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Two-Body Resonances in the Complex Scaled Orthogonal Condition Model

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Abstract. In this work we investigate not only low-lying excited states but also higher excited states of ^{8}Be by using the complex scaled orthogonal condition model (CSOCM). The low-lying 0^{+}, 2^{+}, 4^{+} states of ^{8}Be are measured well but the higher excited states 6^{+}, 8^{+} and 10^{+} of ^{8}Be are not available by experimentally and these higher excited states have been barely studied by theoretical approaches.

Keywords: complex scaling method, orthogonal condition model

1. Introduction

Different methods have been studied by using various approaches in order to describe resonance states accurately. Study of resonances for the scattering problems in the light nuclei has been carried out using various methods, one of which is the complex scaling method (CSM) [1-3]. The theory of the complex scaling was proposed mathematically [2] and it has been extensively applied to the atomic and nuclear physics [4-6]. In the CSM, resonant states of the many-body systems are described using the appropriate L^2 basis functions. The resonance wave functions are obtained as eigenstates together with bound states by carrying out the diagonalization of the complex scaled Hamiltonian.

In this study, we adopt the complex scaled orthogonal condition model (CSOCM) [7] to two-body alpha-alpha system. The orthogonal condition model (OCM) introduced by S. Saito [8] has the main advantage in its appropriate description of the threshold energy independently on effective nuclear interaction. From the viewpoint of a microscopic description of the relative nucleon motion between clusters, it is important to take into account the Pauli exclusion principle in the inter-cluster motion.

In this work we investigate the spectra of ^{8}Be, understanding as an alpha-alpha two-body system. In particular, we focus on the its higher excited 6^{+}, 8^{+} and 10^{+} states because there is no experimental evidence for those higher states. But its low-lying 0^{+}, 2^{+}, and 4^{+} states are experimentally well known. In addition, in this work we apply two different bases states (Gaussian basis function and harmonic oscillator wave function) in order to calculate both low-lying and higher excited states of ^{8}Be nuclei. Furthermore, two different potentials are used for the alpha-alpha interaction: The Buck potential [9] and the Schmid-Wildermuth potential [10]. The difference between both potentials is in the treatment of the Pauli-principle. The Schmid-Wildermuth potential includes Pauli-forbidden states but in the case of the Buck potential Pauli-forbidden states are automatically excluded.

2. Complex scaled orthogonal condition model

2.1. Complex Scaling Method

The CSM has been proposed to take into account the resonance states. In the CSM, the relative coordinate is rotated as like $\vec{r} \rightarrow \vec{r}\, e^{i\theta}$ in the complex coordinate plane. Therefore, the *Schrödinger* equation

$$\hat{H}|\psi\rangle = E|\psi\rangle$$

is rewritten as

$$\hat{H}(\theta)\left|\psi^{\theta}\right\rangle = E^{\theta}\left|\psi^{\theta}\right\rangle, \tag{1}$$

where $\hat{H}(\theta)$ and ψ^{θ} are the complex scaled Hamiltonian and wave function, respectively. The θ is scaling angle being a real number, the $U(\theta)$ operates on a function ψ, that is,

$$\psi(\theta) = U(\theta)\psi(r) = e^{\frac{3}{2}i\theta}\psi(re^{i\theta}). \tag{2}$$

The eigenvalues and eigenstates are obtained by solving the complex scaled *Schrödinger* equation Eq.(1). The eigenvalues of resonance states are found as $E^{\theta} = E_r - i\Gamma_r/2$, where E_r and Γ_r are the energy and decay width of a resonance, respectively. If we do not apply the complex scaling, the original *Schrödinger* equation gives the continuum spectra including resonances on the positive energy axis.

In the CSM, the energy eigenvalues of unbound states are classified into the resonance and continuum states. After applying complex scaling, the resonances separate from the continuum states and the continuum states are obtained on branch cuts rotated down by 2θ as shown in Fig. 1. The branch cuts start from the different thresholds for continuum states. More detailed explanation of the CSM is given in Ref. [4].

