

SEARCH OF $\pm F_0$ SYMMETRY AND IDENTICAL BANDS IN SUPERDEFORMED NUCLEI IN $72 \leq N \leq 86$ REGION

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Abstract—Pairs of conjugate nuclei with the same F -spin and $\pm F_0$ values have identical $N_p N_n$ values. The superdeformed (SD) nuclei having neutron number $72 \leq N \leq 86$ are found to have the same boson number but different F_0 values except of the pair ^{130}Ce , ^{134}Nd , and the pair ^{144}Gd , ^{148}Gd . The behavior of γ -ray transition energies in bands of these pairs of SD nuclei are found to be constant. The smooth dependence of dynamic moment of inertia on rotational frequency gives the existence of identical bands in these two pairs of SD nuclei. The superdeformed band spectra of other nuclei in $72 \leq N \leq 86$ region based on the concept of $N_p N_n$ are also discussed.

Keywords: nuclear structure, superdeformed bands, F -spin multiplets.

1. Introduction

One of the interesting discoveries in nuclear structure investigations are the existence of identical bands (IB) in pairs of even-even as well as in adjacent odd-even nuclei spanning an extended region of spin and deformation. Several explanations have been put forward to understand the origin of this phenomenon in the super deformed (SD) and normal deformed (ND) bands. Twin et al. [1] firstly discovered the discrete line SD bands in $A = 150$ mass region in ^{152}Dy . Nazarewicz et al. [2] and Deleplanque et al. [3] discovered the several other examples of super deformation in neighboring nuclei, it became apparent that some properties of these bands differed considerably. In particular, the dynamic moment of inertia $J^{(2)}$ and excitation frequencies were seen to vary from one example to the next. The most intriguing difference between the properties of superdeformed nuclei in $A = 150$ and $A = 190$ region is the behavior of dynamic moment of inertia $J^{(2)}$ as a function of rotational frequency $\hbar\omega$. One of the outstanding problem in SD nuclei in the mass $A = 190$ region are the so-called “identical bands”, where the transition energies are either identical (within 1 or 2 keV) to those in a reference band of another nucleus or identical to the midpoint energies or to the “quarter points” (average of midpoint energy) of the reference band. Stephens et al. [4] reported that many cases in $A = 190$ mass region have been found where the alignment is quantized in this way. Azaiez et al. [5] found the six identical SD bands in ^{194}Tl . This was the first time that so many SD bands have been observed in a single nucleus. Several ideas have been proposed to understand the identical band phenomenon at normal deformation. Ahmad et al. [6] and Baktash et al. [7] have used different criteria for defining the existence of identical bands. The occurrence of identical bands in neighboring even-even nuclei requires that the moment of inertia in

