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An Alternative Expression of The Poynting Vector Operating in a Confined Region of Evanescent Waves: Natural Function in The Method of Single Expression

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Abstract: An alternative expression of the Poynting vector is presented. This expression is an intrinsic function at the electromagnetic wave description in the method of single expression (MSE). The MSE does not represent solutions of the Helmholtz equation as a sum of counter-propagating waves that permits to operate with an alternative expression for the Poynting vector. At the boundary problems solution carried out numerically by the MSE spatial distributions of electric and magnetic field amplitudes and the Poynting vector are obtained. An alternative expression of the Poynting vector is applicable both in confined media of a positive product of permittivity and permeability and of a negative product, that is relevant to the region of evanescent waves. The expression of the Poynting vector in the MSE is in complete agreement with the traditional representation of the Poynting vector.

Keywords: Evanescent waves, Poynting vector , Helmholtz equation, counter-propagation waves.

1. Introduction

The well-known expression for electromagnetic energy flow is the Poynting-Heaviside vector [1-3]:

$$\vec{P} = \frac{c}{4\pi}\left[\vec{E} \times \vec{H}\right], \tag{1}$$

In media, where electromagnetic wave cannot propagate, for example: in metals and plasma at frequencies less than plasma frequency or in optics at the total internal reflection (TIR) from the boundary of two dielectric media at the overcritical angles of wave incidence the Poynting vector is identically equal to zero [1-3]. However, in thin enough layers of metals [4-6], plasma (reentry plasma [7]) or at frustrated total internal reflection (FTIR) [8, 9] an electromagnetic wave after penetration to the confined region, where no propagating waves exist, indicates some transmitted energy as the fraction of the incident wave. A relevant boundary problem solution reveals that a wave penetrates into the medium where it is transformed to the evanescent field distributions, namely, exponential decreasing and increasing amplitude distributions [9-11]. These are not propagating waves and an application of the expression (1) for this type of a confined medium is a complicate procedure. Though partial energy transmission through the confined region of evanescent waves is under scientific and technical interest for a long time, still no information regarding the energy flow distribution within this type of regions are presented in the literature up to now [9-11].

Traditional methods of boundary problems solution operate with the Poynting vector only outside of the evanescent waves region. As opposed to them in the method of single expression (MSE) [12-15] for any boundary problem solution the Poynting vector is calculated within the media under analysis. The MSE gives an unusual solution to boundary problems as it operates with the alternative expression of the Poynting vector valid in any media including confined regions of evanescent waves as well.

The aim of the current work is a presentation of the alternative expression of the Poynting vector in the MSE suitable for monitoring the power flow distribution within the confined region of evanescent waves. As an example, the boundary problem solution by using the MSE is presented for a layer of negative permittivity at the wave normal incidence.

2. The method of single expression for the normal incidence of a plane wave

The MSE is an alternative correct tool for wavelength-scale analysis of any multilayer and modulated structures comprising dielectric, semiconductor or metallic layers with loss, gain or Kerr-type non-linearity [12-15]. The sketch of the boundary problem of normal incidence of a linearly polarized plane wave on an arbitrary layer bounded by loss-less and gain-less media is presented in Fig.1.

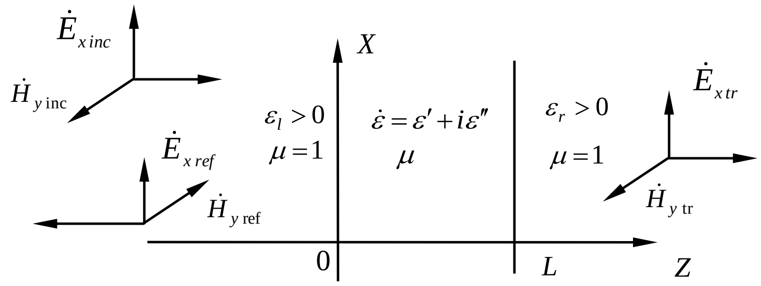


Fig. 1. Normal incidence of linearly polarized plane wave on a layer, from the left. Permittivities of surrounding loss-less and gain-less media and in the layer of thickness L are ε_l, ε_r and $\dot{\varepsilon}$, correspondingly.

The description of the MSE for a plane wave normal incidence on any multilayer or modulated structure is the following. From Maxwell's equations in 1D case the following Helmholtz equation can be obtained for linearly polarized complex electric field component $\dot{E}_x(z)$ propagating along the z axis (*here and hereafter the dot above the letter indicates complexity of the value*):

$$\frac{d^2\dot{E}_x(z)}{dz^2} + k_0^2\dot{\varepsilon}(z)\mu \cdot \dot{E}_x(z) = 0, \tag{2}$$

where $k_0 = \frac{\omega}{c} = \frac{2\pi}{\lambda_0}$ is the free space propagation constant, $\dot{\varepsilon}(z) = \varepsilon'(z) + i\varepsilon''(z)$ is the complex permittivity of a medium, μ is the permeability.

