

A REPRESENTATION OF THE SOLUTION OF GREEN'S BOUNDARY PROBLEM

Vanik E. Mkrtchian

Institute for Physical Research, Armenian Academy of Sciences, Ashtarak 0203, Republic of Armenia

e-mail: vem@daad-alumni.de

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Abstract – A representation of the solution of Green's function boundary problem is found for a class of homogeneous boundary conditions.

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In number of problems of quantum field theory appears a necessity to find Green function of the fields obeying boundary conditions on the moving surfaces. Recently in [1] was developed Keldysh-Schwinger path integral formalism for the Casimir system. This approach provides an opportunity to find the Green functions of the electromagnetic field undergoing boundary conditions when one of the interfaces of the system moves uniformly and parallel to the other. Detailed consideration of the structure of this solution results to rather general consequences related to the Green's boundary problem in generally. In this communication we would like to present this observation.

We consider Green's function equation for a linear operator L_x in the configuration space domain X

$$L_x G(x, x') = \delta(x, x') \quad (1)$$

In (1) x is a combined variable containing discrete indices (spin) and continuous arguments (space-time variables). The right hand side of equation (1) δ is a product of Kronecker symbols of discrete indices and Dirac's functions of continuous variables.

Further on we suppose the Green function undergoes homogeneous boundary condition on the manifold \bar{X} immersed in configuration space domain X

$$M(\bar{x}, x) G(x, x') = 0; \quad (\bar{x} \in \bar{X}; x, x' \in X) \quad (2)$$

Where M is an integral operator and we adopt the Einstein convention: *repeated arguments are implicitly integrated (and summed for discrete variables) over.*

The solution of the boundary problem (1), (2) is given by a sum

$$G(x, x') = G_0(x, x') - \bar{G}(x, x') \quad (3)$$

Where G_0 is an arbitrary solution of (1) and \bar{G} is a solution of the homogeneous equation

$$L_x \bar{G}(x, x') = 0 \quad (4)$$

selected in such way that resulting G in (3) undergoes to boundary condition (2).

Generalizing the results of [1] we come to the following expression for the boundary term \bar{G}

$$\bar{G}(x, x') = g_R(x, \bar{x}_1) g^{-1}(\bar{x}_1, \bar{x}'_1) g_L(\bar{x}'_1, x') \quad (5)$$

Where $g_{R,L}$ are defined as

$$g_R(x, \bar{x}) = G_0(x, x_1) M(\bar{x}, x_1) \quad (6.a)$$

$$g_L(\bar{x}, x) = M(\bar{x}, x_1) G_0(x_1, x) \quad (6.b)$$

In (5) g^{-1} is reciprocal operator of g defined on the manifold \bar{X}

$$g^{-1}(\bar{x}, \bar{x}_1) g(\bar{x}_1, \bar{x}') = g(\bar{x}, \bar{x}_1) g^{-1}(\bar{x}_1, \bar{x}') = \delta(\bar{x}, \bar{x}') \quad (7)$$

where g is defined as

$$g(\bar{x}, \bar{x}') = M(\bar{x}, x_1) G_0(x_1, x'_1) M(\bar{x}', x'_1) \quad (8)$$

Elementary calculation show that (5) satisfies to (4) and besides

$$M(\bar{x}, x_1) \bar{G}(x_1, x) = g_L(\bar{x}, x) \quad (9)$$

This equality proclaims satisfaction of the boundary condition (2) for the solution (3).

As we see the Green's boundary problem is drifted to searching of the reciprocal operator of (8) on the manifold \bar{X} where are defined the boundary conditions.

References

1. V E Mkrtchian and C Henkel, Ann. Phys. (Berlin) **526** (2014) 87 (Preprint quant-ph/1307.0682).