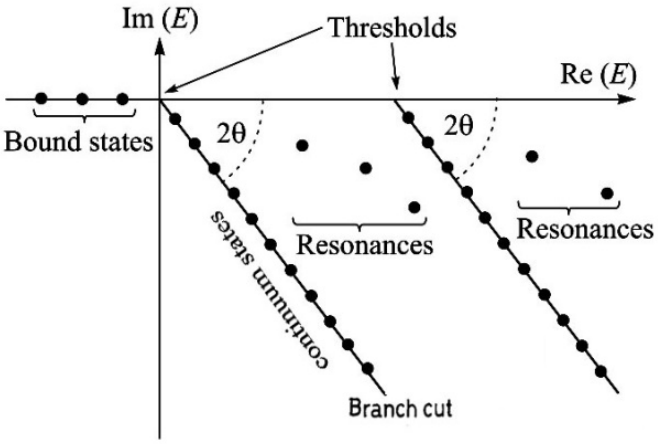

Fig. 1. Schematic energy eigenvalue distribution for a complex scaled Hamiltonian.

2.2. Orthogonal condition model

The OCM is one of the well approximated approaches of the resonating group method (RGM) which was introduced in 1968 by S.Saito and was successfully applied to the two-cluster system. Furthermore, the application of the OCM is not limited just to two-cluster problems but also has been successfully expanded into multi-cluster problems [4]. The orthogonal condition is given in addition to the complex scaled *Schrödinger* equation, that is,

$$\left\langle \chi_F^\theta \middle| \psi^\theta \right\rangle = 0, \tag{3}$$

where χ_F^θ is the Pauli forbidden state and expressed by $\chi_F^\theta = U(\theta)\chi_F$.

The Pauli forbidden states of the α+α system are defined by the following harmonic oscillator wave functions $\phi_{n\ell}(r)$:

$$\chi_F = \phi_{n\ell}(r) \text{ where } \begin{cases} n = 0,1 \ (\ell = 0) \\ n = 1 \quad (\ell = 2) \\ n = all \ (\ell = odd) \end{cases}.$$

The wave function ψ^θ defined as

$$\psi^\theta = \sum_{n=0}^{N} c_n(\theta)\phi_{n\ell}(r). \tag{4}$$

In this work the Gaussian function and harmonic oscillator wave function are used for basis expansion.

3. Alpha-Alpha System

3.1. Two-body model

For the alpha-alpha two-body system the Hamiltonian is expressed as

$$\hat{H} = -\frac{\hbar^2}{2\mu}\nabla^2 + V_{\alpha\alpha}^{Nucl} + V_{\alpha\alpha}^{Coul} + V_{\alpha\alpha}^{PF}, \tag{5}$$

where μ is a reduced mass. The interaction between two alpha particles $V_{\alpha\alpha}^{Nucl}$ we employ the folding potential of the effective nucleon-nucleon interaction the Schmid-Wildermuth potential [10] and the Buck potential [9], $V_{\alpha\alpha}^{Coul}$ is the Coulomb potential and $V_{\alpha\alpha}^{PF}$ is the pseudopotential which is the projection operator to remove the Pauli-forbidden states from the relative motion of alpha-alpha.

In this study, we use two different bases sets as follows:

(i) the Gaussian basis for the radial part which is given as

$$\phi_\ell^i(r) = N_\ell^i r^\ell \exp\left(-\frac{1}{2b_i^2} r^2\right) Y_{\ell m}(r). \tag{6}$$

Here $i = 0,1,2,\ldots,$ and N_ℓ^i is normalization constants expressed as:

$$N_\ell^i = \frac{1}{b_i^{\ell+3/2}} \left\{ \frac{2^\ell}{(2\ell+1)!!\sqrt{\pi}} \right\}^{1/2}, \tag{7}$$

and b_i is the size parameter of the Gaussian function described as $b_i = b_0 \gamma^{i-1}$, where b_0 and γ are the first term and a common ratio in the geometric progression, respectively.
(ii) Harmonic oscillator wave function for radial part given as

$$\phi_{n\ell}(r) = N_\ell^n \left(\frac{r}{b_F}\right)^\ell L_n^{\ell+\frac{1}{2}}\left(\left(\frac{r}{b_F}\right)^2\right) \exp\left(-\frac{1}{2b_F^2} r^2\right) Y_{\ell m}(r), \tag{8}$$

where $L_n^{\ell+1/2}$ are Laguerre polynomials for the angular momentum ℓ and N_ℓ^n denotes the normalization constants as given by

$$N_\ell^n = \left\{ \frac{2\Gamma(n+1)}{b_F^3 \Gamma\left(\ell+n+\frac{3}{2}\right)} \right\}^{1/2}. \tag{9}$$

