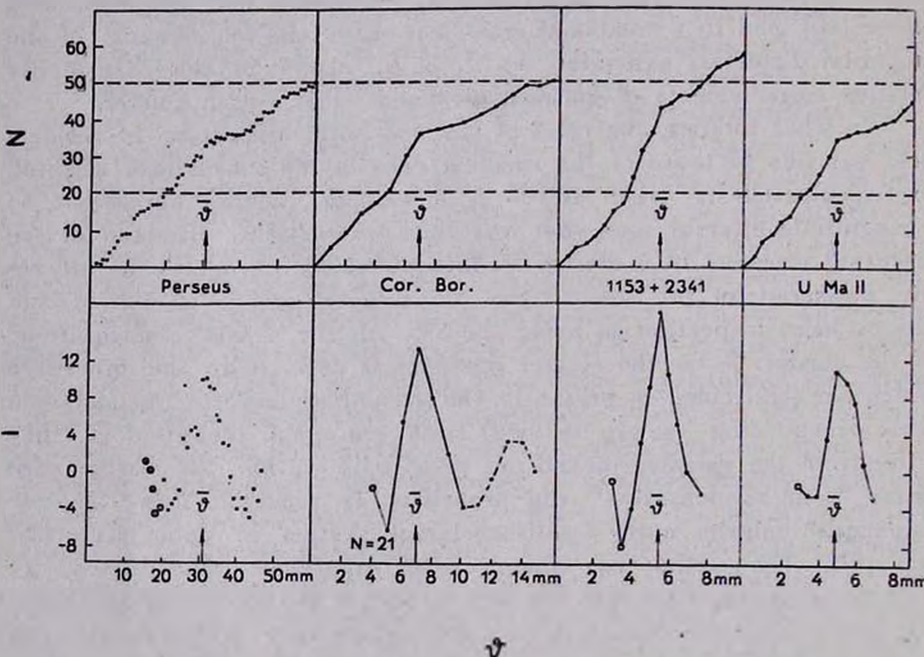

Fig. 1.

of the curve $N(\vartheta)$ generelly occurs at a fairly well-defined radius ϑ, an opportunity arises to derive an accurate charucteristic size of these clusters in accord with the 1st and 2nd requirements mantioned earlier. The perfect fulfilment of the 3rd (last remaining) requirement—

the strict commensurability of the resulting radii—is ensured by the very important observational fact that *these radii are practically independent of moderate changes in the "limiting magnitude" of the counts* (cp. the closing remark of the previous point)*. According to our present usage of the word the condition for the values $\bar{\vartheta}$ to be "commensurable" is *not the strict identity* of the corresponding diameters in space, but *their freedom from systematic observational errors* increasing with the distance and, of course, *the existence of a mean spatial diameter* for clusters of any given type and distance (in full agreement with our observations described in Sections III and IV and also with the generally assumed isotropy of the observable universe at large) as well as a *reasonable scattering* of the individual diameters around this mean (also supported by the data of the following Sections)**. This notion of commensurability implies that identical clusters located at different distances have equal spatial diameters, but evolutionary effects may be present to make the spatial diameters deviate from each other at different epoch of light emission (i. e. at different distance-categories).

The afore-mentioned proposal of defining cluster diameters can be formulated numerically and extended also to rather irregular clusters to give definite sense to the notion of their "diameter" by introducing an appropriate index for slope variation

$$I = 2N(\vartheta) - N(0.7\vartheta) - N(1.3\vartheta) \qquad (1)$$

and calculating the value $\bar{\vartheta}$ at which $I(\vartheta)$ reaches its maximum. This definition still needs to be supplement for the case when there are two or more maxima. To do this properly requires introducing suitable (but otherwise arbitrary) restrictions on the minimum acceptable values of $I(\bar{\vartheta})$ and $N(\bar{\vartheta})$ and on the range of reasonable values for $\bar{\vartheta}$ in every cluster, which in turn may practically be reduced again to restrictions on the minimum and maximum values of $N(\bar{\vartheta})$ to be accepted. The restrictions $I(\bar{\vartheta}) > 5$ and $N(\bar{\vartheta}) = 35 \pm 15$ seem to be the best fit

* The field to cluster ratio is minimal in this range of Δm [6] so that the influence of the field can be disregarded when determining the locus of the "deflection". Obvious foreground objects may, of course, be excluded from the counts.

** Similarly the photometric "commensurability" of the first ranked cluster members is established by the observed small scatter of the empirical points around the existing mean Hubble relation based on them. The *scatter* of these diagrams provides information on the measured objects and the measuring method, whereous the *averages* deduced for small intervals of the red-shift contain the required data of coindogical interest.

for all the clusters investigated in this program and are used throughout the paper. A continuous increase or decrease of the function $I(\vartheta)$ is also required at one side of a reliable maximum of I. The counts are extended at least to $1.7\ \overline{\vartheta}$ in order to detect a definite descending branch of $I(\vartheta)$ at $\vartheta > \overline{\vartheta}$. With the above limiting values indicated, the diagrams $I(\vartheta)$ of a few typical clusters are represented in Fig. 1 (lower part). Were there several well-seperated maxima satisfying all these restrictions, the first one is accepted.

Since a systematic error in Δm brings about a change in $N(\overline{\vartheta})$ as well, the fulfilment of the 3rd requirement might appear doubtful, but the observed deflections at $N < 20$ are always small (expressed in I) and disappear during an appropriate control when the corresponding value N increase over 20 with the increase of Δm. On the other hand the accepted maxima are as a rule well above 20. Usually the second maxima at $\vartheta > \overline{\vartheta}$ also disappear i. e. they cease to meet the restrictions —when $N \sim 35$ is reached at decreasing Δm. If there were a remarkable systematic trend in the values $N(\overline{\vartheta})$ with the distance, a normalization of these values to $N \sim 35$ would be the last step in eliminating systematic errors.

To choose due spacing of the counting circles is a matter of minimal routine. 6 to 10 rings up to $\overline{\vartheta}$ proved to be enough for our purposes. In case of doubt a recourse to superfine spacing is sure to preclude the possibility of any mistake. A refinement of the rings was always carried out in the vicinity of a possible deflection but not published in the present paper.