the two bands be identical. Fallon et al. [8] reported that in $A = 190$ region, pair correlations play an important role in determining the properties of SD bands and therefore may be expected to influence the occurrence of identical bands. In $A = 130$ region, Zhang and Hughes [9] reported the sequences of identical transition energies at normal deformation in odd - A nuclei relative to even-even neighbor and suggested that the identity in moment of inertia in these bands may be the result of precise cancellation between the reduction of pairing and reduction in quadrupole deformation, caused by the odd particle. Bands having identical transition energies have, by definition, identical moment of inertia. Factors that influence the moments of inertia include pairing correlation deformation (quadrupole, hexadecapole, triaxial, etc.), proton–neutron interaction, alignment and so on. Bohr and Mottelson [10] has long been shown that in the traditional pairing theory, an odd nucleon always reduces the monopole pairing correlation in the blocking effect, consequently increasing the moment of inertia. In order to compensate or cancel the reduction of pairing correlation from the blocking effect, there must be at least some factors that act to reduce the moment of inertia in odd - A nuclei. Such a scenario is necessary but not sufficient condition for observing identical bands. In $A = 150 - 190$ region, a considerable number of identical bands have been found in odd - proton nuclei [2, 11-13]. The fact that in SD nuclei, the deformation is larger and more stable and pairing correlations are substantially weakened in these nuclei. Casten et al. [14] studied the low-spin identical bands in $^{156}\text{Dy} - ^{180}\text{Os}$. A simple correlation exists between the nuclei showing the identical spectra and their valence proton (N_p) and neutron number (N_n). Arima and Iachello [15] have given the interacting boson model (IBM), which is helpful to describe the collective level structure of many nuclei. An extended version of IBM-1, known as IBM-2 explicitly takes into account the presence of neutron and proton bosons. A very useful concept in this regard is that of F -spin multiplets. A nucleus with N_p valence protons and N_n valence neutrons which are counted as particles or holes from the nearest closed shell has a total of $N = \frac{N_p + N_n}{2} = N_\pi + N_\nu$ bosons. The N_π proton bosons and N_ν neutron bosons are assigned F -spin, $F = \frac{1}{2}$, with projections $F_0 = \frac{1}{2}$ for proton bosons and $F_0 = -\frac{1}{2}$ for neutron bosons. A given nucleus is then characterized by two quantum numbers, $F = \frac{N_\pi + N_\nu}{2}$ and its projection, $F_0 = (N_\pi - N_\nu)/2$. The usefulness of the F -spin concept has recently been discussed by a number of authors [16-19]. If the Hamiltonian is F -spin invariant, it leads to the occurrence of F -spin multiplets in analogy to isospin multiplets. Such F -spin multiplets can occur either in a series of isobaric nuclei defined by (A, Z) , $(A, Z + 2)$, $(A, Z + 4)$, where both protons and neutrons are

particle-like or both are hole-like, or in series of nuclei given by (A, Z) , $(A+4, Z+2)$, $(A+8, Z+4)$, where one kind of nucleons behave like particles and other kind like holes. The invariance of F -spin has been examined by looking for constancy of excitation energies in a given F -spin multiplet. Jain and Casten [20] have observed the F_0 symmetry in F -spin multiplets in the same rare earth region. The pair of nuclei having same F -value with projections $+F_0$ and $-F_0$ will have the same value of $N_p N_n$ and may be expected that nuclei exhibit the identical transition energies. Mittal and Devi [21] identified the low-spin identical bands in light nuclei by using $N_p N_n$ and F -spin concept.

The aim of the present work is to search the F_0 symmetry and identical bands in SD nuclei in $72 \leq N \leq 86$ region. Search of identical bands is based on the concept $N_p N_n$ product and F -spin multiplets of nuclear structure. Since we focus on the superdeformed (SD) nuclei in $72 \leq N \leq 86$ region, so we will search that whether these nuclei can display identical bands with nearly identical transition energies or not.

2. Result and Discussions

The nuclei in F -spin multiplets have $F_0 = \frac{N_\pi - N_\nu}{2}$ values from $-F$ to $+F$. The nuclei with symmetric $\pm F_0$ values in the F -spin multiplets also have identical $N_p N_n$ values. For example the SD nuclei ^{130}Ce and ^{134}Nd have (N_p, N_n) values equal to $(4, 5)$ and $(5, 4)$, respectively, So that the product $(N_p, N_n) = 20$ and $F_0 = \frac{1}{4}$ in both cases. The (F_0, N_p, N_n) pairs have the similar structure [22].

When we compare $^{132}\text{Ce}(SD-3)$, $^{136}\text{Ce}(SD-2)$, $^{144}\text{Gd}(SD-5)$, $^{148}\text{Gd}(SD-5)$, having different $N_p, N_n = 4, 4: 5, 3: 7, 1: 7, 1$ by using the experimental data [23, 24]. These nuclei do not show constant behavior of γ -ray transition energy as shown in Fig.1. Similarly, when we compare $^{130}\text{Ce}(SD-2)$, $^{134}\text{Nd}(SD-1)$, and $^{150}\text{Gd}(SD-12)$ having $N_p, N_n = 4, 5: 7, 2$, these nuclei also do not show constant behavior of γ -ray transition energies as shown in Fig.2. In Fig.3 we compare the spectra of SD nuclei $^{142}\text{Sm}(SD-3)$, and $^{146}\text{Gd}(SD-12)$ having $N_p, N_n = 6, 1$ and $7, 0$, respectively, which also do not show constant behavior of γ -ray transition energy. In Fig.4 the spectra of γ -ray transition energy of SD nuclei $^{132}\text{Nd}(SD-1)$ and ^{136}Sm having $N_p, N_n = 5, 5$ and $6, 4$, respectively, is also found to be different.