At $\varepsilon'(z) \cdot \mu > 0$, which means: either double positive ($\varepsilon'(z) > 0$ and $\mu > 0$) or double negative ($\varepsilon'(z) < 0$ and $\mu < 0$) medium, general solutions of the equation (2) in the traditional approach are represented as counter-propagating plane waves. In this case the relevant expression for the Poynting vector is:

$$P_z(z) = \frac{c}{8\pi}\mathrm{Re}\left[\dot{E}_x(z) \times H_y^*(z)\right]. \tag{3}$$

At $\varepsilon'(z) \cdot \mu < 0$, which means: either ($\varepsilon'(z) > 0$ and $\mu < 0$) or ($\varepsilon'(z) < 0$ and $\mu > 0$) in the traditional approach solutions of the equation (2) are decreasing and increasing exponential field distributions and this is a so-called region of evanescent waves. In unbounded region of evanescent waves the expression for the Poynting vector is completely imaginary value that brings to the absence of energy transfer, i.e. $P_z(z) = 0$ [1-3]. However, it is well known, it is possible to have energy transfer through a confined region of evanescent waves, i.e. $P_z(z) \neq 0$. Though the traditional expression of the Poynting vector is valid to monitor energy flow in such regions, but its application is a complicate task. The boundary problem solution by the MSE permits to overcome this problem. The essence of the MSE is a presentation of a general solution of the Helmholtz equation for electric field component $\dot{E}_x(z)$ in the form of a single expression:

$$\dot{E}_x(z) = U(z) \cdot \exp(-iS(z)) \tag{4}$$

instead of the traditional presentation as a sum of counter-propagating waves. Here $U(z)$ and $S(z)$ are real functions describing the resulting electric field amplitude and phase, respectively. Time dependence $\exp(i\omega t)$ is assumed but suppressed throughout the analysis. The solution in the form (4) prevails upon the traditional approach of counter-propagating waves and is more general since it is not relied on the superposition principle, hence applicable for non-linear media as well. This form of solution describes all possible distributions of electric field amplitude in space, corresponding to propagating, standing or evanescent waves in a medium of a negative product of permittivity and permeability. This means that no preliminary assumptions concerning the Helmholtz equation's solution in different media are needed in the MSE.

Based on the expression (4) the Helmholtz equation (2) after separation on real and imaginary parts is reformulated to the set of first order differential equations (5) regarding the electric field amplitude $U(z)$, its spatial derivative $Y(z)$ and a quantity $\Pi_z(z)$, which is proportional to the power flow density (the Poynting vector) in a medium:

$$\begin{cases} \dfrac{dU(z)}{dz} = Y(z) \\ \dfrac{dY(z)}{dz} = \dfrac{\Pi_z^2(z)}{U^3(z)} - \varepsilon'(z) \cdot \mu \cdot U(z) \\ \dfrac{d\Pi_z(z)}{dz} = \varepsilon''(z) \cdot U^2(z) \end{cases} \tag{5}$$

here $z = k_0 z$ is the coordinate normalized on the wavelength and $\Pi_z(z) = U^2(z)\dfrac{dS(z)}{dz}$. The actual value of the Poynting vector $P_z(z)$ can be obtained by multiplication of $\Pi_z(z)$ on $\dfrac{c}{8\pi\mu}$:

$$P_z(z) = \frac{c}{8\pi\mu}\Pi_z(z) = \frac{c}{8\pi\mu}U^2(z)\frac{dS(z)}{dz}. \tag{6}$$

The sign of $\varepsilon'(z)$ in the set of equations (5) can be taken either positive or negative describing relevant electromagnetic features of dielectric and metal (plasma), correspondingly. The sign of $\varepsilon''(z)$ indicates loss or gain in a medium.

The set of differential equations (5) is integrated numerically starting from the non-illuminated side of a layer at $(z = L)$, where only one outgoing travelling wave is supposed. The initial values for the integration are obtained from the boundary conditions of electrodynamics at the non-illuminated side of the layer (at $z = L$) as: $U(L) = E_{x\,tr}$, $Y(L) = 0$ and $\Pi_z(L) = \sqrt{\varepsilon_r} \cdot E_{x\,tr}^2 = \Pi_{z\,tr}$, where $\Pi_{z\,tr}$ is proportional to the Poynting vector in the medium of permittivity ε_r beyond the layer (at $z > L$) and $E_{x\,tr}$ is the amplitude of the transmitted wave. In linear problem solution the last can be taken as arbitrary.

