Armenian Journal of Physics, 2014, vol. 7, issue 2, pp. 102-105

PROPAGATION OF ACOUSTIC WAVES THROUGH ELASTIC STRATIFIED MEDIUM WITH HETEROGENEITY IN THE FORM OF RECTANGULAR RESERVOIR

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Received 9 April 2014

Abstract – The problem of propagation of acoustic waves in an elastic-liquid stratified medium with horizontal-heterogeneous structure in the upper stratum is considered. In the presence of heterogeneity in the form of a rectangular reservoir in the upper elastic stratum a solution of the wave equation by the method of reflected waves in combination with the matrix method is constructed. Analysis of derived solutions allows to determine the conditions under which at a certain amplitude of the incident wave the maximum amplitude of acoustic wave in the liquid stratum is gained.

Key words: acoustic waves, stratified medium, method of reflected waves, matrix method.

One of the urgent problems of contemporary science and technology is the study of propagation of elastic waves in stratified media and identifying the opportunities of amplification of the acoustic vibration amplitudes. Within this aspect the acoustic vibration propagation registration in a stratified medium from a remote source of acoustic vibrations is of a great interest. In this case long-wave acoustic vibrations with small amplitudes are mainly registered, i.e. short-wave acoustic vibrations are absorbed in the medium.

To describe the long-wave oscillations with small amplitude emitted from relatively distant source of excitation of acoustic vibrations, the linearized equations of acoustics can be used [1-7], i.e. the wave equation

$$\partial^2\mathbf{u}/\partial t^2 = c_t^2\Delta\mathbf{u} + (c_l^2 - c_t^2)\,\nabla(\nabla\mathbf{u}), \qquad (1)$$

where $\mathbf{u}$ is the displacement vector; c_t and c_l are the velocities of propagated transverse and longitudinal waves, accordingly:

$$c_l^2 = (\lambda+2\mu)/\rho, \quad c_t^2 = \mu/\rho, \qquad (2)$$

where λ and μ are the Lame coefficients, ρ is the density of the medium. The vector of displacement **u** can be expressed by scalar (φ) and vector (**v**) potentials, which satisfy the wave equations

$$\partial^2\varphi/\partial t^2 = c_t^2\Delta\varphi,\ \ \partial^2\mathbf{v}/\partial t^2 = c_l^2\Delta\mathbf{v}. \tag{3}$$

During the registration of long-wave vibrations at locations far from the source it makes sense to explore only the influence of structure of the stratified medium assuming that the wave field on the stratum surface is known. In these cases the model of stratified structure is used, which allows to construct the exact solution of wave equity on (1) in the case of stratified structure with plane-paralleled boundaries.

In order to solve the problem of wave propagation in stratified structure, the matrix method [6, 7] is commonly used. Thus, after accounting the boundary conditions, the solution of the wave equation provides construction of the characteristic matrix of the system, which is the production of the characteristic matrix describing the propagation of waves within the stratum. We will score that the specified method can be used in the practical purposes if the front of incident signal is smaller than the horizontal dimensions of the stratified structure.

While modeling of responses of a reservoir to ultrasonic influences, it is necessary to take into account the resonance and other properties caused by restricted sizes and the geometrical shape of these objects.

For studying the response of the reservoir to acoustic influences we will consider the problem in the following simplified mathematical formulation. The upper stratum of the stratified medium consists of plane-paralleled isotropic strata with known locations and acoustic parameters. In the upper stratum a reservoir with the given shape is located. An acoustic wave with known amplitude-frequency characteristics falls on the bottom stratum. The formed acoustic field in the reservoir must be found.

