Armenian Journal of Physics, 2015, vol. 8, issue 1, pp. 7-16

MOMENTS OF INERTIA FOR EVEN-EVEN $^{120-124}$Te ISOTOPES

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Received 6 November 2014

Abstract - The interacting boson model (IBM-1) has been used to perform the moment of inertia of the nuclear structure of yrast bands of even-even $^{120-124}$Te isotopes. These values are found to be in agreement with previous experimental data. Moreover, IBM-1 was applied to study the systematic moment of inertia as a function of the even neutron number of $^{120-124}$Te isotopes. For even neutrons *N*= 68 to 72 of Te isotopes, nature properties of backbending were indicated from the analysis of the moment of inertia as a function of the square of the rotational angular velocity.

***Keywords*:** Moment of inertia;Te isotopes; even-even nuclei; ground state band;

1. Introduction

The even-even tellurium isotopes Te (*Z*=52) are one of the most important in the nuclear science. In recent years, these isotopes were studied theoretically and experimentally [1,2.]. These studies were emphasized on the interpreting experimental data via different collective models [3]. In addition, many theoretically and experimentally studies showed that the low-lying collective quadrupole *E2* excitations occurred in the even-even nuclei with atomic number *Z*=52 [4]. Many models were used to investigate the electric quadrupole moments of even $^{120\text{-}128}$Te isotopes; such as the framework of the semi-microscopic model [5], the two-proton core coupling model [6], the dynamic deformation model [7] and the interacting boson model-2 [8-10].

The phenomenon in which a plot of twice the moments of inertia versus the square of rotational frequency, for the various spin states has an S-shaped form, is called the backbending [1]. This property was specifically discovered in the ground state rotational bands (GSRB) of even-even rare earth nuclei at high spins. The phenomenon was investigated in many even-even nuclei studies [12-14]. For many deformed nuclei, it was found that the rotational frequency decreases by the sudden change, while the moment of inertia increases by anomalous change. Mariscotti et al. [15] have proposed the variable moment of inertia (VMI) model, which is a popular model among the nuclear science community. This model defines the excitation energy of the state J by the sum of the rigid rotational energy and the potential energy term. It is used to fit the estimated energies with the measured energies.

The backbending was calculated by different models; such as angular momentum projected Tomm-Dancoff approximation [16], neutron-proton interaction as employed by the Bardeen-Cooper-Schrieffer (BCS) model [15], projected shell model [17], and projected configuration interaction (PCI) method. These calculations show that the deformed intrinsic states are directly proportional with the shell model wave function [18]. In previous studies [19,20], we have been studied evolution properties and backbending properties of yrast states for even-even $^{100\text{-}110}$Pd and $^{104\text{-}122}$Cd isotopes.

IBM-1 model has been successfully used to performed the electromagnetic reduced transition probabilities of even-even $^{104-112}$Cd [21], $^{102-106}$Pd [22], and $^{108-112}$Pd [23] isotopes. Also, a detailed study for the characterization of backbending in even even$^{120-130}$Te isotopes were previously performed [24].

To the best of our knowledge, there is no experimental or theoretical research about the moments of inertia of yrast states for even-even $^{120-124}$Te isotopes has been reported. We think that this work is worth to be studied by the IBM-1 model. We have investigated the analysis of the moment of inertia of even $^{120-124}$Te isotopes by the frame-work phenomenological IBM-1 model.

The paper is organized in four sections. Section two is devoted to the method of calculation; section three deals with the results and discussion and in section four we present the conclusion.

2. Theoretical calculation

Interacting Boson Model (IBM-1) of Arima and Iachello [1] has been widely accepted as a tractable theoretical scheme of correlating, describing and predicting low-energy collective properties of complex nuclei. The vibrational model uses a geometric approach, while the IBM employs a severely truncated model space. So that, the nuclei with N nucleons are possible to be calculated, and the experimental results can be compared withthe calculated values by providing a quantitative mechanism [11]. In the first approximation of IBM-1; only one pair with the angular momentum L = 0 (called S-bosons) and L = 2 (called d-bosons) are considered.

The Hamiltonian of the interacting bosons in IBM-1 is expressed as [14]

$$H = \sum_{i=1}^{N} \varepsilon_i + \sum_{i\langle j}^{N} V_{ij} \quad , \tag{1}$$

where ε is the intrinsic boson energy and V_{ij} is the interaction between bosons i and j.