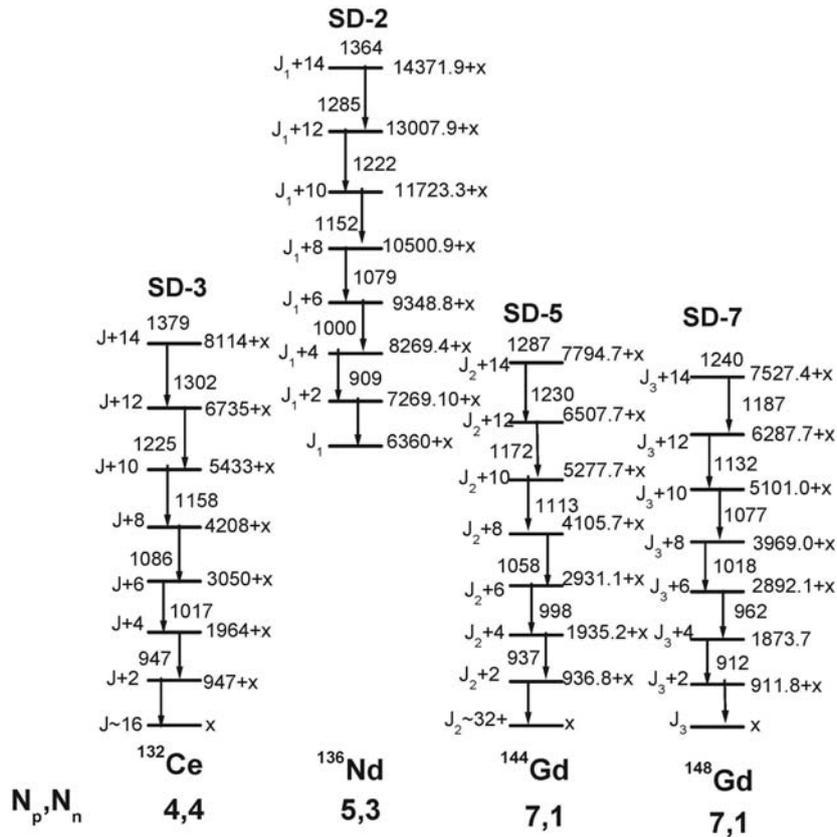


Fig.1. The variation of transition energy in (keV) between the two states with N_p, N_n ; with $N_B = 8$, where N_p and N_n are valence proton and valence neutron boson numbers. The experimental data has been taken from [23,24], however, the values of x , J_1 , J_2 , and J_3 are unknown.

