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COHERENT BREMSSTRAHLUNG FROM 20 MEV ELECTRONS IN THE PRESENCE OF HYPERSONIC WAVES

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Abstract–We investigate coherent bremsstrahlung in the range 1–40 keV from electrons with the energy 20 MeV incident at small angles to crystallographic planes of quartz crystal periodically deformed by acoustic waves. Depending on the parameters of the acoustic wave and incidence angle, the presence of the deformation can either enhance or reduce the bremsstrahlung cross-section.

In the last decade special attention was given to the investigation of interaction of charged particles with crystalline media (see, for instance, [1-5] and references therein). In particular, the coherent bremsstrahlung of high-energy electrons moving in a crystal is one of the most effective methods to produce intense beams of highly polarized and monochromatic photons (for recent experiments in this direction see, for instance, [6] and references therein). Such radiation has a number of remarkable properties and at present it has found many important applications. This motivates the importance of investigations for various mechanisms of controlling the radiation parameters. As such a mechanism, in [7] we have discussed the influence of hypersonic waves excited in a crystal in the process of bremsstrahlung of high-energy electrons for the case of a simplest crystal with a single atom in the elementary base and for sinusoidal deformation field. In order to have an essential influence of the acoustic wave, the generation of high-frequency hypersound is needed. Usually these types of waves are excited by high-frequency electromagnetic fields through the piezoelectric effect in crystals with a complex elementary base having more than one atomic species in the unit cell. In [8] we have generalized the results of [7] for crystals with a complex base and for acoustic waves with an arbitrary profile. In the present paper we use the results of [8] for investigation of the coherent bremsstrahlung from electrons with the energy 20 MeV in a quartz piezocrystal. Throughout of paper the system of units $\hbar = c = 1$ is used.

We consider the bremsstrahlung of high energy electrons moving in a crystal. We shall denote by Δ the volume of the unit cell and by $\mathbf{g}$ the reciprocal lattice vector. When external influences are present (for example, in the form of acoustic waves) the radius-vector of an atom in the crystal can be presented in the form $\mathbf{r}_{n0}^{(j)} = \mathbf{r}_{ne}^{(j)} + \mathbf{u}_n^{(j)}$, where $\mathbf{r}_{ne}^{(j)}$ determines the equilibrium positions of the atom in the situation without the deformation, $\mathbf{u}_n^{(j)}$ is the displacement of the atom caused by the external influence. For a lattice with a complex cell the coordinates of the atoms can be presented in

the form $\mathbf{r}_{ne}^{(j)} = \mathbf{R}_n + \boldsymbol{\rho}^{(j)}$, where $\mathbf{R}_n$ determines the positions of the atoms of one of the primitive lattices, and $\boldsymbol{\rho}^{(j)}$ are the equilibrium positions for the other atoms inside n-th unit cell with respect to $\mathbf{R}_n$.

We assume deformations having periodical structure:

$$\mathbf{u}_n^{(j)} = \mathbf{u}_0 f\left(\mathbf{k}_s \mathbf{r}_{ne}^{(j)}\right), \tag{1}$$

where $\mathbf{u}_0$ and $\mathbf{k}_s$ are the amplitude and the wave vector of the deformation field, $f(x)$ is an arbitrary function with the period 2π, $\max f(x) = 1$. The time-dependence of $\mathbf{u}_n^{(j)}$ in the case of acoustic waves we can disregard, as for the electron energies we are interested in the characteristic time for the change of the deformation field is much greater than the time of passage of the particles through the crystal. For the further use we introduce the Fourier transform of the function $e^{ixf(t)}$:

$$F_m(x) = \frac{1}{2\pi}\int_{-\pi}^{+\pi} e^{ixf(t)-imt} dt, \tag{2}$$

with $m = 0, \pm 1, \pm 2, \ldots$. For sinusoidal deformation fields, $f(z) = \sin(z + \varphi_0)$, one has

$$F_m(z) = e^{im\varphi_0} J_m(z) \tag{3}$$

with $J_m(z)$ being the Bessel function.