Laplace transformations lead to stationary boundary problem. By solving the problem of own vibrations of the reservoir and utilizing the derived eigenfunctions the solutions of the internal and external boundary problems are gained and through cross linking of these solutions on the boundary of the reservoir, taking into account the condition of continuity of displacement function and stress tensor, the general solution is found in the upper stratum. Nevertheless, the coefficients of eigenfunction expansion of the reservoir remain undefined. Utilizing the matrix method, the solution of the wave equation for the remained strata is constructed. The gained solution is decomposed into a series of eigenfunctions of the reservoir and is cross linked with the solution of the upper stratum. The decomposition coefficients are determined from the boundary conditions at the top and the next strata.

In this case, the boundary conditions for the wave equation will have the form

$$\frac{d\ u}{d\ x}\Big|_{x=\pm x_0}=0,\ \ \frac{d\ u}{d\ y}\Big|_{y=\pm y_0}=0,\ \ \frac{d\ u}{d\ z}\Big|_{z=z_0}=\varphi(t),\ \ \frac{d^2u}{d\ z^2}\Big|_{z=0}=0 \tag{4}$$

where u describes the motion of the walls and bottom of the reservoir, $x=\pm x_0$,$y=\pm y_0$, $z=z_0$ are equations of the side walls and bottom of the reservoir, and z = 0 is the equation of free surface of the liquid. The solution of the wave equation is found by the method of variables separation. It is shown that in the presence of an acoustic signal waves with frequencies

$$\upsilon_{nkl}=V\sqrt{\left(\frac{n}{x_0}\right)^2+\left(\frac{k}{y_0}\right)^2+\left(\frac{l+0{,}5}{z_0}\right)^2}\ , \tag{5}$$

(where v is the velocity of the wave propagated in water, n, k, l = 1,2,…) resonant vibrations are formed in the reservoir.

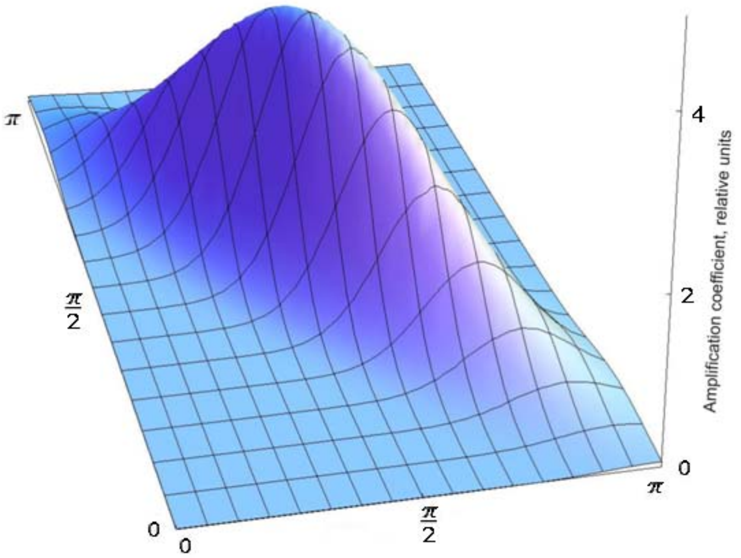

Fig. 1. Dependence of the gain coefficient on the parameters of strata (in relative units)

Thus, the resonant amplification of acoustic vibrations with a certain frequency spectrum can be observed in a rectangular reservoir. Figure 1 shows the dependence of the acoustic vibration enhancement coefficient from the stratum parameters in relative units. In this case the set of resonance frequencies depends on acoustic parameters, thickness and location of the elastic strata under the reservoir. Local enhancement can be gained not only due to the low speed layered sedimentary sequence (due to the interference of multiple reflected and refracted waves), but also because of their geometry.

The analysis shows that in certain cases under the influence of external acoustic waves there can be generated low-frequency resonant vibrations with wavelengths considerably exceeding the reservoir dimensions, i.e. when one part of the reservoir communicates with another via a neck with a small cross-sectional area (similarity with two Helmholtz resonators).

Acknowledgment

I would like to thank Dr. A.H. Mkrtchyan for valuable discussions during the work.

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