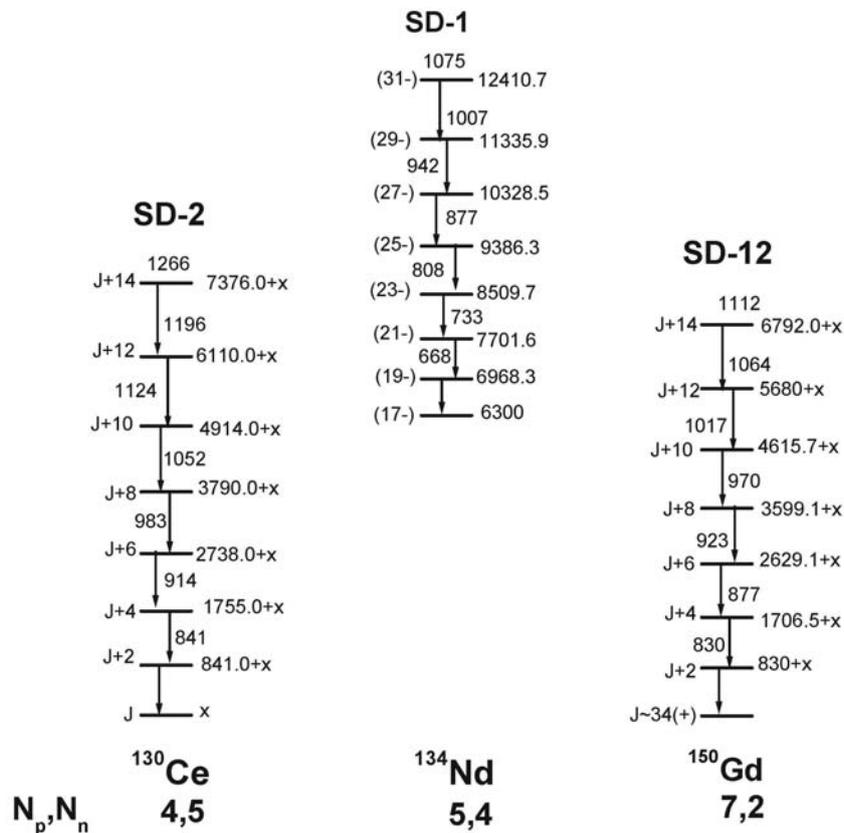


Fig.2. Same as Fig.1 but for ^{130}Ce and ^{134}Nd and ^{150}Gd with $N_B = 9$.

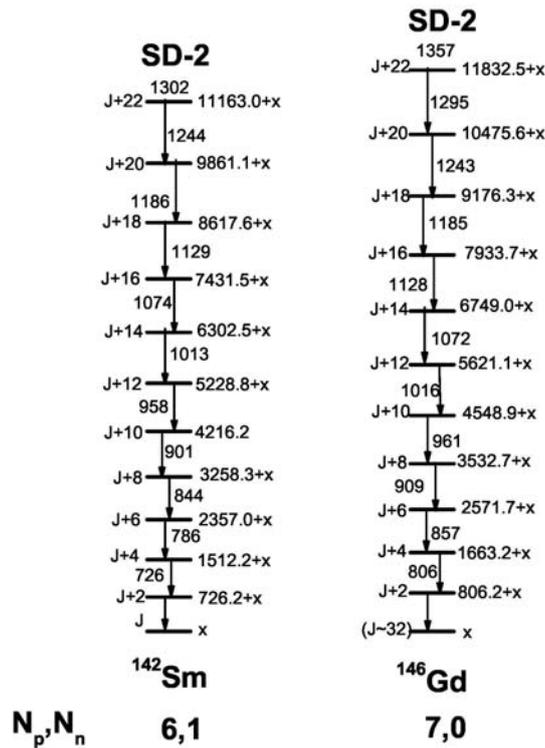


Fig.3. Same as in Fig.1 but for ¹⁴²Sm and ¹⁴⁶Gd with $N_B = 7$.

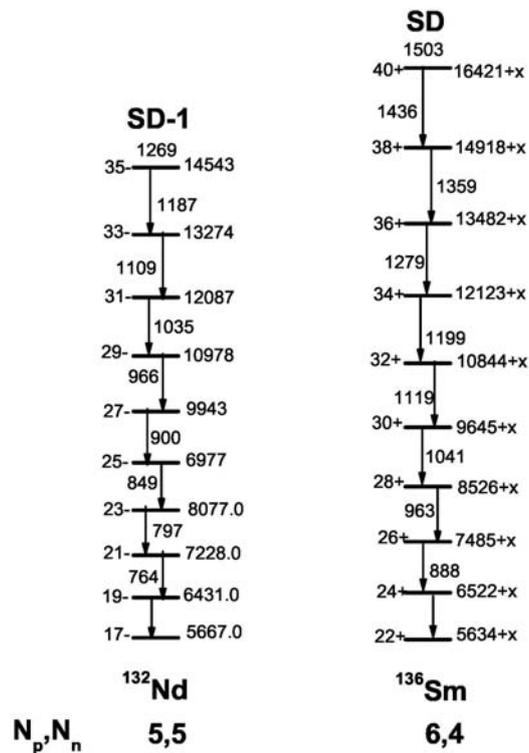


Fig.3. Same as in Fig.1 but for ¹³²Nd and ¹³⁶Sm with $N_B = 10$.