Numerical integration of the set (5) goes step by step towards the illuminated side of the layer taking into account the actual value of the layer's permittivity for the given coordinate at each step of the integration. In the process of integration it is possible to record any variable of the set (5) in order to have full information regarding distributions of electric field amplitude, its derivative and power flow density inside and outside of the structure. From the boundary conditions of electrodynamics at the illuminated side of the structure the amplitude of incident wave $E_{x\,inc}$ and the power reflection coefficient R:

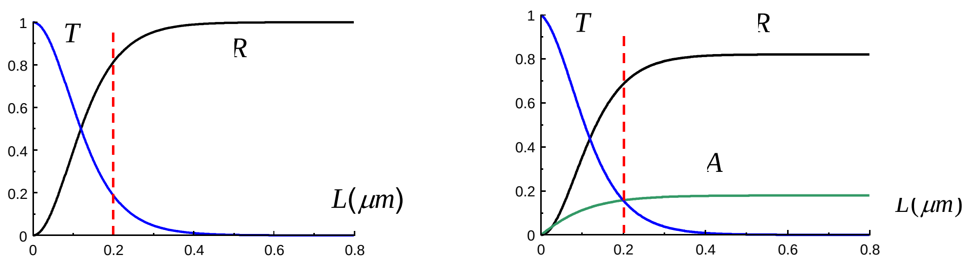
$$E_{x\,inc} = \left| \frac{U^2(0)\cdot\sqrt{\varepsilon_l} + \Pi_z(0) + iU(0)\cdot Y(0)}{2U(0)\cdot\sqrt{\varepsilon_l}} \right| \quad , \; R = \left| \frac{E_{x\,ref}}{E_{x\,inc}} \right|^2 = \left| \frac{U^2(0)\cdot\sqrt{\varepsilon_l} - \Pi_z(0) - iU(0)\cdot Y(0)}{U^2(0)\cdot\sqrt{\varepsilon_l} + \Pi_z(0) + iU(0)\cdot Y(0)} \right|^2$$

are restored at the end of the calculation. Here $U(0)$ is the resultant amplitude of the electromagnetic wave, $Y(0)$ is its derivative and $\Pi_z(0)$ is proportional to the power flow density at the illuminated interface of the layer at $z=0$, $E_{x\,ref}$ is the amplitude of the reflected wave, ε_l is the permittivity of the medium in the front of the structure, at $z<0$. The power flow density in the left medium (at $z<0$) is the sum of two counter-propagating power flows, i.e. incident and reflected ones. In accordance with the energy conservation law $\Pi_z(0) = \Pi_{z\,\mathrm{inc}} + \Pi_{z\,ref}$, where $\Pi_{z\,inc} = \sqrt{\varepsilon_l}\cdot E^2_{x\,inc}$ is proportional to the incident power flow density and $\Pi_{z\,ref} = -\sqrt{\varepsilon_r}\cdot E^2_{x\,ref}$ is proportional to the reflected power flow density. The negative sign of $\Pi_{z\,ref}$ is stipulated by its propagation opposite to z axis. The power transmission coefficient $T = \dfrac{\Pi_{z\,tr}}{\Pi_{z\,inc}} = \dfrac{\sqrt{\varepsilon_r}\cdot E^2_{x\,tr}}{\sqrt{\varepsilon_l}\cdot E^2_{x\,inc}}$ is defined as the ratio of the transmitted $\Pi_{z\,tr}$ to the incident $\Pi_{z\,inc}$ power flows.

3. Monitoring power flow through the confined region of evanescent waves-layer of negative permittivity

Let us consider the linearly polarized plane wave interaction with the layer of negative permittivity where evanescent waves are observed. Continuity of the traditional Poynting vector is fulfilled at the borders, while the relevant expression for the Poynting vector (6) in the MSE is calculated numerically throughout the structure without any issues regarding to the signs of permittivity $\varepsilon'(z)$, $\varepsilon''(z)$ and permeability μ.

As a specific example the MSE is applied for modelling of a plane wave incidence from the left on the layer (see Fig.1) of the thickness L of the negative real part of permittivity $\varepsilon' = -1$, at the absence of loss or gain $\varepsilon'' = 0$ and at the loss $\varepsilon'' = -0.2$ and the positive permeability $\mu = 1$. The calculation results are presented in Fig. 2. Partial transmission through the confined regions of evanescent waves is observed for thin enough layers of negative permittivity (Fig.2a,b). At the absence of loss in the layer ($\varepsilon'' = 0$) by increasing the thickness of the layer the reflectance R tends to the 1 (full reflection) and to zero transmission ($T = 0$), which is the limiting case for unbounded media of negative permittivity when the Poynting vector is zero.

