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EVOLUTION OF B(E2: $8_1^+ \rightarrow 6_1^+$) VALUES OF $^{114, 116, 118, 120, 122}$Cd BY INTERACTING BOSON MODEL (IBM-1)

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In this paper we address the evolution of yrast levels of low-lying structure in the neutron-rich even-even $^{114\text{-}122}$Cd nuclei within the framework of interaction boson model (IBM-1). The reduced transition probabilities B(E2)↓ between 8_1^+ to 6_1^+ states of even-even neutron-rich Cd nuclei for N = 66, 68, 70, 72, 74 have been calculated by IBM-1 and compared with the previous available experimental values. The calculated values of ^{114}Cd, ^{116}Cd, ^{118}Cd, ^{120}Cd and ^{122}Cd are 0.272 e^2b^2, 0.281 e^2b^2, 0.259 e^2b^2, 0.190 e^2b^2 and 0.149 e^2b^2, respectively. The ratio of excitation energies of the first 4^+ and the first 2^+ excited states, $R_{4/2}$, were also calculated for these nuclei. The $^{114\text{-}122}$Cd isotopes in U(5)-O(6) transitional symmetry were investigated. We have studied the systematic B(E2) values as a function of even neutrons from N = 66 to 74.

Key words: Reduced transition probabilities, Ground-state, even-even, Cd, U(5), O(6)

1. Introduction

The interacting boson model (IBM-1) is an excellent interpretive model to understand the nuclear structure [1-2]. The cadmium nuclei, with two protons removed from a strong shell closure, exhibit intriguing aspects of nuclear structure at low excitation energies, namely the coexistence and mixing of vibrational with other collective degrees of freedom arising from the promotion of a proton pair across shell gap [3-4]. The structure of neutron-rich Cd isotopes has been studied the subject of many theoretical and experimental works in recent years. The yrast states up to $I^{\pi} = 8^{+}$ in N = 48 isotones were found two-hole states $\nu g_{9/2}^{-2}$ configurations for the N=50 closed shell. The existences of structure of $\nu g_{9/2}^{-2}$ configurations indicate to find structure of the valance mirror nuclei $\pi g_{9/2}^{-2}$ configurations [5-8]. Therefore, it is interesting to study $\pi g_{9/2}^{-2}$ configurations, which suggest that $(\pi g_{9/2}^{-2})_I^{\pi} = 0^+, 2^+, 4^+, 6^+, 8^+$ configurations dominate the yrast states and their 8_1^+ states are very likely to become isomers. Moreover, B(E2) of the yrast band between 8_1^+ to 6_1^+ plays important role in nuclear structure.

There are a number of theoretical works discussing intruder configuration and configuration mixing in the Cd isotopes. For instance, empirical spectroscopic study within the configuration mixing calculation in IBM [9-11], the IBM configuration mixing model in strong connection with shell model [12,13], conventional collective Hamiltonian approach [14,15] and the one starting from self-consistent mean-field calculation with microscopic energy density function[16]. Long et al. explained the low-lying levels and high-spin states of ^{116,118,120}Cd in the frame work of interacting boson model [17]. We have calculated the ground state energy band up to 8_1^+ levels [18,19] and reduced transition probabilities B(E2) values from 6_1^+ to 4_1^+ and 4_1^+ to 2_1^+ levels in even-even $^{114-122}$Cd isotopes by the framework of IBM-1[20]. In this work, we suggest an approach to search for the dynamical symmetries U(5), SU(3) and O(6) and to calculate B(E2) values between 8_1^+ to 6_1^+ states in even $^{114-122}$Cd isotopes using IBM-1 model [2].

2. Theory and method of calculation.

Yrast state energy band

The Hamiltonian of the interacting bosons in IBM-1 is given by [2,18].

$$H = \sum_{i=1}^{N} \varepsilon_i + \sum_{i\langle j}^{N} V_{ij}, \tag{1}$$

where ε is the intrinsic boson energy and V_{ij} is the interaction between bosons i and j. In the multipole form the Hamiltonian is given by [2,19,20]

$$H = \varepsilon \hat{n}_d + a_0 \hat{P}.\hat{P} + a_1 \hat{L}.\hat{L} + a_2 \hat{Q}.\hat{Q} + a_3 \hat{T}_3.\hat{T}_3 + a_4 \hat{T}_4.\hat{T}_4 \tag{2}$$

Here, a_0, a_1, a_2, a_3 and a_4 are the strength of pairing, the angular momentum and multipole terms. The Hamiltonian as given in Eq.(2) tends to reduce to three limits, the vibration U(5), γ-soft O(6) and the rotational SU(3) nuclei, starting with the unitary group U(6) and finishing with group O(2). In U(5) limit, the effective parameter is ε, in the γ-soft limit, O(6), the effective parameter is the pairing a_0, and in the SU(3) limit, the effective parameter is the quadrupole a_2.