Let us denote by E_1, E_2 the energies of electrons in the initial and final states, respectively, and by ω the frequency of the radiated photon. The minimum longitudinal momentum transfer is given by $\delta = 1/l_c$, where $l_c = 2E_1E_2/(\omega m_e^2)$ is the formation length for the process of bremsstrahlung. The latter determines the effective longitudinal dimension of the interaction region for the phase coherence of the radiation process.

The cross-section of the bremmstrahlung can be presented in the form

$$d\sigma = N_0 (d\sigma_n + d\sigma_c), \tag{4}$$

with $d\sigma_n$ and $d\sigma_c$ being the incoherent and coherent parts of the cross-section per atom, N_0 is the number of atoms in the crystal. The coherent part of the cross-section is determined by the formula

$$d\sigma_c / d\omega = \left(e^2 N / N_0 E_1^2 \Delta\right) \sum_{m,\mathbf{g}} G_m(\mathbf{g}, \omega, E_1), \tag{5}$$

where

$$G_m(\mathbf{g}, \omega, E_1) = \frac{g_{m\perp}^2}{g_{m\parallel}^2}\left[1 + \frac{\omega^2}{2E_1E_2} - 2\frac{\delta}{g_{m\parallel}}\left(1 - \frac{\delta}{g_{m\parallel}}\right)\right] \left|F_m(\mathbf{g}_m \mathbf{u}_0)\right|^2 \left|S(\mathbf{g}_m, \mathbf{g})\right|^2, \tag{6}$$

and the summation is carried out under the condition $g_{m\parallel} \geq \delta$. In (5), e is the electron charge, N is

the number of cells in the crystal, $g_{m\parallel}$ and $g_{m\perp}$ are the parallel and perpendicular components of the vector

$$\mathbf{g}_m = \mathbf{g} - m\mathbf{k}_s, \tag{7}$$

with respect to the direction of the electron initial momentum $\mathbf{p}_1$, the factor $S(\mathbf{g}_m, \mathbf{g})$ is determined by the structure of unit cell and is given by the formula

$$S(\mathbf{g}_m, \mathbf{g}) = \sum_j u_{\mathbf{g}_m}^{(j)} e^{-\frac{1}{2} g_m^2 \overline{u_t^{(j)2}}} e^{i\mathbf{g}\boldsymbol{\rho}^{(j)}}. \tag{8}$$

In the expression (8), $u_{\mathbf{q}}^{(j)}$ is the Fourier transform of the potential for j-th atom, $\overline{u_t^{(j)2}}$ is the temperature-dependent mean-squared amplitude of the thermal vibrations for j-th atom.

The formula (5) differs from the corresponding expression for the bremsstrahlung in undeformed crystals by the replacement $\mathbf{g} \to \mathbf{g}_m$ and by an additional summation over m with the weights $\left|F_m(\mathbf{g}_m \mathbf{u}_0)\right|^2$. This corresponds to the presence of an additional one-dimensional superlattice with the period $\lambda_s = 2\pi/k_s$ and with the reciprocal lattice vector $m\mathbf{k}_s$. In the presence of deformation field the number of possibilities to satisfy the condition $g_{m\parallel} \geq \delta$ in the summation of formula (5) increases due to the term $mk_{s\parallel}$ in the expression for $g_{m\parallel}$. This leads to the appearance of additional peaks in the spectral distribution of the radiated photons. As the main contribution into the coherent part of the cross-section comes from the terms with $g_{m\parallel} \sim \delta$, the influence of the deformation field may be considerable if $\left|mk_{s\parallel}\right| \gtrsim \delta$. By taking into account that the main contribution into the series over m comes from the terms $|m| \lesssim \lambda_s / a$, where a is of the order of the lattice constants, we find the following condition: $u_0/\lambda_s \gtrsim a/4\pi^2 l_c$. At high energies one has $a/l_c \ll 1$ and this condition can be consistent with the condition $u_0/\lambda_s \ll 1$.

