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THz Partially Coherent Undulator Radiation from the Truncated Electron Bunch

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Abstract. The problem of THz coherent undulator radiation from the electron bunch with asymmetric distribution in the longitudinal direction is considered. It is shown that the contribution of coherent part of the total radiation for waves with wavelength much shorter than the characteristic of the longitudinal dimension of the bunch is predominant. It is another way of increasing the efficiency of a free electron laser (FEL).

Keywords: coherent radiation, channeling, nanotube undulator, truncated electron bunch.

1. Introduction

The problem of the finding the new sources of intense and directional radiation in different wavelength regions is urgent, has a great scientific interest, important practical application in biology, chemistry, medical etc. In the paper [1] was propose to use the electron bunch with asymmetric longitudinal distribution in the FEL process. The idea of the stimulated radiation of electron bunch or so-called free electron laser, belong to Madey [2]. The photon beam with 1.5Å wavelength was generated at SLAC using $13.6GeV$ electron bunch and operating on self-amplified spontaneous emission (SASE). The proposal [1] was based on the results of the theoretical work [3], where the radiation has been investigated of any nature from the electron bunch of an arbitrary structure.It was shown that the radiation from the bunch with asymmetric longitudinal distribution of electrons is partially coherent in shorter wavelengths region. The coherence effect of the diffraction radiation was observed in the Japanese experiment [4]. The radiation from an electron bunch with longitudinal asymmetric density distribution in flight over a periodic structure of an arbitrary profile has been considered in [5]. It was shown that the effect of coherence observed in an accelerator experiment [4] may be due to slight asymmetry of the bunch. An analogues problem has been considered in [6] for the radiation from an electron bunch flying over a surface acoustic wave.

The present paper is devoted to the undulator radiation from a truncated electron bunch in the sub millimeter wavelength region.

In 1947 V.L. Ginzburg suggested to use the relativistic electrons in devices with spatially periodic electromagnetic field to generate a micro-radio wave radiation [7]. The first experiments on the radiation, which is formed in the external magnetic periodic field and is called an undulator, were held Motz [8]. The interest in undulator radiation is resumed, when Korkhmazyan proposed to use the undulator to obtain the X-radiation and gamma radiation [9]. In the paper [10] the radiation produced in gas filled undulator both in the optical and in the X-ray regions have been investigated.

2. The total radiation emitted by an electron bunch

The total radiation radiated from a bunch with N electrons per unit a frequency interval per unit a solid angle in a time interval is [11,12]

$$\frac{d^2I}{dod\omega} = \frac{e^2\omega^2\sqrt{\varepsilon}}{4\pi^2c^3}\left|\sum_{j=1}^{N}\int_0^T\left[\vec{n}\times\vec{v}(t)\right]\exp\left[i(\omega t-\vec{k}\vec{r}_j(t))\right]dt\right|^2, \tag{1}$$
$$\vec{r}_j(t) = \vec{r}(t)+\vec{r}_j,$$

where e is the electron charge, ε is the dielectric permittivity, c is the velocity of the light, ω is the radiation frequency, $\vec{k} = \left(\omega\vec{n}\sqrt{\varepsilon}\right)/c$ is the wave vector, $\vec{n}$ is the unit vector in direction of radiation, $\vec{r}(t)$ is the motion law of electrons, $\vec{v}(t)$ its velocity, $\vec{r}_j$ is the distance from origin of the coordinate system to be inside the bunch $do = \sin\vartheta d\vartheta d\varphi$ is the differential of solid angle, ϑ and φ are polar and azimuthal angles of radiation .

It is assumed that the coordinates of anyone electron of bunch are independent random variables. After averaging the quantity (1) over the positions of electrons and making a summation, we obtain (in a analogy to [3])

$$\frac{d^2I}{dod\omega} = \langle\frac{d^2I_N}{dod\omega}\rangle = N\frac{d^2I_1}{dod\omega}\left[G+1-H\right], G = N\cdot H \tag{2}$$

where the second factor is the angular-frequency distribution for a single electron.

Here the coherence factor G is product the number of electrons N and the form-factor H of a bunch, which is presented via the longitudinal F and the transverse Φ form-factors

$$F = \left|\left\langle\exp(-ik_z z)\right\rangle\right|^2, \phi = \left|\left\langle\exp\left(-ik_\perp\rho\right)\right\rangle\right|^2 \tag{3}$$

where $k_\perp$ and ρ are the transverse components of wave vector and coordinate.

If a bunch form-factor $H = 1$, the radiation absolute coherent. When a bunch form-factor $H \gg 1$, but the coherence factor $G \gg 1$, the radiation is partially coherent, because the coherent component of radiation is predominant.

The exponential functions in (3) are averaged by the statistical corresponding distribution functions of electrons in bunch. Then the form-factor is obtaining in the general case.

3. The case of asymmetric Gaussian distribution in the longitudinal direction

For the relativistic electrons the radiation angle ϑ is very small, being of the order of the ratio of the rest energy of the electron to its total energy γ^{-1}.

Let us assume that the electrons of bunch are normally distributed in z and transverse directions with dispersions σ_z^2 and σ_ρ^2. Then for the form-factors we have

$$F(\omega) = \exp\left[-\left(\frac{2\pi\sigma_z\sqrt{\varepsilon}}{\lambda}\right)^2\right], \phi(\omega,\vartheta) = \exp\left[-\left(\frac{2\pi\sigma_\rho\vartheta\sqrt{\varepsilon}}{\lambda}\right)^2\right], \tag{4}$$

where λ is the wavelength of radiation. In the case, when λ is satisfied the condition,

$$\frac{2\pi\sigma_\rho\sqrt{\varepsilon}}{\gamma} \ll \lambda \ll 2\pi\sigma_z\sqrt{\varepsilon}, \tag{5}$$

the coherence factor G exponentially increases through the longitudinal form-factor.