The eigenvalues for the three limits are given by [18]

$$U(5): \quad E(n_d, \nu, L) = \varepsilon\, n_d + K_1\, n_d\, (n_d + 4) + K_4\, \nu(\nu + 3) + K_5\, L(L+1) \tag{3}$$

$$O(6): E(\sigma, \tau, L) = K_3[N(N+4) - \sigma(\sigma + 4)] + K_4\, \tau(\tau + 3) + K_5\, L(L+1) \tag{4}$$

$$SU(3): \; E(\lambda, \mu, L) = K_2\, (\lambda^2 + \mu^2 + 3(\lambda + \mu) + \lambda\mu) + K_5\, L(L+1) \tag{5}$$

Here, K_1, K_2, K_3, K_4 and K_5 are other forms of strength parameters. Many nuclei have a transition property between two or three of the above limits and their eigenvalues for the yrast-line are given by [20]

$$U(5) - O(6): \quad E(n_d, \tau, L) = \varepsilon\, n_d + K_1\, n_d\, (n_d + 4) + K_4\, \tau(\tau + 3) + K_5\, L(L+1) \tag{6}$$

$$U(5) - SU(3): \; E(\varepsilon, \lambda, L) = \varepsilon\, n_d + K_2\, (\lambda^2 + 3(\lambda + \mu)) + K_5\, L(L+1) \tag{7}$$

$$O(6) - SU(3): \; E(\tau, \lambda, L) = K_2\, (\lambda^2 + 3(\lambda + \mu)) + K_4\, \tau(\tau + 3) + K_5\, L(L+1) \tag{8}$$

3. Reduced transition probabilities B(E2)

The reduced transition probability in interaction boson model *IBM-1* [21] is given by equation

$$\text{B(E2; J +2} \rightarrow \text{J)}\!\downarrow = \alpha_2^2\, \frac{1}{4}(J+2)(2N-J)\,, \tag{9}$$

where J is the state that the nucleus translates to it and N is the boson number, which is equal to half of the number of valence nucleons (proton and neutrons). The low-lying levels of even-even nuclei (J_i = 2, 4, 6, 8…….) usually decay by E2 transition to the lower-lying yrast level with $J_f = J_i - 2$. From the given experimental value of transition (2 →0), one can calculate the value of the parameter α_2^2 for each isotopes and use this value to calculate the transition ($8^+ \rightarrow 6^+$).

4. Results and discussion

The transition from the first excited state to the ground state is assumed to be a pure E2, ($2^+ \rightarrow 0^+$) transition. The best parameters for ground-state band in even-even isotopes $^{114-122}$Cd are presented in Table 1. A summary of boson number, 8^+ energy level, gamma-ray transitions 8^+ to 6^+, experimental B(E2)↓ between 2^+ to ground-state and reduced transition probabilities between 8^+ to 6^+ level of even-even nuclei from ^{112}Cd to ^{122}Cd, are presented in Table 2. The calculated results using the framework of IBM-1 are compared with the previous available experimental results.

Table 1: *Parameters in (keV) for even-even $^{114-122}$Cd isotopes

A		K_1	K_4	K_5
114	768.71	-33.62	-16.40	15.69
116	483.66	26.22	-34.54	14.14
118	484.78	26.29	-30.35	9.17
120	292.83	66.94	-29.95	5.72
122	521.45	55.09	-40.52	1.52

*Ref. 18

*Table 2***.** Reduced transition probability B(E2)↓ from level $8_1^+ \rightarrow 6_1^+$

Nucl.	Boson num. $N = n_\pi + n_\nu$	8^+ level* in keV	γ Energy ($8^+ \rightarrow 6^+$) keV	*B(E2) ($2^+ \rightarrow 0^+$) W.U.	B(E2) ($2^+ \rightarrow 0^+$) e^2b^2	$B(E2)_{IBM-1}$ ($8^+ \rightarrow 6^+$) e^2b^2	*$B(E2)_{Ref}$ ($8^+ \rightarrow 6^+$) e^2b^2
^{114}Cd	9=1 +8	2669	678	31(19)	0.102	0.272	0.279(171)
^{116}Cd	8=1+7	2824	798	33.5(12)	0.113	0.281	
^{118}Cd	7=1+6	2591	771	33(3)	0.113	0.259	
^{120}Cd	6=1+5	2886	853	27	0.095	0.190	
^{122}Cd	5=1+4	3062	849	26(14)	0.093	0.149	