The role of coherence effects in the bremsstrahlung cross-section is essential when the electron enters into the crystal at small angles with respect to the crystallographic axes. In this case the main contribution into the coherent part of the cross-section comes from the crystallographic planes parallel to the chosen axis (z axis in our consideration). Assuming that the crystal lattice is orthogonal, the reciprocal lattice vector components are given by $g_i = 2\pi n_i / a_i$, $n_i = 0, \pm 1, \pm 2, ...,$ where a_i, $i = 1,2,3,$ are the lattice constants. Let θ be the angle between the initial electron momentum and the crystallographic z-axis. For the parallel component of the vector $\mathbf{g}_m$ we have the expression

$$g_{m\parallel} = g_{mz} \cos\theta + \left(g_{my} \cos\alpha + g_{mx} \sin\alpha\right) \sin\theta, \tag{9}$$

where α is the angle between the projection of the vector $\mathbf{p}_1$ on the plane (x, y) and y-axis. Coherent effects appear when the electron enters into the crystal at small angles θ. In this case the main contribution to the cross-section is made by the terms with $g_z = 0$ and

$$g_{m\|} \approx -mk_{s\|} + \left(g_y \cos\alpha + g_x \sin\alpha\right)\theta. \tag{10}$$

Two qualitatively different cases should be distinguished. The first one corresponds to the situation where the electron moves far from the crystallographic planes (angles α and $\pi/2-\alpha$ are not small). In this case, for the corresponding cross-section we obtain

$$\frac{d\sigma_c}{d\omega} \approx \frac{e^2 N}{4\pi^2 E_1^2 a_3 N_0} \sum_m \int dg_x dg_y G_m\left(\mathbf{g}, \omega, E_1\right), \tag{11}$$

where the integration range is determined by the condition $g_{m\|} \geq \delta$ with $g_{m\|}$ given by (10).

In the second case, the electron enters the crystal at small angles θ with respect to the crystallographic z axis and near the crystallographic planes (y, z) (the angle α is small). In this case, depending on the electron energy, two subcases should be considered separately. Under the condition $\delta \sim 2\pi\theta/a_2$, for the longitudinal component one has

$$g_{m\|} \approx -mk_{s\|} + \theta g_y \geq \delta, \tag{12}$$

and the summation over g_x can be replaced by the integration, $\sum_{g_x} \to \left(a_1/2\pi\right)\int dg_x$, with the result

$$\frac{d\sigma_c}{d\omega} \approx \frac{e^2 N}{2\pi E_1^2 a_2 a_3 N_0} \sum_{m, g_y} \int dg_x \, G_m\left(\mathbf{g}, \omega, E_1\right). \tag{13}$$

In the second subcase we assume the electron energies for which $\delta \sim 2\pi\theta\alpha/a_1$. Now, the main contribution into the sum comes from the terms with $g_y = 0,$ and the formula for the cross-section will become

$$\frac{d\sigma_c}{d\omega} \approx \frac{e^2 N}{E_1^2 N_0 \Delta} \sum_{m, g_x} G_m\left(\mathbf{g}, \omega, E_1\right), \tag{14}$$

where one has

$$g_{m\|} \approx -mk_{s\|} + \psi g_x, \quad \psi \equiv \alpha\theta,$$

and, as before, the summation goes under the condition $g_{m\|} \geq \delta.$.

We have performed numerical calculations for the bremsstrahlung cross-section for electrons with the energy 20 MeV in the case of SiO_2 single crystal. The corresponding results show that by adjusting the orientation of the crystal relative to the incident electron momentum and the parameters of the external influence, it is possible to enhance the number of bremsstrahlung photons. For the Fourier transforms of atomic potentials we take the Moliere parametrization. The

calculations are carried out for the sinusoidal transversal acoustic wave of the S-type (for the corresponding parameters see, for instance, [9,10]). For this wave the vector of amplitude of the displacement is directed along the X-direction of the quartz single crystal, $\mathbf{u}_0 = (u_0, 0, 0)$, and the velocity is equal 4.687×10^5 cm/sec. The vector determining the direction of the hypersound propagation lies in the YZ-plane and forms the angle 0.295 rad with the Z-axis. As the z-axis we choose the Z axis of the quartz crystal. The corresponding function $F(x)$ is determined by formula (3). The numerical calculations show that, in dependence of the values for the parameters E_1, θ, α, u_0, λ_s, the external excitation can either enhance or reduce the cross-section of the bremsstrahlung process.

