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LONGITUDINAL (C2) FORM FACTORS WITH CORE-POLARIZATION EFFECTS OF SOME FP-SHELL NUCLEI USING TASSIE MODEL

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Abstract – Inelastic electron scattering form factors to 2^+ states in $^{42,44,48}Ca$, $^{46,48,50}Ti$ and $^{50,52}Cr$ with Core-Polarization effects were studied using shell model calculations. Tassie Model was used to calculate the Core-Polarization effects.The GXFP1interactions in the proton-neutron formalism for the fp-shell ($1f_{7/2}$, $2p_{3/2}$, $0f_{7/2}$ and $2p_{1/2}$) are used in the present study. The calculations were performed with Nushell@MSU code for windows. The wave functions of radial single particle matrix elements have been calculated with Skyrme–Hartree Fock potential. The form factors calculation give good agreements with experimental data especially at the first and second maximumdiffraction value for scattered electrons. In our calculations core-Polarization effect were including by employed the effective charges e_p= 1.16e for protons and e_n= 0.7e for neutrons.

Keywords: shell model, (e,e) inelastic electron scattering, form factors, nuclear structure

1. Introduction

The electron scattering process can be explained according to the first Born-approximation as an exchange of a virtual photon carrying a momentum q between the electron and the nucleus. The first Born-approximation is being valid only if $\alpha Z \ll 1$, where Z is the atomic number and α is the fine structure constant. According to this approximation, the interaction of the electron with the charge distribution of the nucleus is considered as an exchange of a virtual photon with zero angular momentum along the direction of the momentum-transfer q. This is called Coulomb or longitudinal scattering. The inelastic electron scattering, which determines transition densities corresponding to the initial and final nuclear state in question, for the three quantities in the nucleus that interact with the passing electron, namely the distribution of charge, current and magnetization[1]. The first experiment on electron scattering of a magnetization distribution is due to Hofstadter and McAllister (1955)[2]. The first elastic electron scattering experiments from spin zero nuclei were done by Hofstadter (1956) [3]. In these experiments, the main conclusion is that the nuclear ground state charge distribution is well fitted by Fermi distribution.The fp-shell nuclei have special interest from the viewpoint of astrophysics, such as the electron capture rate in supernovae explosions, for this reason, a suitable effective interaction for fp-shell nuclei is required[4].

The standard FPD6 [5] two-body interactions are derived for nuclei in the lower part of the *0f1p* shell by fitting semi-empirical potential forms and two-body matrix elements to 61 binding and excitation energy data in the mass range 41 to 49. An excellent reproduction of ground-state magnetic moments and quadrupole moments is obtained with FPD6 interactions. The KB3G interaction is a new mass-dependent version [6]. The results by using the KB3 [7] which consider a monopole modification of the origin Kuo-Brown interaction [8] for ^{52}Fe, ^{50}Cr and ^{50}Mn have been published in refs.[9-11] and gave equivalent results for the KB3G interaction [6].

Recently, nuclear structures of ^{23}Na, ^{25}Mg, ^{27}Al and ^{41}Ca nuclei have been studied using shell model calculations[12]. A set of two-body interactions are used in this work. The universal *sd* of the Wildenthal interaction (USD) in the proton-neutron formalism, universal *sd*-shell interaction A (USDA), universal *sd* interaction B (USDB) and GXFP1 interaction for the fp-shell is used with the nucleon-nucleon (NN) realistic interaction M3Y as a two-body interaction for core polarization calculations. Very good agreements are obtained for all nuclei in this study. Results from electron scattering form factor calculations have shown that the core polarization (CP) effects are essential in obtaining a reasonable description of the data with no adjustable parameters. Electron scattering form factors with transition probabilities have been calculated [13] for different states in ^{10}B, ^{32}Sc and ^{48}Ca nuclei using nuclear shell model calculations. The results with CP effects inclusion modify the form factors markedly and describe the experimental values very well in the range of the momentum transverse (q) values dependence.

The purpose of this work is to calculate the Inelastic Longitudinal (C2) electron scattering form factors for 2_1^+ state of ^{42,44,48}Ca, ^{46,48,50}Ti, and ^{50,52}Cr nuclei using Nushell@MSU code with Tassie model to calculate CP effects [14]. The Hamiltonians GXFP1 interaction has been used to give the ($1f_{7/2}$ $2p_{5/2}1f_{5/2}$ *and* $2p_{1/2}$) shell model wave function. This interaction in the proton-neutron formalism. The single particle matrix elements have been calculated with Skyrme–Hartree Fock (SKX) potential [15].