*Ref. [6,21-28]

5. Boson numbers (N)

A boson represents the pair of valence nucleons and the boson number is counted as the number of collective pairs of the valence nucleons. A simple correlation exists between the nuclei showing identical spectra and their valence neutron proton (N_p), neutron number (N_n). The number of valence proton N_p and neutron N_n has a total $N = (N_p + N_n)/2 = n_\pi + n_\nu$ bosons. At present ^{132}Sn doubly-magic nucleus is taken as an inert core to find boson number of ^{114}Cd to ^{122}Cd nuclei and they are presented in table 2.

6. The $R_{4/2}$ classification

In the collective dynamics of energies of even-even nuclei are grouped into classes, within each class the ratio

$R_{4/2} = \dfrac{E(4_1^+)}{E(2_1^+)}$ of excitation energies of the first 4^+ and the first 2^+ excited states. As pointed out by other similar ratios were characteristics of different collective motions of the nucleus. An harmonic vibrator has $E(4_1^+)/E(2_1^+) = 2.00$, an axially symmetric rotor should have $E(4_1^+)/E(2_1^+) = 3.33$, while X(5) behavior should have $E(4_1^+)/E(2_1^+) = 2.91$. The variation of the $E(4_1^+)/E(2_1^+)$ values as a function of even neutron numbers of Cadmium isotopes for experimental values, IBM-1, U(5), O(6) and SU(3) limits are presented in Fig.1. We identified U(5)-O(6) transitional symmetry in even-even nuclei with Z=48 and N = 66, 68, 70, 72, 74 with a range $2.04 < R_{4/2} < 2.20$. But they are near to U(5) symmetry.

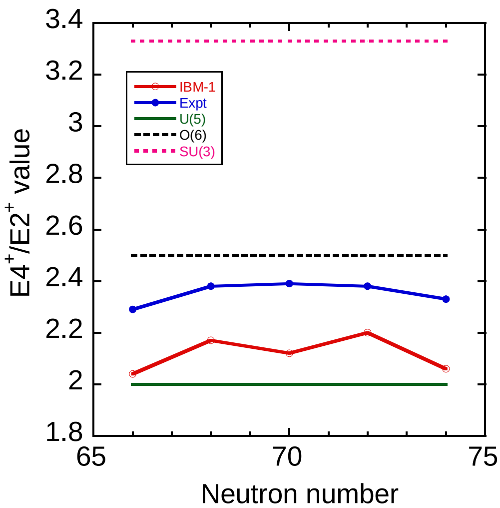

Fig.1. $E(4_1^+)/E(2_1^+)$ values as a function of neutron numbers of Cadmium isotopes $^{114-122}$Cd for experimental values, IBM-1, U(5), O(6) and SU(3) limit.

In Fig.2, we present the energies of the yrast sequences of ground state band using IBM-1 (normalized to the energy of their respective 2_1^+ levels) in these nuclei and compared them with previous experimental values [6, 22-29]. We present the comparisons of the ratios $R_L = E(L^+)/E(2_1^+)$ in the ground-state band (a usually adopted measure of nuclear collectivity), using the neutron numbers N = 66, 68, 70, 72, 74. From figure 1 and 2, we can see that IBM -1 calculation fit the U(5)-O(6) predictions generally. However, we find that the R_L values are consistently smaller in the IBM calculations than in experimental values.