In Fig.5 we compare the ground band spectra of pair of conjugate nuclei having the same $|F_0|$ and $(N_p + N_n)/2$ values i.e., the ¹³⁰Ce having $F_0 = -1/4$ and $(N_p + N_n)/2 = 9/2$ is symmetric with ¹³⁴Nd with $F_0 = +1/4$ and $(N_p + N_n)/2 = 9/2$. The agreement between these two pairs is

impressive. There is no violation of the symmetry in the experimental data in these two nuclei. In Fig.6 we plot the difference between the compared transitions on the vertical axis and γ -ray transition energies on the horizontal axis. The energy differences ΔE_γ are identical within 0.5 keV for all transitions up to spin $10\hbar$, i.e. over 5 transitions, quite similar to those observed in the super deformed bands. We have not compared the superdeformed spectra of these two nuclei due to lack of data information in these nuclei. The dynamic moment of inertia $J^{(2)}$, derived from the transition energies of Fig.5, are plotted against the rotational frequency ($\hbar\omega$) in Fig.7. The smooth dependence of dynamic moment of inertia gives the existence of low-spin identical bands in ^{130}Ce and ^{134}Nd . Therefore, above discussion allows predicting the low-spin identical bands in ^{130}Ce and ^{134}Nd nuclei.

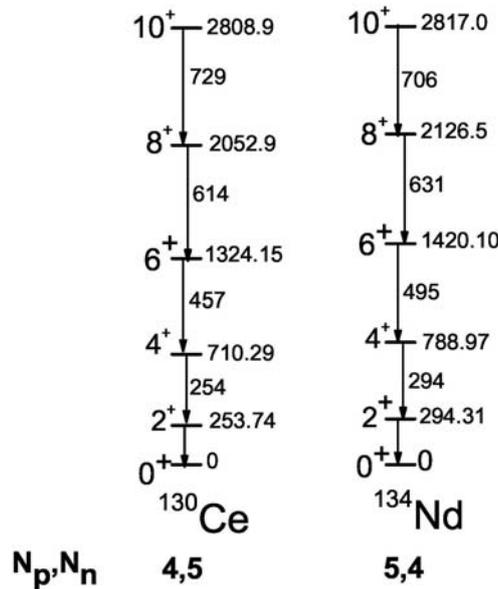


Fig.5. The ground band energy of pairs of conjugate nuclei having the same F_0 and $N_p N_n$ values are compared for their band spectra in $A = 130$ region.

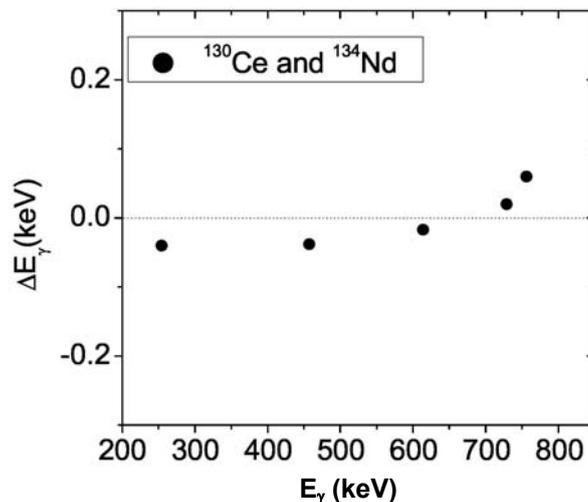


Fig.6. The difference in γ -ray energies ΔE_γ between the g-bands of ^{130}Ce and ^{134}Nd nuclei.