The use of changing differences $\Delta\vartheta = \pm 0.3\vartheta$ in (1) can be justified by pointing out the obvious fact that in doing so we do not violate the requirement of commensurability (and the other requirements) in the sense as described above in detail. The increase of $\Delta\vartheta$ may of course give preference to one of two identical features of a function $N(\vartheta)$ at different values ϑ, but the preference itself is invariant against the change of distance of the cluster. On the other hand any fixed value $\Delta\vartheta$ ought to be chosen such that it be different for every cluster depending just on the cluster diameter, which is to be defined with its aid.

In several cases the observed features may be caused by random fluctuations around the average curve $\tilde{N}(\vartheta)$ of the type of cluster in question, but this holds with the same probability for any distance in an unevolving and homogeneous Universe. Any deviation from the just mentioned properties of the Universe will automatically be revealed by an analysis of the angular-diameter—red-shift relation to be found. The same relation will show that the mean of the values $\bar{\vartheta}$ for a given distance is nearly the same for all types of rich clusters considered.

There is another property of the value $\bar{\vartheta}$ indicating that the "deflection points" are indeed uniquely determined along the whole curves $N(\vartheta)$. The special feature characteristic only of these values can be expressed numerically in the following way: a model curve $M(\vartheta)$ is specified so that it consists of two consecutive linear sections (the first one originating at the point (0,0) with the ratio of their slopes being $2:1$ and the ratio of their projections on the abscissa $3:1$. The integral deviation of the model curve, $M(\vartheta)$, from the observed total number, $N(\vartheta)$, is calculated along the whole domain where $M(\vartheta)$ is defined by using the formula

$$\int_{0}^{1.3\vartheta^*} [M(\vartheta) - N(\vartheta)]\, d\vartheta = f(\vartheta^*).$$

The function $f(\vartheta^*)$ exhibits only one outstanding deflection (which is usually a maximum as well, expressing that the whole model curve rises over the observed curve by a maximum integral measure). The loci of these deflections coincide within reasonable limits with our values $\bar{\vartheta}$ derived earlier in an independent way. Any two possible candidates for the locus of deflection, $\bar{\vartheta}$, on a single curve $N(\vartheta)$ can easily be distinguished by this measure, thus excluding the danger of a misidentification of $\bar{\vartheta}$.

On the basis of all what has been said above we may state that the radii ϑ derived for several clusters chosen at random from two different narrow distance categories represent comparable statistical samples, *characteristic of their distance*. This circumstance makes our method suitable for cosmological investigations, where we are concerned with properties of huge volumes of space rather than with individual objects. At the same time we are probably correct in saying also that our values $\bar{\vartheta}$ are in several cases reliable and *reproducible characteristics of the clusters as well*. This is the case for regular, compact and spherical clusters well-separated from the neighbouring ones. To support this statement we present some data obtained by other authors as a comparison.

Comparison with data from other sources. Counts of the three nearest rich compact clusters made in relatively small magnitude intervals are available in the literature [3, 5, 12]. Strictly speaking the second one is only a mapping and list of all galaxies not fainter than $m_B = 15.7$, which has been converted into counts in circular rings by the author. It is a remarkable coincidence that in spite of the differences in the centres of the counting circles, as well as the limiting magnitudes and spectral sensitivities of the plates used the loci $\bar{\vartheta}$ of the maximal changes of slope as derived from both sources by means of the maxima of index (I) are in satisfactory agreement for these clusters (Table 1). The values I_{max} are also of the same order, which implies a less prominent change of slope in Zwicky's counts, where the limiting magnitudes are somewhat fainter and therefore the values $N(\vartheta)$ larger.

Table 1

Source Cluster	Corona Borealis		Coma		Perseus	
Zwicky's data	$\bar{\vartheta}=7'.5$	$I_{max}=15.2$	$\bar{\vartheta}=22'.4$	$I_{max}=9.8$	$\bar{\vartheta}=40'$	$I_{max}=8.1$
Our data (transformed in min. of arc)	$\bar{\vartheta}=7'.8$	$I_{max}=13.3$	$\bar{\vartheta}=23'.5$	$I_{max}=7.0$	$\bar{\vartheta}=35'$	$I_{max}=11.0$

In case of these larger numbers it is especially convenient to describe the phenomenon of "deflection" of $N(\vartheta)$ in an equivalent form: the value $\bar{\vartheta}$ can be considered as the only common limit of two significantly different distributions of the numbers $n_i = N_i - N_{i-1}$ at its both side. — Zwicky's counts of the Corona Borealis Cluster corrected for the field* are for example: 19.0, 16.8, 11.6, 13.3, 17.0 at $\vartheta < \bar{\vartheta}$, 10.4 in the ring with our $\bar{\vartheta}$ in it, and 4.2, 8.6, 9.2. 10.1, 8.4 at $\vartheta > \bar{\vartheta}$.— The contra-hypothesis that the counts at $\vartheta < \bar{\vartheta}$ and $\vartheta > \bar{\vartheta}$ come from the same distribution can be rejected at a significance level of 0.05 or less for a great many regular clusters. The use of a dividing point different from $\bar{\vartheta}$ would drasticly worsen this significance. This provides another statistical basis to the possibility of defining angular sizes with the aid of the maximum property of $I(\vartheta)$ for at least a number of clusters.

* Using these larger magnitude intervals it is more important to correct for the field. The correction is taken from Zwicky too. The immediate central region may be excluded from the sample because of the typical parabolic form of the function $N(\vartheta)$ at $\vartheta \sim 0$.

Choosing the centre of a cluster. Since the apparent centre of a cluster as indicated by the apparent centre of gravity of the small central group of the brightest member galaxies is not always symmetrically located with respect to the more extended "main condensation" of the cluster and our techniques consist in measuring the radius of this large condensation, the centre of the counting circles may be placed in a slightly eccentric position within the contour of the small central group so as to produce the highest possible maximum of $I(\vartheta)$. To obtain this maximum means of course to find the optimal centre around which the frequency of galaxies shows the most sudden decrease in every direction at nearly equal distances.

Accuracy. The position of the centre of the counting circles marked out by different skilled observers proved to be remarkably stable in the case of regular clusters, which are most valuable in this program. The individual positions usually agreed to better than $\pm$ 10 per cent of $\bar{\vartheta}$ when repeating their independent determination, which is quite sufficient for purposes at hand.