2. Theory

The many particle reduced matrix elements consists of two parts, one is the model space (MS) matrix elements and one for CP matrix elements,

$$\left\langle J_f \left\| \hat{T}_J(q,t_z) \right\| J_i \right\rangle = \left\langle J_f \left\| \hat{T}_J^{MS}(q,t_z) \right\| J_i \right\rangle + \left\langle J_f \left\| \hat{T}_J^{core}(q,t_z) \right\| J_i \right\rangle \quad (1)$$

here the model space matrix elements are given by

$$\left\langle J_f \left\| \hat{T}_J^{ms}(q,t_z) \right\| J_i \right\rangle = \sum_{a,b} X^{J t_z}(i,f,a,b) \left\langle j_a \left\| \hat{T}_J(q,t_z) \right\| j_b \right\rangle \quad (2)$$

where $X^{Jt_z}(i,f,a,b)$ are the One Body Density Matrix elements.

The model space matrix elements can be written as [16]

$$\left\langle J_f \left\| \hat{T}_J^{ms}(q,t_z) \right\| J_i \right\rangle = e_i \int_0^\infty dr\, r^2 j_J(qr) \rho_{J,t_z}^{ms}(i,f,r) \quad (3)$$

where $\rho_{J,t_z}^{ms}(i,f,r)$ is the model space transition density, given by:

$$\rho_{J,t_z}^{ms}(i,f,r) = \sum_{a,b} X^{Jt_z}(i,f,a,b) \langle n_a l_a j_a \| Y_J(\Omega_r) \| n_b l_b j_b \rangle R_{n_a l_a}(r) R_{n_b l_b}(r) \quad (4)$$

The sum extends over the orbits of the model space.The core-polarization matrix elements can be written as

$$\left\langle J_f \left\| \hat{T}_J^{core}(q,t_z) \right\| J_i \right\rangle = \int_0^\infty dr\, r^2\, j_J(qr) \rho_{J,t_z}^{core}(i,f,r) \quad (5)$$

According to Tassie model [17], the core transition density is given by

$$\rho_{J,t_z}^{core}(i,f,r) = \frac{1}{2}(1+\tau_z) N r^{J-1} \frac{d}{dr} \rho_{o,\tau_z}(i,f,r) \quad (6)$$

where N is proportionality constant, and $\rho_{0,\tau_z}(i,f,r)$ is the ground state charge density distribution. The total transition density is given by

$$\rho_{J,t_z}(i,f,r) = e\rho_{J,t_z}^{ms}(i,f,r) + \rho_{J,t_z}^{core}(i,f,r) \quad (7)$$

The Coulomb form factor for this model becomes

$$F(q) = \sqrt{\frac{4\pi}{(2J_i+1)}} \frac{1}{Z} \left[\int_0^\infty r^2 j_J(qr) \rho_{J,t_z}^{ms}(i,f,r) dr + N \int_0^\infty dr\, r^2 j_J r^{J-1} \frac{d}{dr} \rho_o(i,f,r) \right] \times F_{f.s}(q)\, F_{c.m}(q) \quad (8)$$

The radial integral $\int_0^\infty dr\, r^{J+1}\, j_J(qr) \frac{d}{dr} \rho_o(i,f,r)$ can be written as

$$\int_0^\infty \frac{d}{dr}\left\{r^{J+1}\, j_J(qr)\rho_o(i,f,r)\right\}dr - \int_0^\infty dr (J+1)\, r^J j_J(qr)\rho_o(i,f,r)$$

$$-\int_0^\infty dr\; r^{J+1}\frac{d}{dr}\, j_J(qr)\rho_o(i,f,r) \tag{9}$$

The first term gives zero contribution, the second and the third terms can be combined into the integral:

$$-q\int_0^\infty dr\; r^{J+1}\rho_O(i,f,r)\left[\frac{d}{d(qr)}+\frac{J+1}{qr}\right] j_J(qr) \tag{10}$$