The size parameter of relative motion of two alpha-clusters b_F is taken as 0.967 fm which corresponds to a single particle size parameter $b_0 = 1.3975$ fm employed to fit the observed r.m.s. radius of ^{4}He.
In this work, we apply the pseudo potential $V_{\alpha\alpha}^{PF}(r) = \lambda |\chi_F\rangle\langle\chi_F|$, where the strength λ is chosen as 10^7 MeV, which is enough to push up the Pauli-forbidden states into the unphysical energy region.

3.2. Alpha-alpha potentials

The Schmid-Wildermuth potential

The α+α potential is constructed by the folding approach for the effective nucleon-nucleon interaction by the Schmid-Wildermuth potential. An effective two-nucleon force is written as,

$$v_{ij} = V\{W + BP_\sigma(ij) - HP_\tau(ij) - MP_\sigma(ij)P_\tau(ij)\} \cdot \exp\left(-\mu r^2\right) \tag{10}$$

where $P_\sigma(ij)$ and $P_\tau(ij)$ are the spin and isospin exchange operators. In this work we employ the Schmid-Wildermuth potential as a nucleon-nucleon force, which is given by following parameters:

$$V = -72.98 \text{ MeV}; \ \mu = 0.46 \text{ fm}^{-2};$$

$$W = M = 0.4075; \ B = H = 0.0925. \tag{11}$$

The folding potential of the alpha-alpha system is obtained from such a nucleon-nucleon force and also the Coulomb force. Its explicit form is

$$V_{\alpha\alpha}^{Nucl}+V_{\alpha\alpha}^{Coul}=2X_D\left[\frac{2\nu_\alpha}{2\nu_\alpha+\frac{3\mu}{2}}\right]^{\frac{3}{2}}V\exp\left(-\frac{\nu_\alpha\mu}{\nu_\alpha+\frac{3\mu}{4}}r^2\right)+\frac{4e^2}{r}erf\left(r\sqrt{\frac{4}{3}\nu_\alpha}\right), \qquad (12)$$

where $X_D=2.445$ and $erf(x)$ is the error function. We use a harmonic oscillator constant $\nu_\alpha=\frac{M\omega}{2\hbar}=0.2675$ fm^{-2} which is obtained by using $r_{rms}=1.63$ fm of the alpha-cluster.

In Eq. (12) the simplified notations can be applied:

$$V_0=2X_DV\left[\frac{2\nu_\alpha}{2\nu_\alpha+\frac{3\mu}{2}}\right]^{\frac{3}{2}},$$

$$\beta=\frac{\nu_\alpha\mu}{\nu_\alpha+\frac{3\mu}{4}},$$

$$\alpha=\sqrt{\frac{4}{3}\nu_\alpha}.$$

The Buck potential

The Buck potential introduced in Ref. [9] which is independent on energy and angular momentum and the Pauli-forbidden states are automatically excluded. The parameters for the Buck potential are given as following:

$$V_0=122.6225 \text{ MeV}; \quad \beta=0.22 \text{ fm}^{-2}; \quad \alpha=0.75 \text{ fm}^{-1}.$$

4. Numerical Results

In numerical calculations of few-body cluster systems, we often use a vibrational method with appropriate bases, such as Gaussian basis function and harmonic oscillator wave function. For two-body cluster systems, the Gaussian expansion method was shown to be powerful with various types of matrix elements calculated with high precision [4-6]. In the complex scaling method, calculations with high precision are needed to maintain the stability of the complex eigenvalue problems.

