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Estimation of Phase Noise Impact on Coherence Length in FM-CW Radars With Voltage Controlled Oscillators

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Abstract: In this paper we analyze phase noise “propagation” within FM-CW radar originated from voltage controlled oscillator (VCO), which is of great importance from the viewpoint of effective moving target indication (MTI).

Keywords: Phase noise, voltage controlled oscillators, homodyne receiver, power spectrum density.

1. Introduction

Generally, circuit and device noise can perturb both the amplitude and phase of oscillator’s signal. Because usually amplitude fluctuations are greatly attenuated, the phase noise generally dominates [1-3]. In radars with VCOs in use, the stability and noisy spectrum of tuning voltage is of great interest. Obviously, noisy spectrum of tuning voltage is somehow transformed into another phase spectrum at the output terminal of VCO. In other words, VCO operates like a “noise transformer”. Much noise we apply to the tuning terminal, more phase noise we have at output terminal. The purpose of this paper is to estimate the coherence time of Doppler type homodyne receiver due to VCO’s phase noise.

2. Transformation of frequency noise into phase noise

Let’s assume we have Doppler homodyne receiver and the power spectrum density (PSD) of phase noise of oscillator is $L_1(\omega)$. The PSD of the reflected signal hence will be [4]

$$L(\omega) = 2L_1(\omega)(1-\cos\omega t_0) = 4L_1(\omega)\sin^2\frac{\omega t_0}{2} \tag{1}$$

where t_0 is time delay of reflected signal, and ω is cyclic frequency offset. Instantaneous frequency of the system can be presented as

$$\Omega(t) = \omega_0 + \omega_1(t), \tag{2}$$

where $\omega_1(t)$ is a noisy component of the frequency. In order to determine Doppler shift (beating frequency) we need to estimate the spectrum of frequency noise. So, instantaneous phase of oscillator is expressed as

$$\varphi(t) = \int_{-\infty}^{t} \Omega(t)dt\ . \tag{3}$$

Substituting (2) into (3) we have

$$\varphi(t) = \varphi_0 + \omega_0 t + \int_{-\infty}^{t} \omega_1(t)dt \,. \tag{4}$$

If the PSD of frequency noise $L_1^{\omega}(\omega)$ is already known, hence according to the integration property of Fourier transform, the PSD of phase noise $L_1(\omega)$ in stationary case will be

$$L_1(\omega) = \frac{L_1^{\omega}(\omega)}{\omega^2} \,. \tag{5}$$

Collecting all (5), (2) and (1) it is easy to obtain

$$L(\omega) = 4\frac{L_1^{\omega}(\omega)}{\omega^2}\sin^2\frac{\omega t_0}{2} = t_0^2 L_1^{\omega}(\omega)\sin c^2\frac{\omega t_0}{2} \,, \tag{6}$$

where $\sin cx = \dfrac{\sin x}{x}$ is non-normalized cardinal sine function.

By definition, the coherence time is defined as the width of equi-area rectangular having the height equal to the maximum value of correlation function (Fig. 1)

$$\tau_{coh} = \frac{1}{R_0}\int_0^{\infty} R(\tau)d\tau \,, \tag{7}$$

where $R(\tau)$ is correlation function.

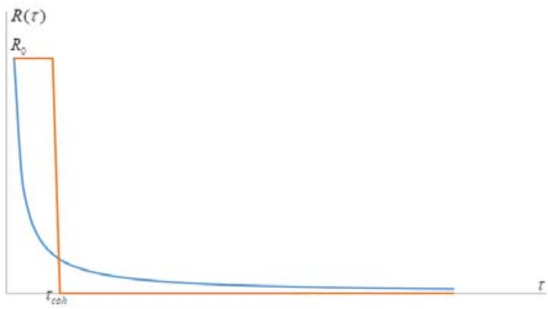

Fig. 1. Definition of coherence time

According to Wiener-Khinchin theorem

$$R(\tau) = \frac{1}{2\pi}\int_0^{\infty} L(\omega)e^{-i\omega\tau}d\omega \,. \tag{8}$$

Let's assume $L_1^{\omega}(\omega)$ is like white Gaussian noise

$$L_1^{\omega}(\omega) = L_0^{\omega} \,. \tag{9}$$

Substituting (6) and (9) into (8) and taking into account the duality property of Fourier transform finally we have for correlation function

$$R(\tau)=\frac{1}{2\pi}\int_0^\infty t_0^2 L_1^\omega(\omega)\sin c^2\frac{\omega t_0}{2}e^{-i\omega\tau}d\omega=4\pi t_0 L_0^\omega tri(\tau/t_0), \tag{10}$$

where $tri(x)$ is normalized symmetric triangle function.
Then substituting (10) into (7) we obtain

$$R_0=R(\tau=0)=4\pi t_0 L_0^\omega, \tag{11}$$

$$\tau_{coh}=\int_0^\infty tri(\tau/t_0)d\tau=\frac{t_0}{2}. \tag{12}$$

Taking into account (6) and (9), the velocity of "phase drift" σ^2 can be expressed as

$$\sigma^2=\frac{1}{2\pi}\int_0^\infty L(\omega)d\omega=\frac{L_0^\omega t_0}{\pi}\int_0^\infty \sin c^2(x)dx=L_0^\omega\frac{t_0}{2}, \tag{13}$$

and for absolute "phase drift" φ_{drift} we find from (12)

$$\varphi_{drift}=\sigma^2\tau_{coh}=\frac{L_0^\omega t_0{}^2}{4}. \tag{14}$$

3. Conclusion

The criterion at which the sufficient "phase drift" results to coherency breaking in the system [5], can be expressed as

$$\varphi_{drift}\sim\pi, \tag{15}$$

or for maximum "unbroken" coherent time delay finally we obtain

$$t_0\sim\sqrt{\frac{4\pi}{L_0^\omega}}. \tag{16}$$

Hence, the coherent time delay inversely depends on the square root of power spectrum density of generator's frequency noise.

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