It is a more difficult matter to estimate the uncertainties in the final value $\bar{\vartheta}$ resulting from all the errors of the whole procedure described. Repeated independent determination of $\bar{\vartheta}$ may, however, throw some light on this question. There is a sound basis for considering that the values $\bar{\vartheta}$ are correct to within 15 per cent for about one half of the objects (regular cases), which are given the statistical weight 2 (or 1—2) in Table 2, of the following Section, while other clusters of higher degree of irregularity are denoted by 1 or 0 in the corresponding column of the Table.

Finally we note, that by changing the centre of the counting circles within the area of the group of the brightest members the radius $\bar{\vartheta}$ sometimes "disappears" (in the sense that the function $I(\vartheta)$ does not reach its adopted limit), but seldom, if ever, can the locus of a maximum satisfying the criteria be appreciable altered.

Studying small numbers of galaxies. One contraversial point in the proposed method still needs to be accounted for: the use of a rather poor statistical sample in each cluster. The author's investigations reveal the surprising fact that the unavoidable uncertainty caused by the deficiency of the sample would only be increased by a considerable enlargement of the number of member galaxies studied. Using larger numbers (i. e. larger magnitude intervals or size ratios) the deflection

Table 2

	Cluster	Counts (above) within ring limits given in *mm* on P.S.S. (below)	$\bar{n}$	Weight
1	Virgo	3 0 0 1 1 3 7 1 5 6 0 1 2 2 3 3 4 0 10 20 30 40 50 60 70 80 90 100 110 120 130 140 150 160 170	100	0
2	Perseus	2 3 4 5 2 1 4 4 5 4 2 1 0 1 3 2 4 1 1 2 0 3 6 9 12 15 18 21 24 27 30 33 36 39 42 45 48 51 54 57 60	31	2
3	Coma	2 2 1 5 4 4 4 4 3 1 3 1 0 1 3 1 3 0 3 0 0 2 4 6 8 10 12 14 16 18 20 22 24 26 28 30 32 34 36 38 40	21	2
4	1627+3938	2 3 2 3 2 2 2 3 6 2 2 2 4 2 4 3 3 3 2 2 0 2 4 6 8 10 12 14 16 18 20 22 24 26 28 30 32 34 36 38 40	18	0
5	Hercules	5 3 3 0 2 3 2 1 0 5 4 4 0 0 2 2 1 1 4 1 1 3 2 0 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26	15	1
6	UMa I	9 3 2 1 4 1 6 2 2 3 3 1 1 1 0 2 1 3 4 2 2 0 2 2.5 3 3.5 4 4.5 5 5.5 6 6.5 7 7.5 8 8.5 9 9.5 10 11 12 13 14	7.0	1
7	0106—1536	3 5 3 2 2 4 3 4 3 0 3 1 2 3 3 2 0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16	9.0	1
8	Leo	3 0 2 0 0 1 2 6 9 2 3 3 4 3 3 2 0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16	9.0	1
9	Cor. Bor.	5 4 5 3 4 9 6 1 1 1 2 2 3 3 0 1 1 5 1 2 0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20	7.0	2
10	0348+0613	1 2 3 0 2 5 4 3 4 3 9 2 3 2 2 6 4 3 3 5 0 0.5 1 1.5 2 2.5 3 3.5 4 4.5 5 5.5 6 6.5 7 7.5 8 8.5 9 9.5 10	5.5	1
11	1513+0433	5 5 4 3 6 5 6 2 2 2 5 5 1 1 1 1 1 2 2 2 3 0 1 2 3 4 5 6 7 7.5 8 8.5 9 9.5 10 10.5 11 11.5 12 13 14 15 16	9.5	1—2
12	Boötes	3 3 4 2 3 1 3 1 4 3 3 1 2 1 1 2 2 1 1 2 0 0.5 1 1.5 2 2.5 3 3.5 4 4.5 5 5.5 6 6.5 7 7.5 8 8.5 9 9.5 10	5.5	1
13	UMa II	3 4 2 3 2 5 2 4 5 5 1 1 0 1 1 1 3 2 0 0.5 1 1.5 2 2.5 3 3.5 4 4.5 5 5.5 6 6.5 7 7.5 8 8.5 9	5.0	2
14	1153+2341	3 2 1 2 3 4 2 7 7 5 7 1 2 0 3 2 4 1 1 2 0 0.5 1 1.5 2 2.5 3 3.5 4 4.5 5 5.5 6 6.5 7 7.5 8 8.5 9 9.5 10	5.5	2

15	1534+3749	1 5 6 3 3 3 5 4 5 4 6 5 2 2 1 3 3 3 4 5 0 0.5 1 1.5 2 2.5 3 3.5 4 4.5 5 5.5 6 6.5 7 7.5 8 8.5 9 9.5 10	6.0	2
16	0025+2223	2 2 0 2 4 5 3 5 4 4 0 1 2 2 4 3 2 0 0 0.5 1 1.5 2 2.5 3 3.5 4 4.5 5 5.5 6 6.5 7 7.5 8 8.5 9	5.0	1—2
17	1228+1050	2 1 1 4 2 5 4 2 4 0 1 2 0 1 1 2 0 0.5 1 1.5 2 2.5 3 3.5 4 4.5 5 5.5 6 6.5 7 7.5 8	4.5	0—1
18	0138+1840	4 3 6 3 5 4 2 5 5 1 0 1 4 5 2 4 1 0 0.5 1 1.5 2 2.5 3 3.5 4 4.5 5 5.5 6 6.5 7 7.5 8 8.5	4.5	1—2
19	1309—0105	5 5 2 3 1 5 4 4 1 0 1 1 3 2 0 0.5 1 1.5 2 2.5 3 3.5 4 4.5 5 5.5 6 6.5 7	4.0	2
20	Coma II.	8 4 2 4 1 3 5 4 1 2 0 3 3 1 0 0.5 1 1.5 2 2.5 3 3.5 4 4.5 5 5.5 6 6.5 7	4.0	1—2
21	0925+2044	4 3 2 1 5 3 4 3 3 1 1 2 3 4 1 1 1 3 0 0.5 1 1.5 2 2.5 3 3.5 4 4.5 5 5.5 6 6.5 7 7.5 8 8.5 9	4.5	0—1
22	1253+4422	4 3 2 8 6 7 2 2 2 3 4 4 0 0.5 1 1.5 2 2.5 3 3.5 4 4.5 5 5.5 6	3.0	2
23	Hydra II.	3 3 0 5 1 5 3 3 0 0 3 2 2 2 0 0.5 1 1.5 2 2.5 3 3.5 4 4.5 5 5.5 6 6.5 7	4.0	1
24	0024+1654	4 4 2 3 2 3 2 1 3 0 0 1 0 2 2 0 2u 4u 6u 8u 9u 10u 11u 12u 13u 14u 15u 16u 17u 19u 21u u=.137	1.8	2
25	1448+2619	3 4 4 4 4 5 6 3 0 2 1 1 6 4 3 0 2v 4v 6v 8v 9v 10v 11v 12v 13v 14v 15v 16v 18v 20v 22v v=.25	3.0	1
26	1410+5224	6 4 3 5 5 7 10 2 4 4 min2 min4 0 1x 2x 3x 4x 5x 6x 7x 8x 9x 10x 11x 12x x=.22	1.5	1