In this study, we have used two different bases sets: (i) the Gaussian basis function and (ii) the harmonic oscillator wave function to solve the complex scaled *Schrödinger* equation for alpha-alpha two cluster system. The Buck-potential [9] and the Schmid-Wildermuth potential [10] are applied for the alpha-alpha interaction. In the case of the Buck-potential, all bound solutions are the unphysical Pauli-forbidden states because this system does not have any bound states. But in the case of the Schmid-Wildermuth potential, we have to take into account the Pauli-forbidden states correctly.

In Table I we give not only calculated results of the energies with decay widths for the 0^+, 2^+, 4^+, 6^+, 8^+ and 10^+ states of ^{8}Be, but also experimental data and two different potential parameters. The experimental data are taken from Ref. [11]. The resonance energies with decay widths are

calculated by using the complex scaling method, which permits to extract resonances as poles on the complex energy plane.

As seen in the table, two different bases functions used in the Schmid-Wildermuth potential give very similar spectra, not only for the energies, but also for the corresponding decay widths for all states. The calculated values for the low-lying 2^+ and 4^+states show that reasonably good agreement with measured data. The computed decay widths for the experimentally unknown 6^+, 8^+ and 10^+ higher states are rather big, however, they can be recognized as resonances on the complex energy plane. In all states the calculated values of the energies and the decay widths are agree very nicely with the same potential when we apply two different bases functions (the Gaussian basis function and the harmonic oscillator wave function).

Table I. Two different potential parameters, experimental and calculated resonance energies with corresponding decay widths of ^{8}Be.

States	**EXP**		**Gaussian Basis function**				**Harmonic oscillator wave function**	
	[11]		***Present work***					
	E_r(MeV)	Γ_r(MeV)	E_r(MeV)	Γ_r(MeV)	E_r(MeV)	Γ_r(MeV)	E_r(MeV)	Γ_r(MeV)
0^+	9.18×10^{-2}	5.57×10^{-6}	9.13×10^{-2}	$\sim10^{-6}$	6.41×10^{-1}	3.8×10^{-5}	6.06×10^{-1}	3.0×10^{-5}
2^+	2.94	1.51	2.75	1.24	3.01	1.2	2.90	1.4
4^+	11.35	~3.5	11.78	3.65	11.75	4.4	11.7	4.4
6^+	-	-	33.4	37.2	30.5	35.7	30.5	36.8
8^+	-	-	51.5	92.4	51.6	120	51.6	120
10^+	-	-	70.7	160	70.0	180	70.0	180
Potential parameters			Buck potential [9]		Schmid-Wildermuth potential [10]			
V_0	-	-	122.6225		106.09			
β (fm^{-2})	-	-	0.22		0.2009			
α (fm^{-1})	-	-	0.75		0.5972			

As we can see from the table, two different potentials (the Buck and the Schmid-Wildermuth) are applied to the Gaussian basis functions and one resonant pole in every partial wave of $\ell = 0, 2, 4, 6, 8$ and 10 is obtained. In the case of $\ell = 0$, there is a slight difference in the calculated energy and decay widths for two different potentials. In order to clarify a reason of this a slight difference, it may necessary to check the convergence of solutions and increase the number of employed basis functions.

In addition, the both potentials in the two different bases functions predict resonance energies with decay widths at the 6^+, $8^+$and 10^+ states, but there is no experimental evidence to support those calculated results, however, the energies and decay widths of the 6^+ and 8^+states are comparable with other computed recent results [12].

For the case of $\ell = 10$, there is no measured data and also there is no other theoretical prediction, however, we calculated a resonance energy with a broad decay width by using two different potentials and two different basis functions. The calculated results by both potentials and both bases functions are very similar each other and we can see from table this state is clearly obtained.

5. Discussion and Summary

Positions and widths of low-lying and higher excited states of ^{8}Be are calculated by using the CSOCM and the two-body model. In the numerical calculation, we have used to different bases sets and two different potential parameters for alpha-alpha two-body interaction. Different basis sets and two potentials provide very similar results and both potentials predict the 6+, 8+ and 10+ higher excited states. The results of recent calculation indicate that reasonably good agreement with measured data for the low-lying excited states including ground state. The next higher levels are calculated and compared with recent other theoretical results and present calculated results are quite close with the previous theoretical prediction.

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