GENERAL EXPRESSION OF TEMPERATURE DEPENDENCE FOR HOLE AND ELECTRON CONCENTRATION IN MANY-VALLEY SEMICONDUCTORS

A.I. VAHANYAN[*]

Department of Physics of Semiconductors and Microelectronics, Yerevan State University,

1 Alex Manoogian St., Yerevan 0025, Republic of Armenia

[*]e-mail: aivahanyan@yahoo.com

Abstract – On the basis of charge neutrality equation, the expression for the temperature dependence of the charge carrier concentration in a non-degenerate many-valley semiconductor is derived.

Keywords: many-valley semiconductor; charge carrier concentration; charge neutrality equation

1. Introduction

In solid solutions when the starting compounds of such solutions are of qualitatively different band structure, with the change of compositions two or more valleys may by closely located according to their energy [1]. Such semiconductors behave either like two-valley ones or many-valley ones, accordingly. In the present work we derive a general expression for the temperature dependence of electron concentration in a many-valley semiconductor.

2. Derivation of general expression

We consider a many-valley n-type semiconductor which has m valleys arranged in different points of the quasi-momentum k-space and located at δE_i energy distance from a chosen zero level.

Let us consider a non-degenerate *n*-type compensated semiconductor with one type of singly ionized donors and acceptors. For simplicity, assume that the constant energy surfaces of each valley are of spherical symmetry in the vicinity of the minimum. It must be emphasized that the donor level corresponds to the valley from which the level is split off [2, 3] and its energy level is calculated from the bottom of the corresponding valley.

Note that the general expression for the charge neutrality condition in case of the many-valley semiconductors can be written as

$$\sum_i n_i + \sum_i n_{di} - N_d = \sum_i p_i + \sum_i n_{ai} - N_a \,, \tag{1}$$

where n_i is the concentration of free electrons of the ith valley, p_i is the concentration of free holes, n_{di} and n_{ai} are the concentrations of non-ionized donors and acceptors bounded with the ith valley, respectively, N_d and N_a are the concentrations of singly ionized donors and compensating acceptors, respectively.

If the band gap of the semiconductor is so large or the temperature is so low that in the considered temperature range the intrinsic conductivity is of no importance yet, and, taking into account the fact that in that case the Fermi level is located in the region near donor levels (at *n*-type conductivity) and, therefore, the concentration of free holes and non-ionized acceptors can be neglected, then the charge neutrality equation becomes

$$\sum n_i + \sum n_{di} = N_d - N_a \,. \tag{2}$$

If the counting of energy is performed with respect to the bottom of each ith valley, the energy distance from the Fermi level can be represented as $(-\delta E_i + E_F)$ (Fig. 1). Then the probability of occupation of the donor level related to the ith valley is expressed as follows:

$$P(E) = \frac{1}{1 + \beta_i^{-1} \exp\left(\dfrac{E - E_F + \delta E_i}{kT} \right)} \,, \tag{3}$$

where $E = -\varepsilon_{di}$ and β_i is the number taking into account spin degeneracy of the ith level.
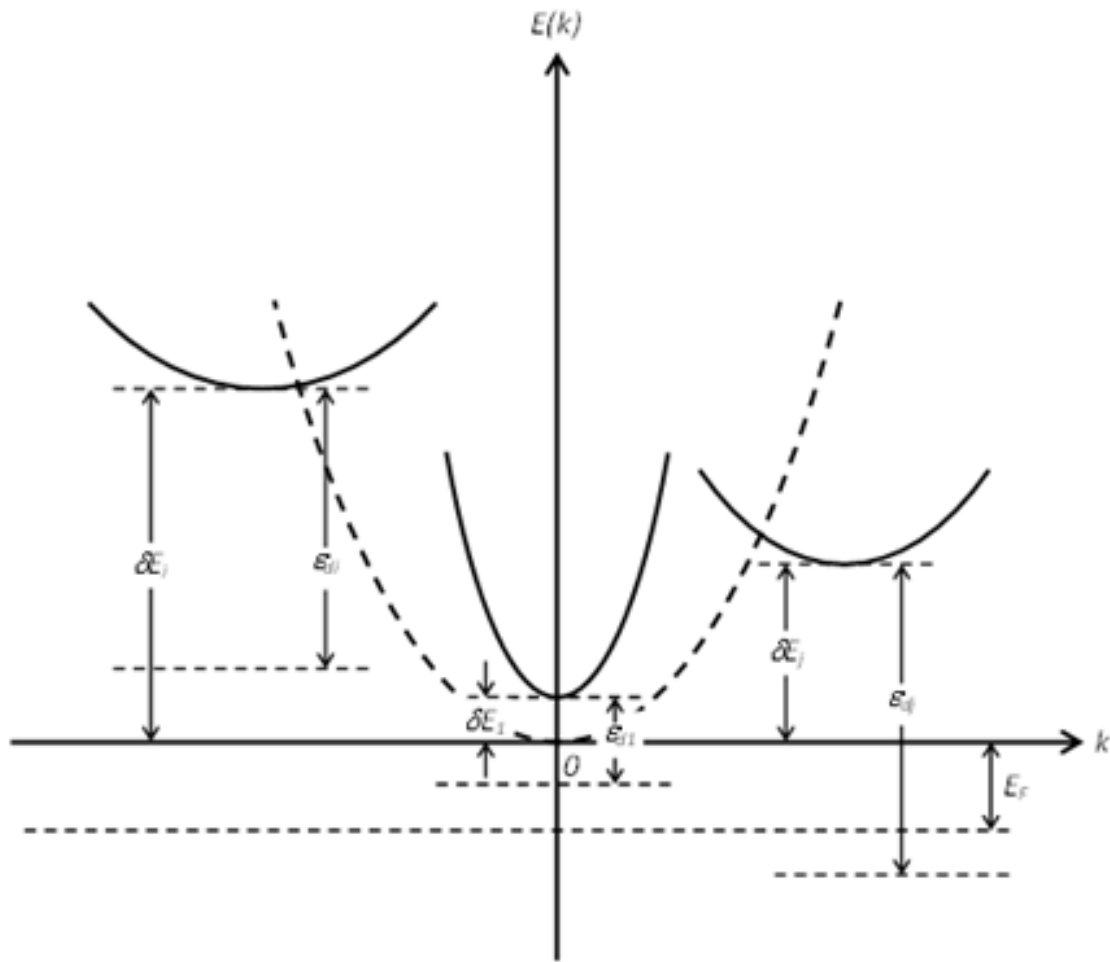
Fig. 1. Energy diagram of a many-valley *n*-type semiconductor