Notes: 1) Circles around NGC 4686, otherwise $\overline{\mathfrak{N}} \gtrsim 100$. 2) Supported by Zwicky's data. Cf. [12] and Fig. 1. 3) This value $\overline{\mathfrak{N}}$ refers to intervals of 1 *mm* 4) Hardly separable double system. See [14]. 5) Data refer to the more regular (northern) part. 9) Supported by Zwicky's data [3, 12]. Cf. Fig. 1. 10) Extremely faint and small for its red-shift [15]. 12) Ambiguous distinction of member galaxies. 14) Very rich [8], yet of "normal" size. Cf. Fig. 1. 15) Foreground galaxies slightly interfere. 17) Virgo Cluster galaxies (foreground) interfere. 19) Extremely rich [8], yet of "normal" $\overline{\mathfrak{N}}$ and $N(\overline{\mathfrak{N}})$. 21) Ambiguous distinction of member galaxies. 22) Small richness [8]; small $\overline{\mathfrak{N}}$; normal $N(\overline{\mathfrak{N}})$. 23) Foreground group seems to be projected on it. 24 Supported by Zwicky's data [3, 12]. 25) Too bright and large for its red-shift, Cf. Fig. 4. 26) Counts on a reproduction of a 200″ Palomar plate.

of the diagram $N(\vartheta)$ — a distinctive feature of the space distribution of the brightest cluster members — gradually disappears [11] and a less conspicuous feature in the radial distribution of galaxies near to the centre takes its place (cp. also Zwicky's counts [3, 4]). At those smaller radial distances the number of counted members is just as small as in our case, but the resulting characteristic size is much less definite owing to the necessary error in identifying the centre and the individual differences in the distribution of galaxies in clusters. A minimal field to cluster ratio is also an avantage of the small magnitude range [6]. Consequently the only way of improving the statistics is to *increase the number of clusters* themselves, included in the investigation, in perfect accord with the original design of the present paper. Common properties of several clusters may thus be detected at a satisfactory significant level even in case of poor sampling in each cluster.

It is also worth noting that the phenomenon underlying the existence of a deflection in $N(\vartheta)$, an essential physical separation of the brightest galaxies from the fainter ones, fits well in the known properties of ellipticals, the typical members of compact clusters [6, 13].

3. *Counts of galaxies in clusters.*

Equipment. Estimation of the isophotal diameters of the galaxian images as well as the counts themselves were made on the red P. S. S. prints with the aid of a variable magnifying power microscope possesing a turnable fine scale in the focal plane of the ocular lens. The resulting counts proved to be practically independent of the amplification, which was finally chosen to have different values in the interval from 3.3 × to 16 × for clusters of different distances. The P. S. S. prints were inspected through a special plane-parallel cast glass sheet put on them, which had a precise set of equidistant concentrical circles etched in the bottom side. Their spacing of one millimeter (1.12 minutes of arc of the sky) could be subdivided into two or more equal intervals by means of the ocular scale.

Explanation of Table 2. Some of the most important results of the counts of galaxies in all rich* clusters with known red-shifts are listed in Table 2. The clusters are designated either by their adopted names or by their equatorial coordinates in the first column of the Table. The second column defines the ring-shaped domains used at counting (below in each line) and contains the numbers, n, of those

* Clusters of Abell's richness group higher than 0 are termed "rich".

galaxies within the indicated rings, that meet the criterion for inclusion in the study as outlined in Section 2 (between the ring limits in an upper position). The characteristic radial distance $\bar{\vartheta}$ is given in the next column. Also given in Table 2 is the estimated statistical weight of $\bar{\vartheta}$ described in Section 2*. The tabulated values, n, are uncorrected for the continuous field. Any plausible field correction proportional to the area of the ring would make the same change of slope more prominent for every cluster. Typical relations $N(\vartheta) = \sum_{\vartheta}^{\vartheta} n$ and $I(\vartheta)$ of a few fairly regular clusters are plotted in Fig. 1.

4. *Discussion.* Table 2 of Section 3 contains the new observational relation of cosmology: a correlation between the red-shift and angular diameter for rich clusters of galaxies also represented in graphical form in Fig. 2. It covers a range $0.004 < z < 0.46$, reaching the most distant cluster to have its velocity measured.

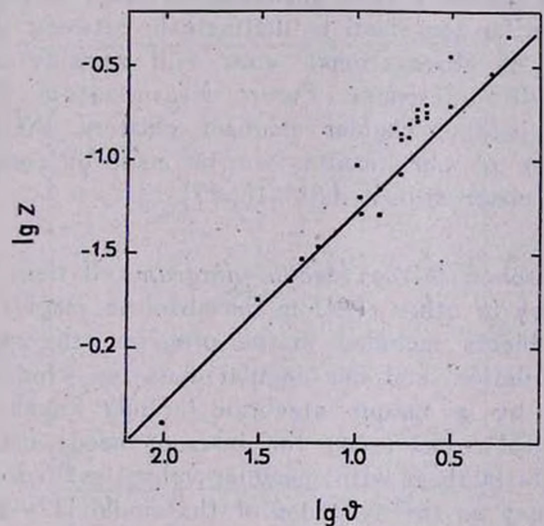

Fig. 2.