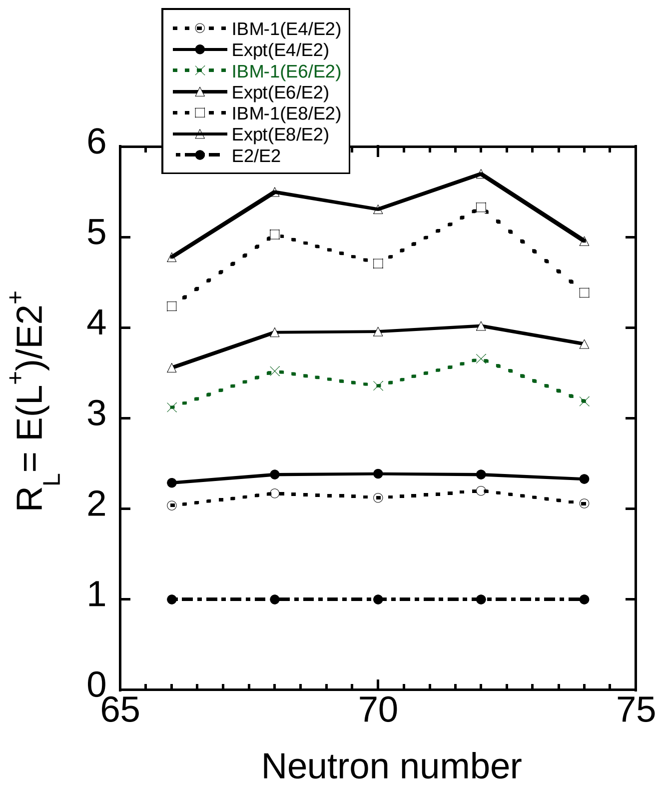

Fig. 2. The yrast sequences of ground state band of $R_L = E(L+)/E(2_1^+)$ as a function of neutron numbers (normalized to the energy of their respective 2_1^+ levels) in $^{114\text{-}122}$Cd nuclei.

7. Reduced transition probabilities B(E2)

The values of the reduced transition probabilities have been fitted the calculated absolute strengths B(E2) of the transitions within the ground state band to the experimental ones. The value of the effective charge (α_2) of the IBM-1 was determined by normalizing to the experimental data $B(E2;2_1^+ \to 0_1^+)$ of each isotope by using Eq (1). From the given experimental value of the transitions $(2 \to 0)$, we calculated value of the parameter α_2^2 for each isotope and used this value to calculate the transition ($8^+ \rightarrow 6^+$). The B(E2) values are presented in table 2, where the experimental data is compared with the present calculations and other previous works [6, 22-29]. The theoretical and experimental values of B(E2) values are plotted as a function of even neutrons represented in fig.3. The calculated reduced transition probabilities using IBM-1 as a function of neutrons number slowly increase from 0.272 e^2b^2 to 0.282 e^2b^2 in the neutron number from 66 to 68, and then decrease to 0.149 e^2b^2 for up to neutron number 74. In fig.3, results of the present work are compared with the available previous experimental values and shows good agreement for N=66. Moreover, the general agreement between the calculation and their previous experimental values for B(E2; $8^+ \rightarrow 6^+$) transition in Cd isotopes show a little different for N = 66. Using IBM-1 in fig.3, B(E2) values for the transition 8^+ to 6^+ decrease smoothly after the neutron number N = 68. The B(E2) values of the Z=48 isotopes with N < 68 differ significantly from those with N > 68. This difference probably originates from the orbital occupied by valence neutron; in the ground state with Z=48, the valence protons occupy hole-like states in the Z=50 closed shell, with a main configuration $\pi g_{9/2}^{-2}$. The valence neutrons occupy mainly particle-like states in the 50-82 shells. Due to the proton-neutron interaction, the nucleus is deformed. In $^{114}Cd_{66}$ and $^{116}Cd_{68}$ isotopes the valance neutrons occupy in the $2d_{3/2}$ orbitals while for $^{118,120,122}Cd$ isotopes, the valance neutrons occupy in the $3S_{1/2}1h_{11/2}$ orbitals. The nuclei ^{114}Cd by IBM-1 model nicely reproduced the experiment data and were fit satisfactory.

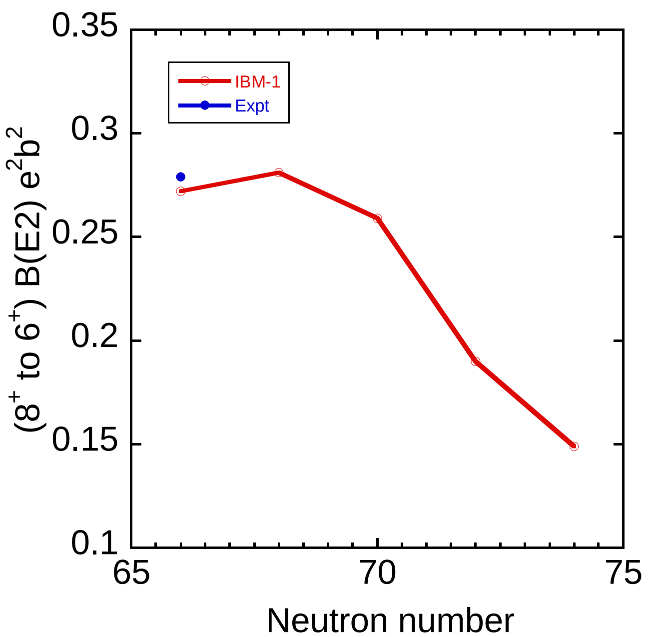

Fig. 3. Reduced transition probabilities B(E2: $8^+ \rightarrow 6^+$) as a function of even neutron numbers of $^{114-122}Cd$ isotopes.