From the recursion relation of spherical Bessel function [16]

$$\left[\frac{d}{d(qr)}+\frac{J+1}{qr}\right] j_J(qr) = j_{J-1}(qr) \tag{11}$$

one can obtain the following integral:

$$-q\int_0^\infty dr\; r^{J+1} j_{J-1}(qr)\rho_O(i,f,r) \tag{12}$$

Hence, the form factor takes the form:

$$F_J(q) = \sqrt{\frac{4\pi}{(2J_i+1)}}\frac{1}{Z}\left\{\int_0^\infty r^2 j_J(qr)\rho_{J,t_z}^{ms}(i,f,r)dr\right.$$

$$\left. -N\, q\int_0^\infty dr\; r^{J+1}\rho_o(i,f,r)\, j_{J-1}(qr)\right\} F_{f.s}(q)F_{c.m}(q) \tag{13}$$

The model space transition charge density is calculated according to equation (4) where $\rho_O(r)$ is the ground state charge density

The form factor at the photon point $q = k$, is related to the reduced transition strength $B(CJ)$. So, the proportionality constant N can be determined from the form factor evaluated at $q = k$, and can be shown to be equal to

$$N = \frac{\int_0^\infty dr\, r^2\ j_J(qr)\rho^{ms}_{J,t_z}(i,f,r) - \frac{\sqrt{(2J_i+1)B(CJ)}}{(2J+1)!!}\ k^J}{k\int_0^\infty dr\, r^{J+1}\ \rho_o(i,f,r)\ j_{J-1}(kr)} \tag{14}$$

$$j_J(kr) = \frac{(kr)^J}{(2J+1)!!} \tag{15}$$

$$j_{J-1}(kr) = \frac{(kr)^{J-1}}{(2J-1)!!}$$

By putting equation (15) in equation (14), one obtains

$$N = \frac{1}{(2J+1)} \frac{\int_0^\infty dr\ r^{J+2}\ \rho^{ms}_{J,t_z}(i,f,r) - \sqrt{(2J_i+1)B(CJ)}}{\int_0^\infty dr\ r^{2J}\rho_O(i,f,r)} \tag{16}$$

,

with $(2J+1)!!=(2J+1)\,(2J-1)!!$

The experimental values of $B(CJ)$ are used to calculate the proportionality constant.

3. Results and discussion

The shell model calculations in the present paper are performed using NuShell@MSU code for Windows within *fp*-model space (*1f$_{7/2}$ 2p$_{3/2}$ 1f$_{5/2}$ 2p$_{1/2}$*) with the ^{40}Ca as the inert core. Inelastic electron scattering form factors have been calculated based on the GXFP1effective interactions. The radial wave function for the single-particle matrix elements have been calculated with the SKX potential. In our calculations Core-Polarization effect [17] was included by employing the effective charges e_p=1.16e for protons and e_n=0.7e for neutrons.The values of the experimental longitudinal C2 electron scattering form factor [18] are compared in Fig.1 with values from interaction GXFP1 and FPD6 in upper panel (a) and lower panel (b), respectively, for 2^+ states inthe ^{42}Ca nucleus. Two effective interactions give a good agreement for all momentum transfer values, but the calculations using the GXFP1 predicted the experimental data more than the FPd6 interaction in high momentum transfer (q >1.5 fm^{-1}).

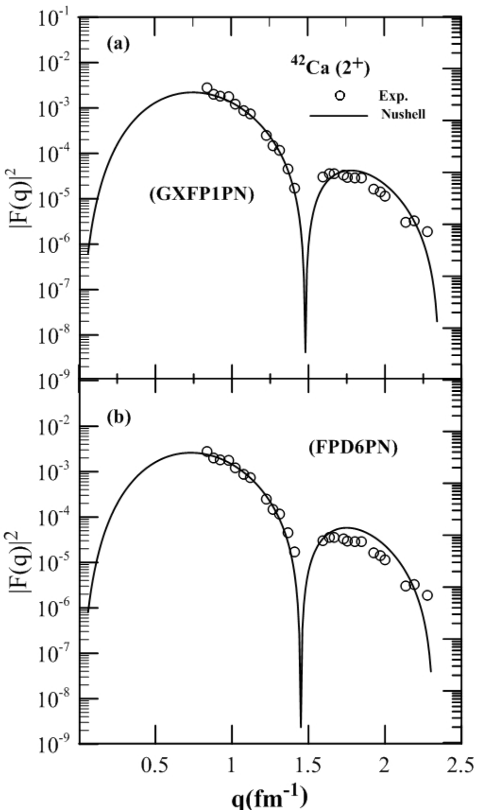

Fig.1. Comparison of the experimental inelastic C2 form factors $|F(q)|^2$with values from the interactions GXFP1 and FPD6 for 2^+ state in the^{42}Ca nuclei. The upper panel (a) represents that the calculated form factors with GX1FP effective interaction while the lower panel (b) represent that the calculated with FPD6 effective interaction. The experimental data are taken from ref.[18].