The first conclusion that follows from Fig. 2 is that the values ϑ really serve as a good characteristic measure of the clusters and a useful means of cosmological studies, because the scatter of the points is moderate. The gross properties of the diagram (log $\bar{\vartheta}$, log z) cor-

* Clusters of weight 0 and 0—1 give information of lower values. They are included in the program for the sake of completeness and comparison.

respond to what have been expected theoretically, most of the distant clusters being somewhat larger than it would follow from the simple intuitive law of perspectivity: $\bar{\vartheta} = \text{const}/z$. However some difficulties arise when one tries to understand the details of the diagram: contrary to all expectations the empirical points seem to align themselves around two parallel linear sections with an apparent discontinuity near $\log z = -1.0$. After a careful re-examination of all possible sources of uncertainty the author found that it was impossible to attribute this peculiar feature to any feasible observational error or selection (i. e. changes of the average richness or red-shift of the continuous spectra etc.). Even an intentional choice of the most improbable parameters during the measuring procedure cannot produce systematic errors of the required order. An attempt to eliminate this peculiarity by any suitable and arbitrary change of the limiting "magnitude" of the counts was also a complete failure. Two possibilities remain: either the fine structure of the diagram is to be interpreted in terms of random scatter or it is to be regarded a real phenomenon. The number and weight of our points are far too small to distinguish between these alternatives. A great deal of observational work still remains to be done before drawing definite inferences. Future measurements should extend to further well-separated regular compact clusters. At this stage of the research a test of our results can be made by comparing them with other related observational data [16, 17].

Comparison with the Hubble diagram. If there are no changes (of evolutionary or other type) in the absolute magnitude and intrinsic size of the objects included in the program, the connection between the Hubble relation and the angular-diameter — red-shift relation can be expressed by a unique algebraic formula known to be valid for any cosmological model (even for inhomogeneous and anisotropic ones and for objects in them with peculiar velocities) independently of any particular theory on the dynamics of the model [17—20]:

$$m = 10 \log (1 + z) - 5 \log \bar{\vartheta} + \text{const.} \qquad (2)$$

With the aid of this formula we can calculate a fictitious relation $(\bar{m}, z)$ exactly corresponding to our relation $(\bar{\vartheta}, z)$ and compare it to that obtained by direct photometry. The additive constant of eq. (2) is to be chosen so as to guarantee the best possible fit of the two empirical relationships to each other. Supposing the observations are not in error, any departure O—C of the directly observed diagram (m, z) from the calculated one expresses a change in brightness and/or size of the objects involved. On the other hand any coincidence between them is a strong indication favouring the reality and cosmological significance of the corresponding feature. The reason for the latter lies in the fact that there is but little chance for observational errors or cosmogonical corrections to be in perfect agreement with the cosmological connection between m, z and $\bar{\vartheta}$ expressed by eq. (2). Thus we have a general means of separating cosmogonical effects (any evolution of the content of the Metagalaxy) from cosmological ones and testing the reality of features of cosmological significance either on the Hubble diagram or on the diagram ($\bar{\vartheta}$, z). The method of doing this consists of comparing these diagrams after having transformed one of them to the system of the other one by the help of eq. (2). (To avoid any prejudice regarding the "most probable" continuous representation of the empirical relations, each empirical point of one of the relations should be transformed and the two "point diagrams" are to be confronted directly).

The diagram ($\bar{m}$, z), which results from the transformation of the diagram ($\bar{\vartheta}$, z) shown in Fig. 2, is given in Fig. 3. The additive constant

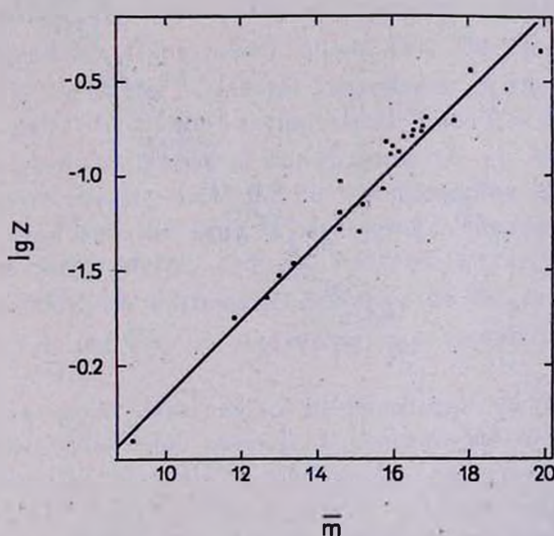

Fig. 3.

in eq. (2) is taken to be $+19.20$. One should not be surprised to find that the jump of the diagram is much less prominent in system ($\bar{m}$, z) than in ($\bar{\vartheta}$, z). It is a well-known fact that the apparent sizes of remote objects observed in an expanding universe are too large compared to the inverse of the present, momentary geometrical distance. This makes the upper part of the diagram ($\bar{\vartheta}$, z) deflect to the left to an extent increasing with distance. On the other hand the apparent intensities are reduced by the so-called number—and energy—effects causiug an opposite deflection in the relation (m, z) [21]. Consequently the anomalies in the derived fictitious Hubble relation ($\bar{m}$, z) give the impression of a wave around the asymptotical line rather than a split of the diagram into two branches.

The most recent information on the Hubble relation available for the author at the time of writing this paper is that coming from unpublished observations by Sandage and Baum. It is presented only in graphical form in [22] and [23]. Fig. 4 is constructed by superim-