The multi-pole form of the IBM-1 Hamiltonian [16] is given by the following equation:

$$H = \varepsilon \hat{n}_d + a_0(\hat{P}.\hat{P}) + a_1(\hat{L}.\hat{L}) + a_2(\hat{Q}.\hat{Q}) + a_3(\hat{T}_3.\hat{T}_3) + a_4(\hat{T}_4.\hat{T}_4), \tag{2}$$

where $\hat{n}_d = (d^+ . \tilde{d})$, $\hat{P} = \frac{1}{2}(\tilde{d} . \tilde{d}) - \frac{1}{2}(\tilde{s} . \tilde{s})$,

$$\hat{L} = \sqrt{10}\,[d^+ x\, \tilde{d}]^{(1)},$$

$$\hat{Q} = [d^+ x\, \tilde{s} + s^+ x \tilde{d}\,]^{(2)} - \frac{1}{2}\sqrt{7}\,[d^+ x\, \tilde{d}\,]^{(2)},$$

$$\hat{T}_3 = [d^+ x\, \tilde{d}]^{(3)}, \quad \hat{T}_4 = [d^+ x\, \tilde{d}]^{(4)},$$

$\hat{n}_d$ is the number of d boson, p is the pairing operator for the S and d bosons, J is the angular momentum operator, and Q is the quadrupole operator.T_3 and T_4 are representing the octupole and hexadecapole operators, respectively.

The IBM-1 Hamiltonian tends to be reduced to three limits; which are the vibration U(5), γ-soft O(6) and the rotational SU(3) nuclei. It will be starting with the unitary group U(6) and finishing with group O(2) [15]. The effective parameters are: ε,the pairing a_0 and the quadrupole a_0 for the U(5) limit, the γ-soft limit O(6) and the SU(3) limit, respectively.

The eigen-values for the three limits are given as follows [17]:

$$U(5):\quad E(n_d,\nu,L)=\varepsilon\, n_d+K_1\, n_d\,(n_d+4)+K_4\,\nu(\nu+3)+K_5\,L(L+1)\ ,\qquad (3)$$

$$O(6):E(\sigma,\tau,L)=K_3[N(N+4)-\sigma(\sigma+4)]+K_4\,\tau(\tau+3)+K_5\,L(L+1),\qquad (4)$$

$$SU(3):\ E(\lambda,\mu,L)=K_2\,(\lambda^2+\mu^2+3(\lambda+\mu)+\lambda\mu)+K_5\,L(L+1)\ ,\qquad (5)$$

where K_1, K_2, K_3, K_4 and K_5 are other forms of strength parameters.

2.1. *Moment of inertia (ϑ) and gamma energy E_γ*

The relation between the moment of inertia (ϑ) and the gamma energy Eγ is given by [19]

$$2\vartheta/\hbar^2=\frac{2(2I-1)}{E(I)-E(I-2)}=\frac{4I-2}{E_\gamma},\qquad (6)$$

while the relation between Eγ and ħω is given by [25]

$$\hbar\omega=\frac{E(I)-E(I-2)}{\sqrt{I(I+1)}-\sqrt{(I-2)(I-1)}}=\frac{E_\gamma}{\sqrt{I(I+1)}-\sqrt{(I-2)(I-1)}}\ .\qquad (7)$$

3. Results and discussions

The strength parameters for different levels in IBM-1 for even $^{120-124}$Te isotopes are given in Table 1. The transition level, the gamma ray energy, the moment of inertia, the square of rotational energy for the ground state band of even-even $^{120-124}$Te isotopes are presented in Table 2 [26-28].

Table 1. Strength parameters for different levels in IBM-1 for $^{120-124}$Te isotopes.

Nucl.	N	*States*	*Limits*	$\varepsilon\,(keV)$	$K_1\,(keV)$	$K_4\,(keV)$	$K_5\,(keV)$
^{120}Te	8	2-12	U(5)	495.018	9.414	-3.879	5.642
^{122}Te	7	2-12	U(5)	451.183	39.386	-24.092	2.058
^{124}Te	6	2-12	U(5)	514.511	52.332	-42.200	-.776

Table 2. Excitation energies, moment of inertia and square of rotational frequency for even $^{120-124}Te$ isotopes[26-28].

Nucl.	I	$I(I+1)$	$E_{exp}(I)$ keV	$E_{cal.}$ keV	$2\vartheta/\hbar^2$ Exp MeV^{-1}	$2\vartheta/\hbar^2$ Cal MeV^{-1}	$(\hbar\omega)^2$ Exp $(MeV)^2$	$(\hbar\omega)^2$ Cal $(MeV)^2$
^{120}Te	2	6	560.4	560.4	10.714	10.707	0.0784	0.0785
	4	20	1161.1	1177.1	23.294	22.701	0.0903	0.0951
	6	42	1775.7	1849.9	35.772	32.699	0.0946	0.1132
	8	72	2652.4	2579.0	34.246	41.147	0.1918	0.1329
	10	110	3543.4	3364.3	42.648	48.392	0.1984	0.1542
	12	156	4459.4	4205.9	50.218	54.658	0.2097	0.1777
^{122}Te	2	6	560.4	560.09	10.639	10.713	0.0795	0.0784
	4	20	1161.5	1175.2	22.690	22.760	0.0952	0.0946
	6	42	1776.1	1833.4	38.596	33.424	0.0812	0.1083
	8	72	2652.8	2538.6	41.394	42.541	0.2100	0.1243
	10	110	3273.8	3290.8	61.191	50.518	0.0964	0.1414
	12	156	3978.8	4090.5	65.248	57.522	0.1243	0.1599
	14	210	4888.8	4758.7	59.34		0.2070	
^{124}Te	2	6	602.7	602.7	9.967	9.956	0.0906	0.0908
	4	20	1248.6	1219.5	21.705	22.698	0.1040	0.0951
	6	42	1747.0	1850.4	36.756	34.871	0.2139	0.0995
	8	72	2664.4	2495.3	44.117	46.529	0.1156	0.1039
	10	110	3267.1	3154.4	86.363	57.654	0.0484	0.1086
	12	156		3827.5		68.341		0.1133