In fig. 4, we compare the ratio R = B(E2: $L^+ \rightarrow (L\text{-}2)^+$)/B(E2: $2^+ \rightarrow 0^+$) of IBM-1 and the previous experimental values in the ground state bands (normalized to the B(E2: $2^+ \rightarrow 0^+$)) as a function of even neutron number in these nuclei. We found that the R values were consistently smaller in the IBM calculations than in the experimental values. However, we could see that the best agreement is closed to the calculations with the neutron number N = 66. Actually, in IBM-1 the proton and neutron bosons are not distinguishable as long as valence protons and neutrons are both hole-like or both particle-like [2]. The large B(E2) values in ^{114}Cd and ^{116}Cd nuclei are the main indicator of vibration characters.

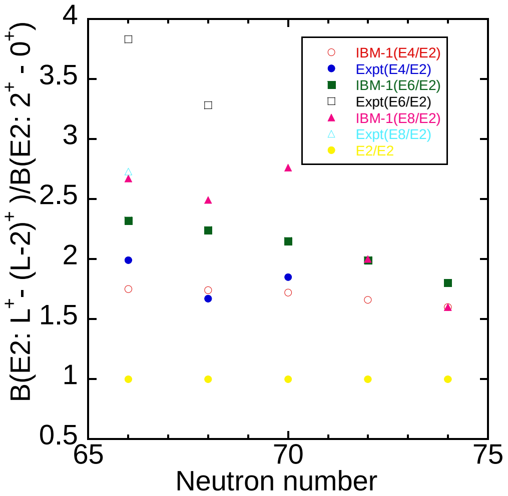

Fig. 4. Comparison of the B(E2) values in IBM-1 and experimental value. The ratio R = B(E2: $L^+ \rightarrow (L\text{-}2)^+$)/B(E2: 2+ → 0+) in the ground state bands (normalized to the B(E2: 2+ → 0+)) in these nuclei.

8. Conclusion

The evolution of nuclear low-lying yrast states in the even neutron-rich $^{114\text{-}122}$Cd isotopes were investigated within the interaction boson model (IBM-1). As a measure to quantify the evolution, we calculated the energy ratios of yrast states and the B(E2) transition rates of some of the low-lying quadrupole collective states in comparison to the available experimental data. It is seen that ground state band up to 8^+ levels and electric quadrupole reduced transition probability of those nuclei are in good agreement with the previous experimental results. The even-even $^{114\text{-}122}$Cd isotopes in U(5)-O(6) transitional symmetry were also investigated.

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References

1. Turkan N & Maras I (2010) E(5) Behaviour of the Ge isotopes. Math. and comput. Appli., **16;** 2**:** 428-438

2. Iachello F, Arima A (1987) The interacting boson model, Cambridge Univ. Press, Cambridge, 1987

3. Kumar R, Sharma A, and Gupta J.B., (2010) Character of quasi-bands in 150 Sm using IBM. Armenian J. Phys. **3**; 3:150-154

4. Aphahamian A, Brenner DS, Casten RF and Heyda K, (1984) Observation of 0^+ states in ^{118}Cd and the systematics of intruder states. Phys. Lett. B **140**;(1-2): 22-28

5. Makishima A, Asai M, Ishii T, Hossain I, Ogawa M, Ichikawa S and Ishii M (1999) The $(\nu g^{-2}_{9/2})8^+$ isomers in $^{82}Se_{48}$ and $^{80}Ge_{48}$ populated by deep-inelastic collisions. Phys. Rev. **C 59;**5: 2331-2334

6. Abdulla H Y, Hossain I, Ahmed I M, Ahmed S T, Karwan W Q, Kasimin M K, Chong J C, Viswanathan KK and Ibrahim N (2011) Calculation of 8^+ isomers of even-even Nuclei ^{76}Ni to ^{94}Pd for N=48 neutrons. *Int J. Phys. Sci. **6;4:** 901-907*