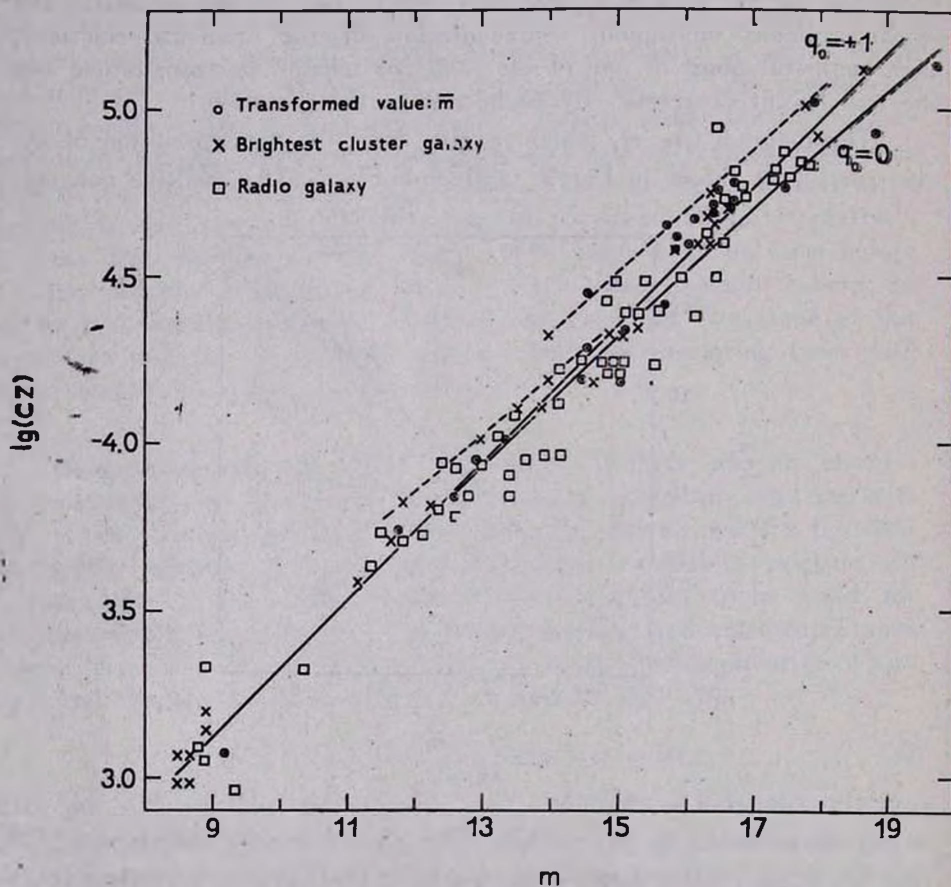

Fig. 4.

posing Fig. 11 of [22] on Fig. 3 of the present paper. The important result that emerges from Fig. 4 is that *the relation ($\bar{m}$, z) obtained* *from our results by transformation (2) does coincide with the usual Hubble relation (m, z)* within reasonable limits, i. e. within the scatter of the latter. Thus our relation $(\overline{\vartheta}, z)$ and the well-known Hubble relation mutually support each other as regards their over-all appearance. The observed relation $(\overline{\vartheta}, z)$ however much surprising it seemed to be at first sight, represents just one of the neglected possibilities permitted by the recent Hubble diagram (and by all its predecessors during the last fourteen years). It is important to note that all the Hubble diagrams published from 1956 till 1967 [15, 20, 24, 25] exhibit a minute wave or jump at $z > 0.1$. After having the relation $(\overline{m}, z)$ at hand we should even more carefully treat the question of the decrease or almost complete disappearance of this feature (without detailed explanation) on the latest diagrams [22, 23]. As stated above a coincidence in the fine structure of the diagrams (m, z) and $(\overline{m}, z)$ would be a test of reality of the corresponding feature.

The only tabulated data on the Hubble relation are still those obtained by photographic techniques as early as in 1956 [15]. They admit a numerical treatment and an opposite transformation into a fictitious diagram $(\overline{\overline{\vartheta}}, z)$ by means of eq. (2). The fine structure of the relation $(\overline{\overline{\vartheta}}, z)$ — i. e. the horizontal departures of the individual points or averaged pairs from the asymptotical line $\vartheta = \text{const}/z$ — shows a definite correlation with that of our diagram $(\overline{\vartheta}, z)$, the sample correlation coefficient ranging from 0.8 to 0.9 depending on the magnitudes used (m_V or m_{pg}) and the type of averaging. The opposite hypothesis that they are uncorrelated can be rejected at levels of significance from 0.01 to 0.001! (A systematic deviation of the relations $(\overline{\overline{\vartheta}}, z)$ and $(\overline{\vartheta}, z)$ or (m, z) and $(\overline{m}, z)$ worsening the correlation will be dealt with later).

One more point deserves to be mentioned. In comparing the fine structure rather than the over-all appearance of these diagrams it is vital to employ as accurate a photometric aperture as possible. The usual method of choosing the aperture as some plausible continuous function of the red-shift [26, 27] is certain to smooth out any really existing jump or irregularity on the Hubble diagram. According to Sandage [26] such uncertainties may amount to ± 0.1 mag., or even more in special cases, consequently the error of the differences of the magnitudes at both sides of an unexpected jump of the proposed type may come up to 0.3 mag. Moreover the author's investigations based on de Vaucouleur's data show that this limit of uncertainty ought to be increased. These small systematic errors are of essential importance when considering the fine structure of the diagram (m, z) and its connection with that of the relation $(\bar{\vartheta}, z)$. Obviously any correction for them would result in a closer correlation between the two diagrams compared.—One can easily see, that in the neighbourhood of a really existing jump on these diagrams the old method of determining the photometric apertures [15] brings about a smaller systematic error.— If the wave on the Hubble diagram and the diagram $(\bar{\vartheta}, z)$ were ever verified, one would, of course, be led to suppose an irregularity in the space distribution of red-shifts (velocities) — the common variable on the two similar looking diagrams.

Comparison with the number-count — red-shift relation. While in agreement with the Hubble relation, our relation $(\bar{\vartheta}, z)$ is definitely incompatible with the number counts of clusters of galaxies, if no strong geometrical or cosmological effects or large-scale inhomogeneities are admitted. For a homogeneous cosmological model with negligible space curvature the following relation between the counted numbers of clusters, N_{cl}, their diameter, ϑ, and red-shift, z, should apply independently of the kinematics of the model [17]

$$\lg N_{cl} = 3 \lg \frac{1+z}{\vartheta} + \text{constant.} \tag{3}$$

After calculating averages for point-triplets our relation $(\bar{\vartheta}, z)$ has been converted into a fictitious relation $\bar{N}(z)$ by the help of eq. (3) and compared with the relation $N_{cl}(z)$ obtained by Abell [8]. (See Fig. 5). The disagreement of the two diagrams is obvious whatever the choice of the additive constant may be. There is a sudden deflection from the most reliable (right hand side) part of these diagrams at the same value of the red-shift in both cases. This gives another strong support for the *existence of inhomogeneities up to a distance corresponding to* $z \sim 0.1$ (cp. also [16]). Moreover Abell used counting circles inversely proportional to the red-shifts, which according to Fig. 2 should cause a systematic loss of clusters after $z \sim 0.1$. Correcting for this effect makes the discrepancy increase. For the compact clusters (in Abell's sample), less subjected to systematic errors at counting, the discrepancy becomes even more prominent. Both curves might, of course, be smoothed out, but even in this case we seem to be justified in concluding that either the space section of the space-time strongly deviates from the Euclidean one (at those small distances where $z \sim 0.1$) or *there are inhomogeneities* in the velocity — and density — distribution of medium distant cluster or *cosmogonical changes* are observed.