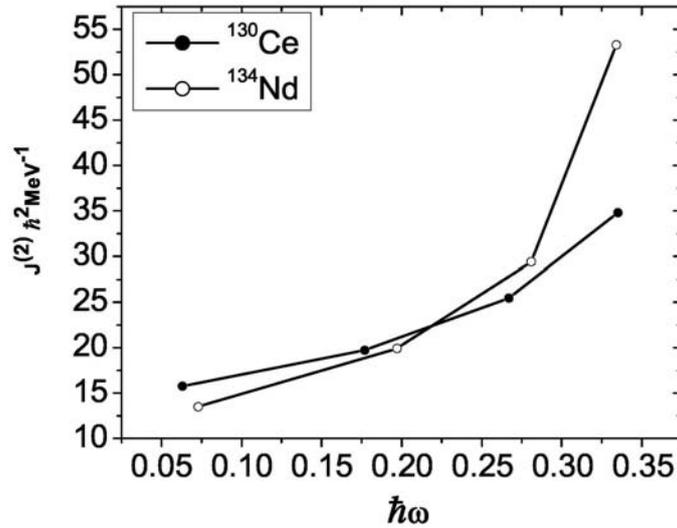


Fig.7. The dynamic moment of inertia $J^{(2)}$ as a function of rotational frequency ($\hbar\omega$) for ground bands of ^{130}Ce and ^{134}Nd .

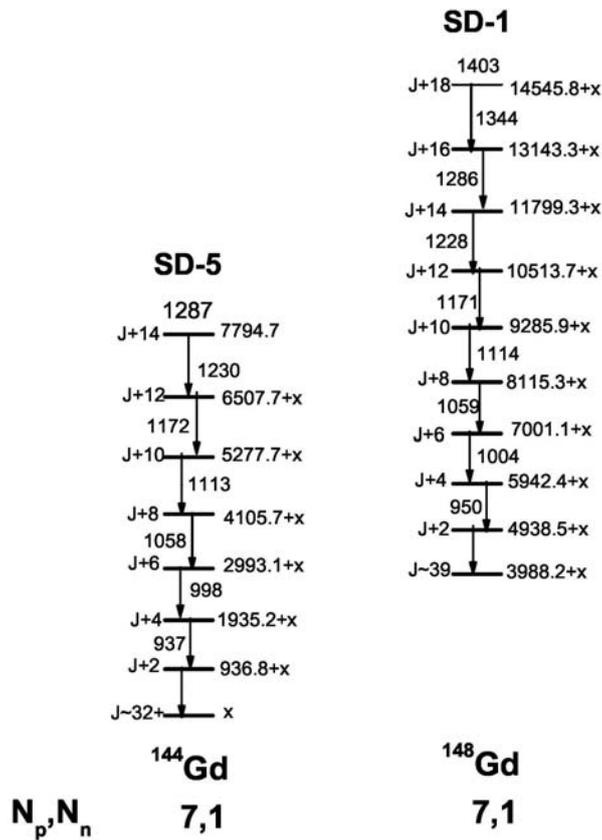


Fig.8. The superdeformed spectra of pairs of conjugate nuclei ^{144}Gd and ^{148}Gd having the same F_0 and $N_p N_n$ values are compared for their band spectra in $A = 130$ region.

In Fig.8 we compare the SD spectra of pair of conjugate nuclei having the same $|F_0|$ and $\frac{N_p + N_n}{2}$ values, i.e. $^{144}\text{Gd}(SD-5)$ having $F_0 = 3$ and $\frac{N_p + N_n}{2} = 8$ is symmetric with the SD spectra of $^{148}\text{Gd}(SD-1)$ with $F_0 = 3$ and $(N_p + N_n)/2 = 8$. The agreement between the

experimental SD spectra of these two nuclei is impressive. There is no violation of the symmetry in the spectra of these two SD nuclei. In Fig.9 we plot the difference between the γ -ray transition energies ΔE_γ of these two SD nuclei, i.e. $^{144}\text{Gd}(SD-5)$ and $^{148}\text{Gd}(SD-1)$ and γ -ray transition energies on the horizontal axis. The energy difference ΔE_γ is approximately equal to zero for 5 transitions. In Fig.10 we have plotted the dynamical moment of inertia $J^{(2)}$ against the rotational frequency $\hbar\omega$. The approximately same value of dynamic moment of inertia of these two SD nuclei, i.e. $^{144}\text{Gd}(SD-5)$ and $^{148}\text{Gd}(SD-1)$ gives the existence of identical SD bands in these two nuclei.