3.1. The moment of inertia

The positive parity yrast levels are connected by a sequence of the stretched E2 transition with energies. A very steep increase in energy occurs for certain *I* values and the even bending backwards curve (backbending). For constant rotor, the transition energy $\Delta E_{I,I-2}$ should be increase linearly with I as the relation ($\Delta E_{I,I-2} = \hbar^2(4I-2) / 2\vartheta$), but it is found to be decreasing for certain *I* values. The moment of inertia $2\vartheta / \hbar^2$ and rotational frequency $\hbar\omega$ are calculated from Eq. (1), (6) and (7). The ground state bands up to 12, 14 and 12 units of angular momentum are investigated for the moment of inertia in even $^{120-124}$Te nuclei respectively. The moments of inertia are plotted versus the even neutron number as can be seen in Fig.1. It is seen that calculated moment of inertia are agreeable to experimental data. One can conclude that $2\vartheta / \hbar^2$ as a function of neutrons do not change upto spin 4^+. Figure 2 presents $2\vartheta / \hbar^2$ as a function of the square of the rotational energy in even $^{120-124}$Te nuclei. A linear proportional occurs between $2\vartheta / \hbar^2$ and ω^2 in the lowest order according to VMI model, as shown in Fig. 2. The moment of inertia are rapidly increases at $8^+$and 6^+states for N=72 and N=70, respectively. The backbending phenomena appear clearly in Fig. 2.Table 2 presents the results of collective $\Delta I = 2$ ground band level sequence for the variation of shapes for Te isotopes with even neutron from N=68-72.

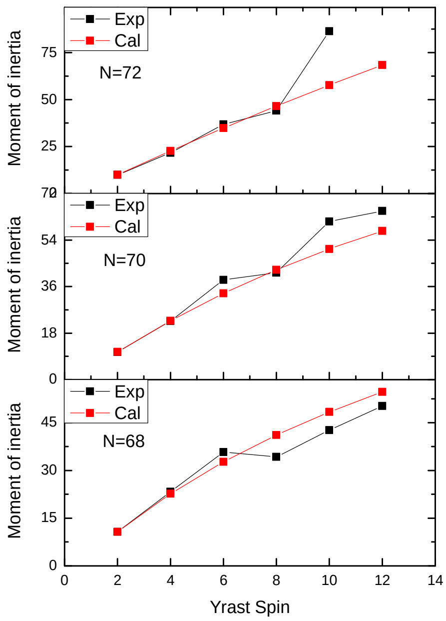

Fig. 1. Moments of inertia as a function of yrast spin for even $^{120\text{-}124}$Te isotopes.

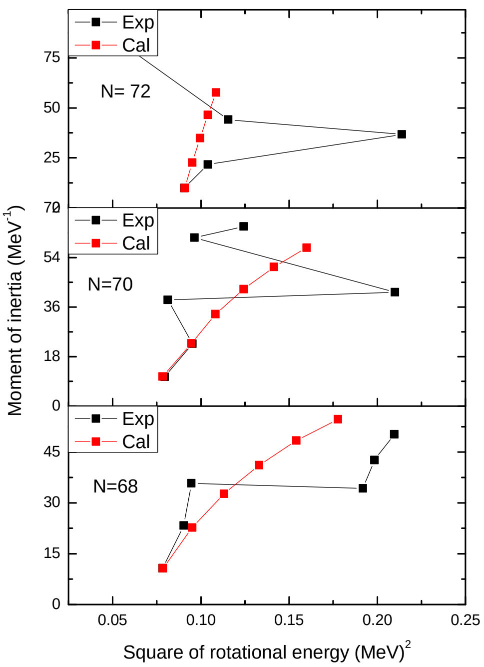

Fig. 2.Moments of inertia as a function of square of rotational energy for even $^{120\text{-}124}$Te isotopes.

4. Conclusions

The IBM-1 calculations have been used to investigate the moment of inertia for even-even $^{120-130}$Te isotopes. These results are comparable with the experimental resultsand extremely useful for compiling nuclear data table.

Acknowledgment

The authors thanks to king Abdul Aziz university to provide a research fund.

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