Consequences of a possible "local" inhomogeneity. All that has been said above makes it reasonable to consider briefly the case when the homogeneous "cosmological substratum" is represented on our diagrams only at $z > 0.1$, the left hand side of the diagrams describing only local irregularities. The *Hubble constant* needed to characterize

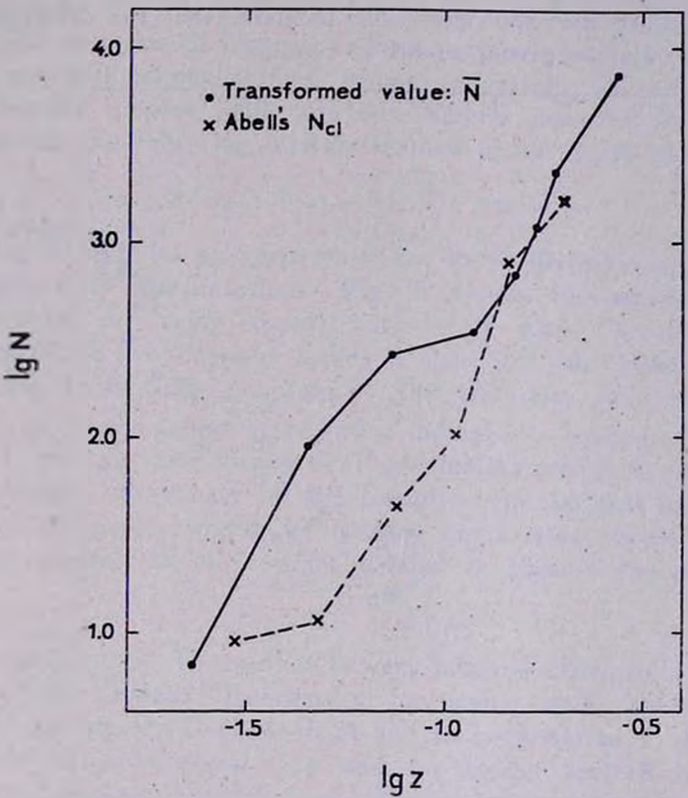

Fig. 5.

those distant domains of the space might *increase as much as 50 per cent* compared to the presently adopted values (see the dots in Fig. 4), while the *deceleration parameter*, q_0, *ought to decrease from about 1 to 0 or less,* thus causing *no decrease in the resulting time scale.* Another important consequence of disregarding the medium distant clusters would be obtaining an *empirical evidence for the evolution of clusters of galaxies.* The diagrams (m, z) and $(\overline{m}, z)$ appear to have different slopes at $\lg z > 4.3$, which — if real — is certainly not a cosmological effect. It can only mean that the remote clusters differ from the near-by ones either because of their evolution or spatial inhomogeneities, (This is why the values H_0 corresponding to the diagrams (m, z) and $(\overline{m}, z)$ are not exactly equal). Since the deviation of the two diagrams does not seem to depend on the direction of observations, the evolutionary interpretation is tentatively suggested. Clearly the plotted values are far too uncertain to claim that the discrepancy is real, but the data, as given, would so require.

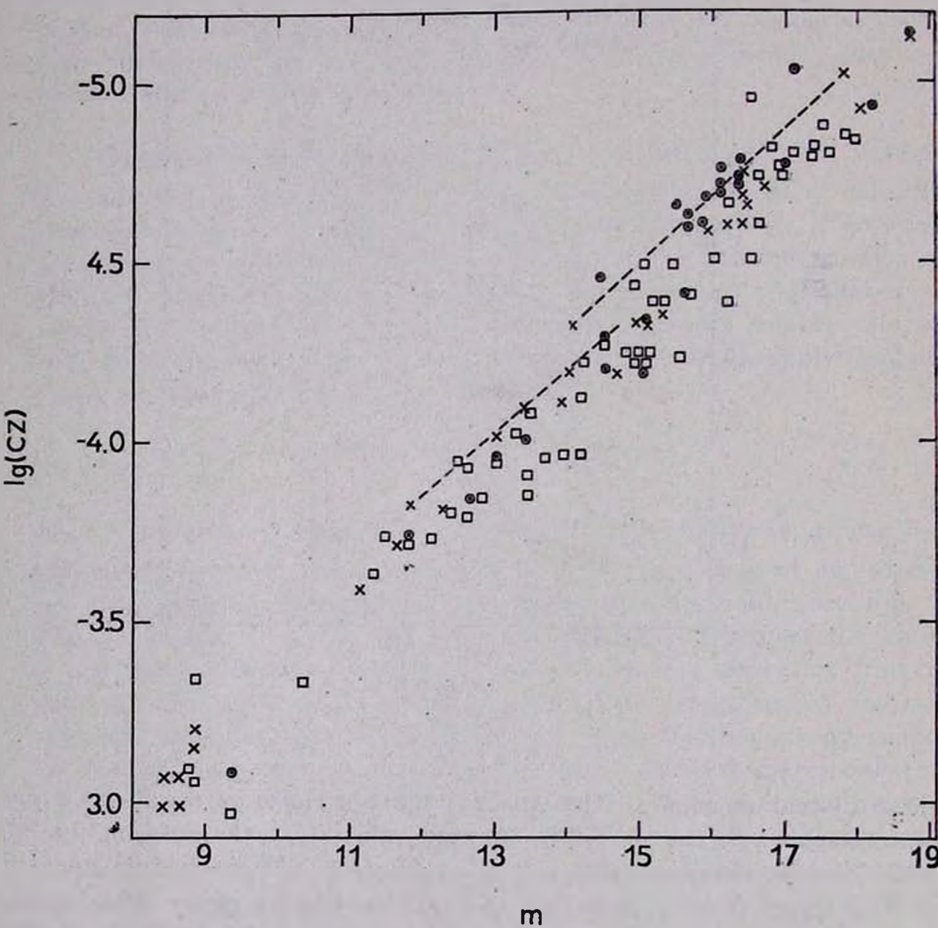

Fig. 6.