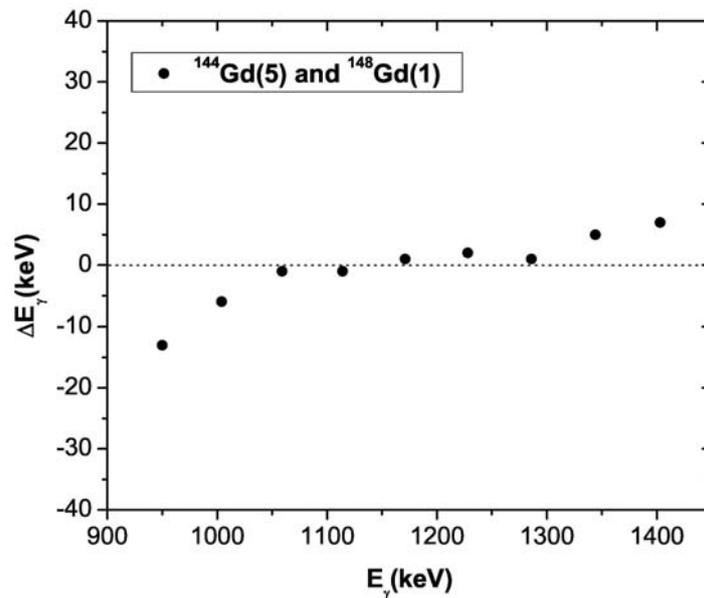


Fig.9. The difference in γ -ray energies ΔE_γ between the SD bands of ^{144}Gd and ^{148}Gd nuclei.

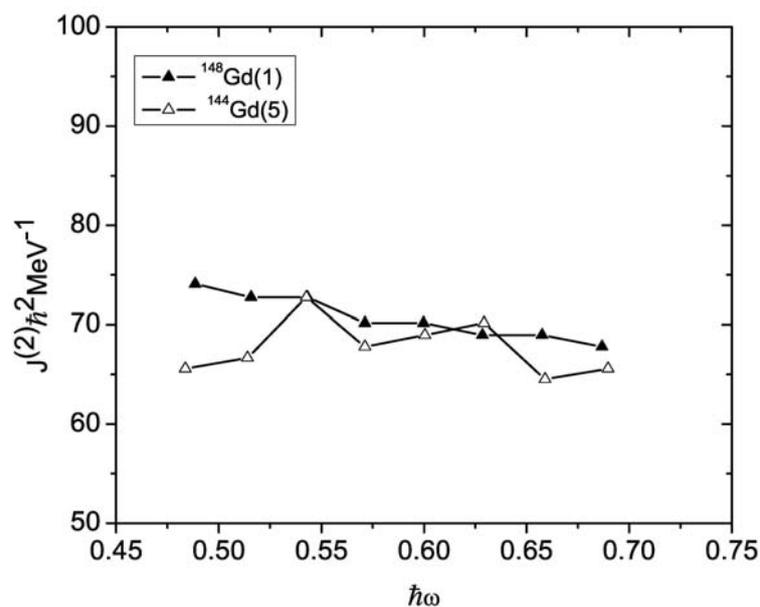


Fig.10. The dynamic moment of inertia $J^{(2)}$ as a function of rotational frequency ($\hbar\omega$) for SD bands of ^{144}Gd and ^{148}Gd .