Fig.2 shows the longitudinalC2 form factors for the transition from the ground state (0_1^+) to 2_1^+states with theoretical excitation energy 1.247 MeV and 3.791 MeV for ^{44}Ca and ^{48}Ca nuclei, respectively. The calculations were performed using the GXPF1 effective interaction. The maximum diffraction value of ^{44}Ca and ^{48}Ca are the same value at q= 0.7fm^{-1}. The second maximum diffraction value for scattered electrons at the momentum transfer q=1.8fm^{-1} and 1.6 fm^{-1} for ^{44,48}Ca nuclei, respectively.The results are well described the experimental data [15] for the most of the momentum transfer, especially at the first and second maximum diffraction region.

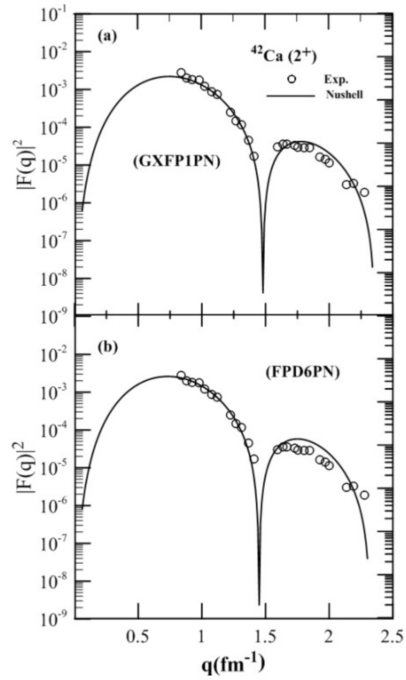

Fig.2. Inelastic longitudinal C2 form factors for the transition to the 2^+ state in ^{44}Ca and ^{48}Ca nuclei calculated with the GXFP1 effective interaction. The experimental data are taken from ref.[18].

The results of C2 form factors for the 2^+ statein the ^{46}Ti nucleus with the GXFP1effective interaction are shown in Fig. 3.

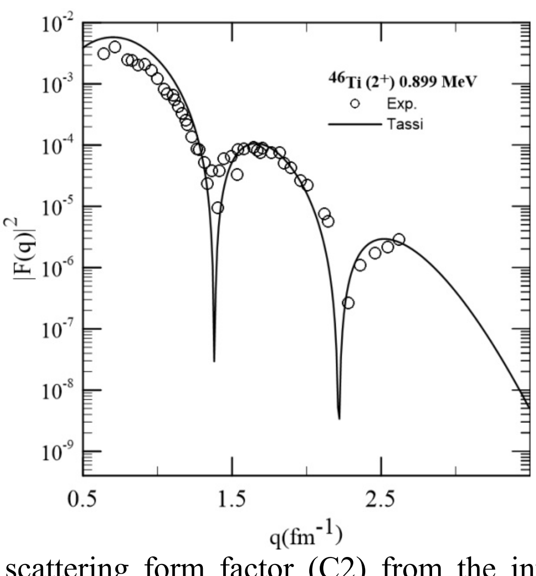

Fig.3. Inelastic electron scattering form factor (C2) from the interactions the GXFP1 for the transition to the 2^+ state in the ^{46}Tinuclei. The experimental data are taken from ref.[18].

The results with neutron effective charge at 0.7 give a very good agreement in first, second and third peaks comparing with experimental data [18]. The maximum diffraction value is at q= 0.7 fm^{-1}. The calculations with using GXFP1 give a good agreement with experimental data inthe entire momentum transfer region between(0.5 < q < 3) fm^{-1}. Fig.4 shows comparison between experimental and theoretical longitudinal (C2) form factorsfor ^{50}Ti nucleus. The first and second peaks are reasonably well reproduced comparing with the experimental data, while in the thirdpeak;the form factor results are deviated from the experimental data.