7. Gorska M, Grawe H, Foltescu D, Fossan D, Grzywacz R, Heese J, Maier K, Rejmund M, Roth H, Schubart R (1995). Proton-neutron interaction at N=Z First observation of the Tz=1 nucleus $^{94}Pd_{48}$ in-beam. Zeitschrift für Physik A Hadrons and Nuclei **353**;3: 233-234

8. Mrginean N, Bucurescu D, Rossi A C, Skouras L, Johnstone L, Bazzacco D, Lunardi S, de Angles G, Axiotis M (2003) *Phys. Rev. C.*, **67(6)** 61310

9. Smbataro M (1982). A study of Cd and Te isotopes in the Interacting Boson Approximation. *Nucl. Phys. A*, **380**:365-382

10. Deleze M et al., (1993) Systematic study of the mixed ground-state and “intruder” bands in $^{110, 112, 114}$Cd. *Nucl. Phys. A* **551(2)**: 269-294

11. Garrett P E, Green K L, and Wood J L (2008) Breakdown of vibrational motion in the isotopes $^{110-116}$Cd. *Phys. Rev. C,* **78**: 044307

12. Heyde K , Van Isacker P, Waroquier, and Wenes (1928) Systematic study of the mixed ground-state and “intruder” bands in $^{110, 112, 114}$Cd. *Phys. Rev. C,* **25**:3160
13. Isacker P V, P̧ittel S, Frank A, Duval P.D. (1986) Microscopic study of configuration mixing in the interacting boson model. *Nucl. Phys. A,* **451**:202-218
14. Inci I, Bonatsos D, Boztosun I (2011) Electric quadrupole transitions of the Bohr Hamiltonian with the morse potential. *Phys. Rev. C,* **84**: 024309
15. Bonatsos D, Georgoudis P E, Lenis D, Minkov N, and Quesne C. (2011) Bohr Hamiltonian with Deformation-Dependent mass term for the Davidson potential. *Phys. Rev. C,* **83**: 044321
16. Prochniak L et al. (2012) *Int. J. Mod. Phys. E,* **21**; 8: 1250036
17. Long GL, Zhut SJ and Sun HZ (1995) Description of $^{116, 118, 120}$Cd in the interacting boson model. J. Phys. G; Nucl. Part. Phys. **21:** 331-338
18. Hossain I, Abdullah HY, Ahmed I M, Saeed MA and Ahmed ST, (2012) Study on ground state energy band of even $^{114\text{-}124}$Cd isotopes under the framework of interacting boson model (IBM-1). Int. J. Modern Phys. E, **21**; 8: 1250072-(1-6).
19. Hossain I, Abdullah HY, Ahmed I M, Saeed MA, Ground-state energy band of even $^{104\text{-}122}$Cd isotopes under the framework of interacting boson model-1(2013): a review. *Journal of Theoretical and Applied Physics,* **7**:46
20. Hossain I, Abdullah HY, Ahmed IM, Saeed MA and Ahmed ST, (2012) Calculation of Reduced Transition Probabilities B(E2) in ^{114,116,118,122}Cd by interacting boson model (IBM-1). Armenian J. Phys., **5**; 3: 101-104.
21. Casten RF, Warner DD (1988) The interacting boson approximation. Rev. Mod. Phys. **60:** 389-469
22. Venkova T, Andrejtscheff W (1981). Transition strengths B (E2) in the yrast bands of doubly even nuclei, Atomic Data and Nucl. Data Tab. **26**;2: 93-136
23. El-Khoshi M (1993). *II Nuovo Cimento A* (1971-1996) **106**;7: 875
24. Raman S, Malakey C, Milner W, Nestor C (1987). Transition probability, B (E2) from the ground to the first excited 2^+ state of even-even nuclides. Atomic data Nucl. Data Tab. **36**; 1:1-96
25. Blachot J (2002). Nuclear data sheet for A =114, nuclear data sheet, **97**: 593-763
26. Blachot J (2010). Nuclear data sheet for A =116, nuclear data sheet, **111**:717

27. Kitao K (1995). Nuclear data sheet update for A =118, nuclear data sheet, **75**; 1: 99-198

28. Kitao K (2002). Nuclear data sheet for A =120, nuclear data sheet, **96**, 2: 241-390

29. Tamuza (2007). Nuclear data sheet for A = 122. Nuclear data sheets, **108**: 455-632