2.1. Identical Spectra of SD Nuclei with $N_p N_n$ scheme

Casten [25] studied the structure of transitional nuclei in terms of the product of valence proton and valence neutron boson. Casten noted that 2_1^+ energies in a wide group of nuclei depend mostly on the parameter $N_p N_n$: $E(2_1^+, N_p, N_n) \approx f(N_p N_n)$, since the nuclei having identical $N_p N_n$ and $|N_p - N_n|$ values are found to have identical moment of inertia function

$$\theta_k \propto [f(N_p N_n)]^{1/k}, \quad k = 1, 2, \tag{1}$$

where

$$\theta_1 = \frac{3}{\hbar^2} E(2_1^+), \quad \theta_2 = \frac{4\hbar^2}{E(4_1^+) - 2E(2_1^+)}, \tag{2}$$

$$f(N_p N_n) = N_p N_n (N_p + N_n). \tag{3}$$

The function $f(N_p N_n)$ is also called the saturation factor (SF). We calculate the value of SF using Eq.(3) for $72 \leq N \leq 86$ nuclei. Saha and Sen [26] obtained the $(SF)_{\max} = 9520$ and classified the value SF into two parts for $Z = 66 - 76$ and $N = 90 - 106$ nuclei, i.e. (i) if $SF < 5000$ and (ii) $SF > 5000$. But in the present region of study we observed that the maximum value of saturation factor comes out to be 2000 in case of ^{132}Nd SD nucleus so that

$$(SF)_{\max} = 2000. \tag{4}$$

With the help of this $(SF)_{\max}$ we have calculated the saturation parameter (SP) as

$$SP = \left(1 + SF / (SF)_{\max}\right)^{-1}. \tag{5}$$

Table 1. The structure factor (SF), structure parameter (SP)

SD nuclei	$SF = f(N_p N_n)$	SP
^{130}Ce	1440	0.581
^{132}Ce	1024	0.661
^{132}Nd	2000	0.5
^{134}Nd	1440	0.581
^{136}Nd	960	0.675
^{136}Sm	1920	0.510
^{142}Sm	336	0.856
^{144}Gd	448	0.8169
^{146}Gd	0	1
^{148}Gd	448	0.8169
^{150}Gd	1008	0.6648

The values of SF , SP are summarized in Table 1. The functional and linear dependence of moment of inertia of SF factor is expressed as

$$\theta = SF, \quad (6)$$

$$\theta \propto SF \left(1 + \frac{SF}{(SF)_{\max}} \right)^{-1}. \quad (7)$$

The nuclei having nearly equal values of SF show the low-spin and superdeformed identical spectra. The ^{130}Ce is identical in SF with ^{134}Nd and ^{144}Gd is identical in SF with ^{148}Gd . This shows a smooth dependence of moment of inertia in these SD nuclei.

3. Conclusion

In the above description we observed that ground band energies and corresponding γ -ray transition energies of two nuclei ^{130}Ce and ^{134}Nd in $72 \leq N \leq 86$ region are constant. Secondly, these two nuclei have the same F_0 values from $-F$ to $+F$. We observe that the nuclei with symmetric F_0 values in an F -spin multiplet have identical $N_p N_n$ values. ^{130}Ce and ^{134}Nd nuclei show low-spin identical bands. We have not compared the band spectra of these two nuclei at higher spin because of lack of information of experimental data of these two superdeformed nuclei. Also the γ -ray transition energies of two SD nuclei $^{144}\text{Gd}(SD-5)$ and $^{148}\text{Gd}(SD-1)$ in $72 \leq N \leq 86$ is found to be constant. These two SD nuclei have identical F_0 and $N_p N_n$ values. Also, the values of dynamical moment of inertia of these two SD nuclei are approximately the same. It suggests that F_0 symmetry do not loose the symmetry at higher deformation also. Finally, with the help of $N_p N_n$ scheme we observed that the nuclei ^{130}Ce and ^{134}Nd , ^{144}Gd and ^{148}Gd have equal SF and SP , respectively, which further means that these nuclei show identical spectra. Hence, ^{130}Ce and ^{134}Nd nuclei show low-spin identical bands and $^{144}\text{Gd}(SD-5)$ and $^{148}\text{Gd}(SD-1)$ show identical superdeformed bands in $72 \leq N \leq 86$ region.

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