The concentration of the non-ionized donor levels is defined as

$$n_{di} = \frac{N_d - \sum_{j}' n_{dj}}{1 + \beta_i^{-1} \exp\left(\dfrac{-\varepsilon_{di} - E_F + \delta E_i}{kT}\right)},$$

(4)

where $\sum' n_{dj}$ is the concentration of the non-ionized donors associated with the rest of the valleys. The prime over the sum symbol denotes the term $j = i$ is omitted.

Obviously,

$$\sum_{i} n_{di} = \sum_{j}' n_{dj} + n_{di}.$$

(5)

Rearrangement of Eq. (4) gives

$$n_{di} = \frac{N_d - \sum\limits_i n_{di} + n_{di}}{1 + \dfrac{\beta_i^{-1} N_{ci} \exp\left(-\dfrac{\varepsilon_{di}}{kT}\right)}{N_{ci} \exp\left(\dfrac{E_F - \delta E_i}{kT}\right)}} \, . \tag{6}$$

Here, N_{ci} is the effective density of states of the ith valley and equals

$$N_{ci} = M_i \cdot 2 \left(\frac{2\pi m_i kT}{h^2}\right)^{\!3\!/\!2}, \tag{7}$$

where M_i is the number of equivalent valleys. Taking into account the expression

$$n_i = N_{ci} \exp\left(\frac{E_F - \delta E_i}{kT}\right) \tag{8}$$

and denoting

$$B_i = \beta_i^{-1} N_{ci} \exp\left(-\frac{\varepsilon_{di}}{kT}\right), \tag{9}$$

we get a formula

$$n_{di} = \frac{\left(N_d - \sum\limits_i n_{di} + n_{dj}\right) n_i}{n_i + B_i}, \tag{10}$$

or taking into account Eq. (2), the latter can be written as

$$n_{di} = \frac{n_i(n + N_a)}{B_i} = \frac{n_i(n + N_a)}{\beta_i^{-1} N_{ci} \exp\left(-\dfrac{\varepsilon_{di}}{kT}\right)} \, . \tag{11}$$

This relationship makes it possible to calculate the concentration of non-ionized donor levels bounded to the ith valley.

Substituting Eq. (11) into the charge neutrality equation (2) and taking into account that $n = \sum n_i$, we obtain

$$n + (n + N_a) \sum_i \frac{n_i}{B_i} = N_d - N_a \tag{12}$$

or

$$\frac{n(n+N_a)}{N_d - N_a - n} = \frac{1}{\frac{1}{n}\sum_i \frac{n_i}{B_i}}.$$
(13)

Denoting the relative occupancy of the ith valley by $c_i = n_i/n$, one can obtain finally the following

relation for Eq. (13):

$$\frac{n(n+N_a)}{N_d - N_a - n} = \left[\sum_i \frac{c_i}{\beta_i^{-1} N_{ci} \exp\left(-\frac{\varepsilon_{di}}{kT}\right)} \right]^{-1}.$$
(14)

This relation is defined as the general expression for the temperature dependence of the electron

concentration in a many-valley semiconductor in a low-temperature region. From Eq. (14), at $i = 1$, as

a particular case, the well-known expression [4] follows:

$$\frac{n(n+N_a)}{N_d - N_a - n} = \beta^{-1} N_c \exp\left(-\frac{\varepsilon_d}{kT}\right).$$
(15)

3. Conclusions

Thus, we have derived the general expression for the temperature dependence of the electron

concentration in the non-degenerate many-valley *n*-type semiconductor at the low-temperature region.

Using the above reasoning, one can obtain similar expressions for the case of many-valley *p*-type

semiconductors:

$$\frac{p(p+N_d)}{N_a - N_d - p} = \left[\sum_i \frac{c_i}{\beta_i^{-1} N_{vi} \exp\left(-\frac{\varepsilon_{di}}{kT}\right)} \right]^{-1}.$$
(16)

References

1. A. Suiyokwaishi, *Trans. Min. Metall.* **7**, 293 (1972).

2. V. Pol, *Impurity levels associated with several extremes of the conduction band. Proc. of the IX International Conference on Semiconductor Physics.* Nauka, Moscow, 1968.

3. A.Y. Vul, G.L. Bir, Yu.V. Shmartsev, *Fiz. Tekh. Poluprovodn.* **12**, 2331 (1970).

4. J.S.Blakemore, *Semiconductor Statistics*. Dover, New York, 2002.