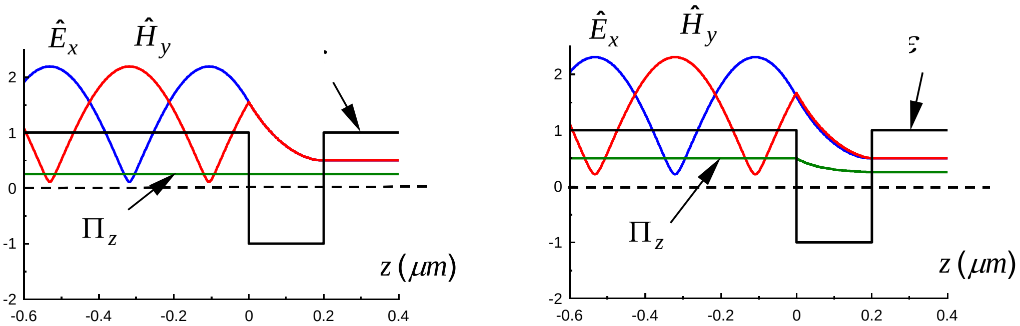


a) b)

Fig.2. Reflectance R and transmittance T from the layer of negative permittivity versus its thickness L at the plane wave normal incidence; a) at $\varepsilon' = -1$, $\varepsilon'' = 0$, b) $\varepsilon' = -1$, $\varepsilon'' = -0.2$; $A = 1 - R - T$ is the loss in the layer. $\lambda_0 = 850\,nm$.

At the loss ($\varepsilon'' = -0.2$) in the layer of the thickness L by an increase of its thickness the reflectance tends to the value less than 1 by the expense of loss in the layer (the curve indicated by letter A in Fig. 2b.).

To go deeply in the physics of the observed dependences it is useful to consider the layer of specific thickness, for example, of $L = 200\,nm$ (Fig.3).



a) b)

Fig.3 Distributions of electric $\hat{E}_x$ and magnetic $\hat{H}_y$ field amplitudes and power flow vector $\Pi_z(z)$ within and outside of the layer of the fixed thickness $L = 200\,nm$. $E_{x\,tr} = 0.5\,ESU(CGSE)$, $\lambda_0 = 850nm$, $\varepsilon_l = \varepsilon_r = 1$; a) at the absence of loss $\varepsilon'' = 0$, $R = 0.8121$, $T = 0.1879$; b) at the loss $\varepsilon'' = -0.2$, $R = 0.6856$, $T = 0.1568$, $A = 0.1576$.

A superposition of incident and reflected waves at $z < 0$ creates the region of partial standing wave (oscillating amplitudes of electric $\hat{E}_x$ and magnetic $\hat{H}_y$ components). Within the layer an exponential decrease of both amplitudes is observed (Fig.3a,b).

A continuity of the power flow at the boundaries is clearly observed in the modelling. At the absence of loss or gain ($\varepsilon'' = 0$) the power flow within the layer is constant and equal to the transmitted power flow (Fig. 3a). The transmitted power flow $\Pi_{z\,tr}(L) = \sqrt{\varepsilon_r} E_{x\,tr}^2$ is equal to $\Pi_z(L) = U^2(L) \cdot \left. \frac{dS(z)}{dk_0 z} \right|_{z=L}$ at the output of the layer ($z = L$) and within the layer ($0 \le z \le L$). At the illuminated side of the structure in accordance with the energy balance law the sum of incident and reflected power flows is equal to the power flow within the layer.

At the loss in the layer ($\varepsilon'' = -0.2$) an exponential decrease of the Poynting vector in the region of evanescent waves is observed (Fig.3b). Thus, the loss in the region of evanescent waves is also possible to monitor by the MSE.

4. Conclusion

The expression of the Poynting vector (6) in the MSE is a valid alternative to the traditional one, especially useful for the region of evanescent waves and making it a unique instrument for analyzing energy flow within any media.

The MSE makes also possible to monitor power flow in the confined region of evanescent waves at electric and magnetic loss or gain by application of complex values $\dot{\varepsilon} = \varepsilon' + i\varepsilon''$ and $\dot{\mu} = \mu' + i\mu''$ [16,17] and at the plane wave oblique incidence [18-20].

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