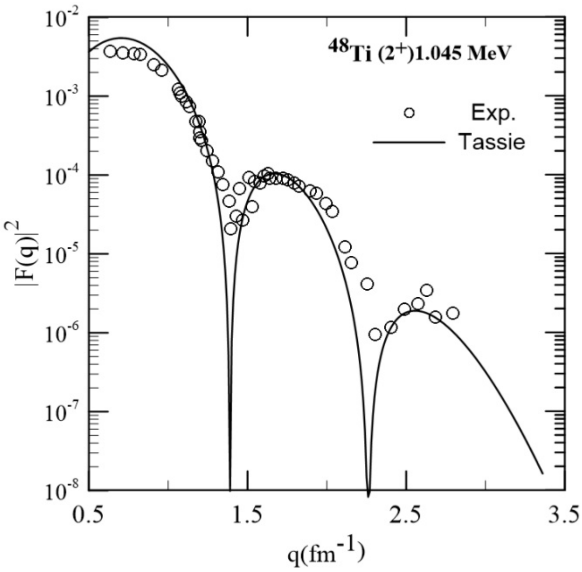

Fig.4. Inelastic electron scattering form factor (C2) from the interactions the GXFP1 for the transition to the 2^+ state in the ^{48}Ti nuclei. The experimental data are taken from ref. [18].

The theoretical longitudinal C2 form factor of $^{50,\ 52}$Cr are shown in Figs. 5 and 6, respectively. In those figures, the experimental data form factors are taken from Ref.[18]. One can see that the theoretical results for ^{50,52}Cr agree very well with the experimental data in the entire momentum transfers region. The magnitude of the maximum of inelastic longitudinal C2 form factor for ^{42,44,48}Ca nuclei as a function of neutron number (N) are compared in Fig.6 with values from the calculation with interactions GXFP1, KB3G and FPD6 as well as with the experimental data.

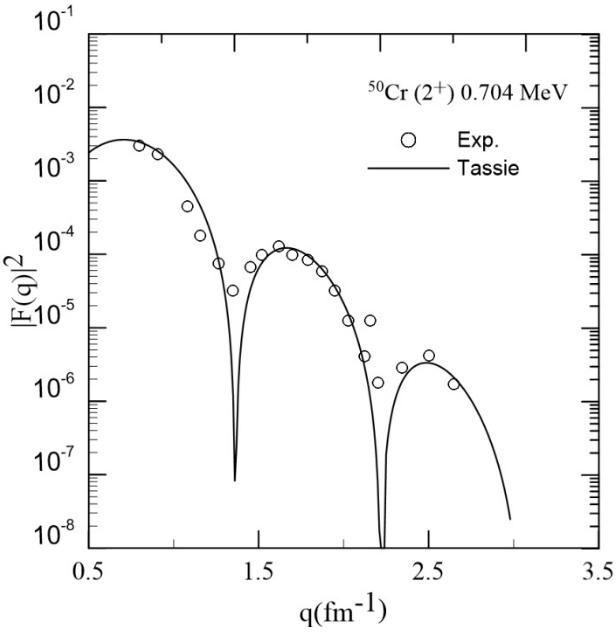

Fig.5. Inelastic electron scattering form factor (C2) from the interactions GXFP1for the transition to the 2^+ state in the ^{50}Cr nuclei.The experimental data are taken from ref.[18].

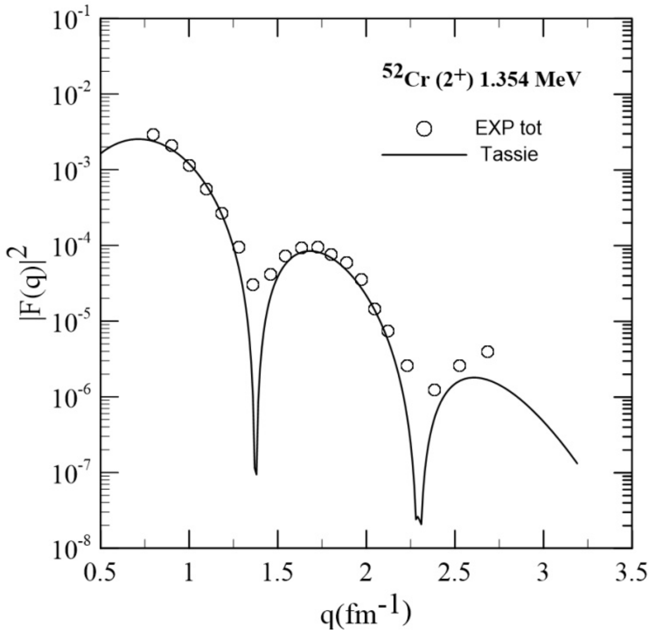

Fig.6. Inelastic electron scattering form factor (C2) from the interactions GXFP1for the transition to the 2^+ state in the ^{52}Cr nuclei. The experimental data are taken from ref. [18].