In connection with evolution the following is to be noted. Working with a fixed measuring technique when determining radii for, probably, evolving objects we cannot categorically exclude the possibility that the maxima of the index 1 defined by eq. (1) are more and more often misidentified with the increase of light travel time (i. e. distance) thus causing a false discontiunity in Fig. 2. In this case our suggestion regarding the evolution of clusters of galaxies would become much better substantiated.

Confrontation of the static and expansional world models with the observed data. Up to now we have interpreted the phenomenon of the red-shift in terms of Doppler effect. (More exactly a combined kinematical and gravitational effect whose separation depends on the frame of reference adopted [28]). In a non-Doppler cosmology we should be led to expect the following formula instead of eq. 2 [29—31]

$$m = 2.5 \lg (1 + z) - 5 \lg \overline{\vartheta} + \text{const.} \tag{4}$$

This in principle provides an apportunity for us to distinguish between the two alternative interpretations. Fig. 6 shows the equivalent of Fig. 4 constructed by using eq. (4), i. e. for a static world model. A comparison of the two diagrams makes it clear that our empirical data strongly favour the expanding picture of the Universe. Interpreted in a static world model the values $\overline{\vartheta}$, m and z indicate a jump in the evolution (or in the intrinsic properties) of clusters which is extremely unlikely. Moreover the succes of the Doppler type formula (2) in eliminating the discrepancy would be nothing but a mere chance in this case. Further research is, of course, needed to show if the effect is significant.

Acknowledgements. The author is very much indebted to Dr. Allan R. Sandage of the Palomar Observatory for having made available to him a copy of the cluster Cl 0024+1624 and to academician V. A. Ambartsumian for valuable discussions and for having invited him for a stay at the Byurakan Astrophysical Observatory where this work was partially done.

Konkoly Observatory,
Budapest, Hungary

СООТНОШЕНИЕ УГЛОВОЙ ДИАМЕТР—КРАСНОЕ СМЕЩЕНИЕ ДЛЯ БОГАТЫХ СКОПЛЕНИЙ ГАЛАКТИК

Г. ПААЛ

Разработан простой метод определения „характеристического размера“ скоплений галактик и применен ко всем богатым скоплениям с известными красными смещениями, с целью установления нового наблюдательного соотношения для космологии, зависимости угловой

диаметр — красное смещение. Получено эмпирическое указание в пользу существования крупномасштабных неоднородностей в распределении скоплений галактик, эволюции скоплений, правдоподобности реального расширения Вселенной и возможности исправления значения „постоянной“ Хаббла, а также параметра торможения универсального расширения.

REFERENCES

1. *G. de Vaucouleurs*, C. R. Acad. Sci. (Paris), 227, 586, 1948.
2. *C. D. Shane, C. A. Wirtanen*, A. J., 59, 302, 1954.
3. *F. Zwicky*, Encycl. of Phys. LIII, 401, Springer, 1959; Morphological Astronomy, 137, Springer, 1957; Proc. Third Berkeley Symp. on Math. Stat., III, 113, 1956.
4. *F. Zwicky*, P. A. S. P., 68, 331, 1956.
5. *F. Zwicky a. o.*, Catalogue of Galaxies and of Clusters of Galaxies, I, California Inst. of Technology, 1960.
6. *G. O. Abell*, Problems of Extra-Galaxies Research, IAU Symp. No. 15, 213, MacMillan Company, N. Y., 1962.
7. *K. Rudnicki*, Acta Astr., 14, pp. 279, 283, 1964.
8. *G. O. Abell*, Ap. J., Suppl. ser., 3, 211, 1958.
9. *E. Hubble*, Ap. J., 71, 213, 1930.
10. *G. de Vaucouleurs*, Ann. Astrophys., 11, 247, 1948.
11. *G. Paal*, Acta Phys. Hung., 21, 373, 1966.
12. *F. Zwicky*, Proc. Nat. Acad. Sci. U.S.A., 28, 1942, pp. 317, 355.
13. *J. L. Sersic*, Astrofizika, 4, 105, 1968.
14. *R. Minkowski*, A. J., 66, 558, 1961.
15. *M. L. Humason, N. U. Mayall, A. R. Sandage*, A. J., 61, 97, 1956.
16. *G. Paal*, Mitt. Sternw. Ung. Akad. Wiss., No. 54, 1964.
17. *G. Paal*, Astr. Z., 42, 19, 1965.
18. *J. Kutstian, R. K. Sachs*, Ap. J., 143, 379, 1966.
19. *O. Heckmann, E. Schücking*, Encycl. of Phys., LIII, 512, Springer, 1959.
20. *A. R. Sandage*, Ap. J., 133, 381, 1961.
21. *E. Hubble*, Ap. J., 84, 534, 1936.
22. *A. R. Sandage*, Galaxies and the Universe, Vetlesen Symp., 108, ed L. Woltjer, Columbia University Press, 1968.
23. *A. R. Sandage*, Observatory, 88, 91, 1968.
24. *G. C. McVittie*, Fact and Theory in Cosmology, Appendix, Eyre and Spottiswoode, London, 1961.
25. *A. R. Sandage*, Ap. J., 141, 1560, 1965.
26. *A. R. Sandage*, Proc. of the Meeting on Cosmology for the Galileo Celebration, Vol. 2, Tomo 3, 104, ed. L. Rosino, Florence, 1966.
27. *W. A. Baum*, Probl. of Extra-galactic Res., IAU Sump. No. 15, 396, 1962.
28. *E. Schücking*, High Energy Astroph., I, 226, ed. C. DeWitt, Gordon and Breach, 1966.
29. *E. Hubble, R. C. Tolman*, Ap. J., 1935,pp. 315, 324.
30. *J. D. North*, The Measure of the Universe, 229—242, Clarendon Press, 1965.
31. *M. S. Eigenson*, Vnegalaktic. Astr. (in Russian), Moscow, 1960, p.p. 339—342.