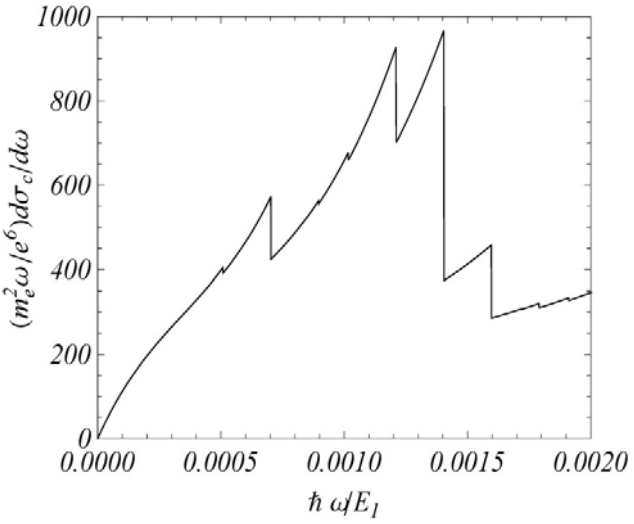

Fig.1. The coherent part of the cross-section evaluated by formula (14), as a function of ω/E_1 for the electron energy $E_1 = 20$ MeV and $\psi = 0.002$ in the case $2\pi u_0/a_1 = 0.5$. The deformation field is generated by the acoustic wave of the *S*-type with the frequency $\nu_s = 5$ GHz.

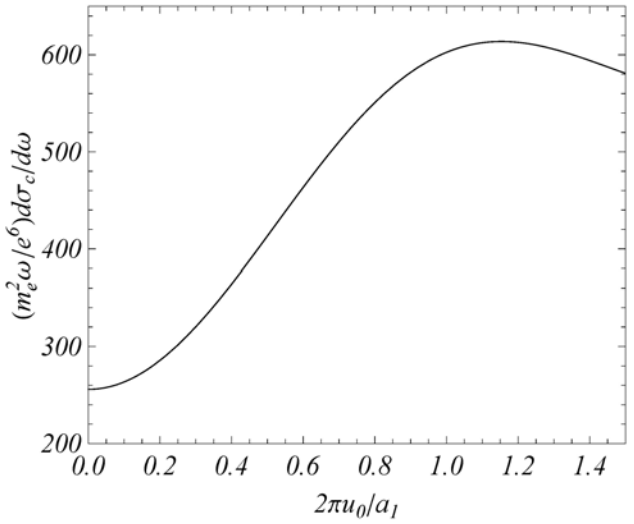

Fig.2. The cross-section $(m_e^2\omega/e^6)d\sigma_c/d\omega$, evaluated by formula (14), as a function of $2\pi u_0/a_1$ for the electron energy $E_1 = 20$ MeV, the photon energy $\omega/E_1 = 0.0015$ and $\psi = 0.002$.

For simplicity, in the numerical calculations we consider the case corresponding to (14). Similar features are observed for the other cases. In Fig.1 we have plotted the quantity $\left(m_e^2\omega/e^6\right)d\sigma_c/d\omega$, evaluated by formula (14), as a function of the ratio ω/E_1, for $2\pi u_0/a_1 = 0.5$ in the case $\psi = 0.002$. We have assumed that the deformation field is generated by the transversal acoustic wave of the S-type with the frequency $\nu_s = 5$ GHz. In Fig.2 the cross-section $\left(m_e^2\omega/e^6\right)d\sigma_c/d\omega$, evaluated by formula (14), is presented as a function of the relative amplitude of the deformation, parameter $2\pi u_0/a_1$, for the energy of photon corresponding to $\omega/E_1 = 0.0015$. The values of the other parameters are the same as those for Fig.1.

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