We notice that the results are reproduced well by the three interactions, especially for A=44. Fig.7 displays the maximum values of longitudinal form factor (C2) of 2^+ state for ^{46,48,50}Ti with three effective interactions. From this figure, we notice that the best maximum values are with the GXPF1and KB3 effective interactions for N=44 and 46 comparing with experimental data[18].

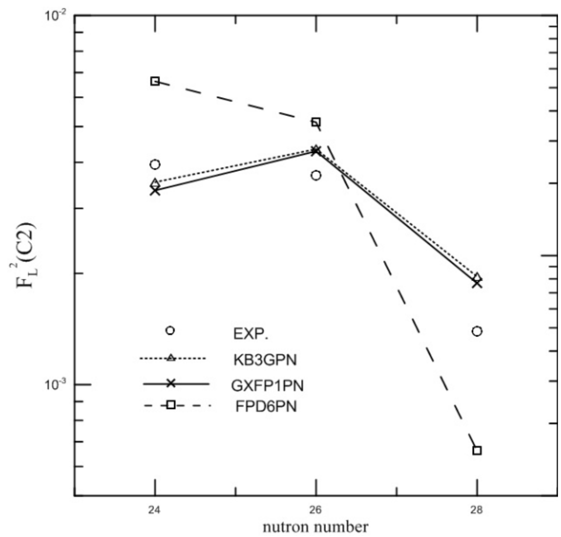

Fig.7. Comparison of the first maximum of the experimental [18] longitudinal C2 form Factors with values from the interactions GXFP1, KB3G and FPD6for the transition to the 2^+ state in ^{46,48,50}Ti nuclei.

The B(E2; $0^+ \rightarrow 2^+$) results using GXFP1 effective interaction are compared with experimental data in table 1. The results are calculated with effective chare e_p=1.16e and e_n=0.7e for protons and neutrons, respectively.The B(E2; $0^+ \rightarrow 2^+$) results for^{48}Ca,^{48,50}Tiand ^{50,52}Crnuclei are closer to the experimental data.

Table 1. The B(E2, $0^+ \rightarrow 2^+$) values for some Ca, Ti, and Cr isotopes calculated with the effective charges e_p=1.16e for protons and e_n=0.7e for neutrons are tabulated. The B(E2, $0^+ \rightarrow 2^+$) values are in units of $e^2 fm^4$.

Nucleus	B(E2) (e^2 fm^4)	
	(Exp.)[a]	GXFP1
^{42}Ca	84.11 (7)	60.70
^{44}Ca	94.11 (4)[b]	52.40
^{48}Ca	18.65 (6)	16.30
^{46}Ti	189.9 (10)	140.90
^{48}Ti	125.31 (4)[c]	124.90
^{50}Ti	57.99 (7)	65.40
^{50}Cr	211.38 (42)[d]	218.60
^{52}Cr	125.43 (42)[d] 130.04 (23)	81.90

[a] Experimental data are taken from Ref. [19], [b] Reference [20], [c] Reference [21], [d]Reference [22].

4. Conclusion

Inelastic longitudinal (C2) From factorsand B(E2, $0^+ \rightarrow 2^+$)have been calculated for 2^+ state in some fp-shell nuclei using the GXFP1, KB3G and FPD6 effective interaction as a residuals interaction for ^{42,44,48}Ca, ^{46,48,50}Ti and ^{46,48,50}Cr nuclei. The calculations have been performed using Nushell@MSU code under windows.The CP effects have been calculated using Tassie model. The longitudinal C2 form factors for the^{42}Ca nucleus using the GXFP1 gives a good agreement for all momentum transfer values. The calculation for 2^+ state of ^{46,48}Ti and ^{50,52}Cr nuclei with GXFP1PN and KB3PN give a good agreement comparing with the experimental data in the momentum transfer range ($0.5 < q<3$) fm^{-1}.By using the GXFP1 effective interaction for the transition from 0^+ to 2^+ of ^{44}Ca, ^{46,48}Ti and ^{52}Cr, we are able to reproduce the third peak. The thirdpeak in the form factors of the ^{50}Ti gives poor agreement with experimental data.

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