4. The case of Gaussian longitudinal asymmetric distribution

Let us assume that the electron distribution function in the z direction is the asymmetric Gaussian function:

$$f(z) = \frac{2}{\sqrt{\pi}}\frac{1}{(1+p)\sigma_z}\left[\exp\left(-\frac{z^2}{2p^2\sigma_z^2}\right)\eta(-z) - \exp\left(-\frac{z^2}{2\sigma_z^2}\right)\eta(z)\right], \tag{6}$$

where the parameter $p(0 < p < 1)$ determines the degree of bunch asymmetry, $\eta(z)$ is the unit step function.

In the case, when $\xi \gg 1(\xi = (2\pi\sigma_z)/\lambda),$ for an asymmetric bunch, the dependence of coherence factor (gain) G on ξ is power-law function. The asymptotic expressions for the function have the form

$$\begin{aligned} G(\xi) &= \frac{2N}{\pi\xi^2}, \ p\cdot\xi \ll 1, \\ G(\xi) &= \frac{4N}{\pi\xi^4}, \ p\cdot\xi = 1, \\ G(\xi) &= \frac{2N}{\pi\xi^2}\frac{1}{(p\cdot\xi)^4}, \ p\cdot\xi \gg 1. \end{aligned} \tag{7}$$

Consequently, the greater the asymmetry of the truncated bunch and the smaller its longitudinal dimension, the greater the spontaneous emission amplification.

5. The influence on the efficiency of FEL

The motion of an electron happing with a velocity $\langle v_z \rangle$ along the axis of undulator and is accompanied by transverse oscillation with the frequency $\Omega = (2\pi\langle v_z \rangle)/l$, where l is the period of magnetic structure. The transverse oscillation of electron lead to longitudinal oscillation because the electron energy

$$E = mc^2 \left[1 - \frac{v_z^2(t) + v_\perp^2(t)}{c^2}\right]^{-\frac{1}{2}} = mc^2\gamma \tag{8}$$

is maintained in the magnetic field. Here m is the electron mass, $v_z(t)$ and $v_\perp(t)$ are the longitudinal and transverse velocities of electron.

The radiation of a ultra-relativistic electron ($\gamma \gg 1$), when $\langle v_\perp^2 \rangle / c^2 \ll 1$, occurs mainly at small angles $\vartheta \le 1/\gamma$ to the undulator axis. Then the average $\langle v_z^2 \rangle$ can be expressed as

$$\frac{\langle v_z^2 \rangle}{c^2} = 1 - \frac{1 + \mathrm{K}^2/2}{\gamma^2}, \tag{9}$$

where the undulator parameter K in the periodic magnetic field with amplitude B and space period l is equal to

$$\mathrm{K} = \frac{eBl}{2\pi mc^2}. \tag{10}$$

The radiation frequency ω at the first harmonic, which observed at the angle ϑ is determined by the Doppler-effect

$$\omega = \frac{2\Omega\gamma^2}{1 + \frac{1}{2}\mathrm{K}^2}. \tag{11}$$

It is proposed to use truncated electron bunch in FEL process $(\vartheta = 0)$. The electron bunch will be truncated if, for example, in a moment of reflection of a half bunch at the smooth metal surface to spoil thus reflection property of the laser action. The radiation wavelength at zero angle formed in an undulator with the parameter $\mathrm{K} = 1.17$ and $l = 2.79$ is equal to

$$\lambda = \frac{l}{2\gamma^2}\left(1 + \frac{1}{2}\mathrm{K}^2\right) = \frac{2.3}{\gamma^2}. \tag{12}$$

The below given numerical results of the gain have been calculated with the help of for the following bunch parameters of LCLS and undulator.

The electron bunch parameters are following: $N = 1.56 \cdot 10^3, \sigma_z = 9\mu m, \sigma_\rho = 6.12\mu m, E$ for energies of $5, 20, 50 MeV$.

The undulator parameters are: $l = 2.73cm, \mathrm{K} = 1.17$.

When the energy of bunch is $5MeV(\gamma = 9.885)$ the radiation occurs in the sub-millimeter region $\lambda = 0.24mm(1.25THz)$, then the parameter $\xi = 0.24$. In this case the radiation absolutely coherent.

If the bunch energy is equal to $20MeV(\gamma = 39)$, the spontaneous radiation at zero angle is formed on the wavelength $\lambda = 15\mu m(20THz)$, therefore $\xi = 3.77$ and, if $p = 0.265(p \cdot \xi = 1)$, then $G \approx 9 \cdot 10^7$. The radiation is partially coherent.

For the electron bunch energy $E = 50MeV(\gamma = 97.8)$, we have the wavelength $\lambda = 2.4\mu m(125THz)$ and the parameter $\xi = 23.5$. When $\xi = 0.2(p \cdot \xi = 5)$, the radiation is partially coherent with $G \approx 2 \cdot 10^5$.

Thus, if it is technically possible to obtain a sufficiently asymmetric bunch it is possible to essentially increase the efficiency of FEL.

6. Conclusion

In this paper we propose to use a truncated electron bunch in the process of FEL for the purpose of increasing the efficiency of FEL. It is shown that the undulator radiation of a truncated electron bunch with energy $50MeV$ is partially coherent in THZ region.

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