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A CORRESPONDENCE BETWEEN IBA-1 AND IBA-2 MODELS OF EVEN ISOTOPES OF Sm

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Abstract–The level scheme of the ^{148,150,152,154,156,158}Sm isotopes have been investigated using both IBA-1 and IBA-2 versions of Interacting Boson Approximation Model. IBA-1 and IBA-2 Hamiltonian parameters are obtained as well as the extraction of the energy levels. Also, the electric quadrupole transition probabilities $B\left(E2:J_i \to J_f\right)$ of the Sm isotopes were calculated. In calculations, the theoretical energy levels and the electric quadrupole transition probabilities have been obtained by using PHINT code. Good agreement was found from comparison between the calculated energy levels and the electric quadrupole transition probabilities $B\left(E2\right)$ of the Sm isotopes with the experimental data.

***Keywords*:** IBA, PHINT, energy levels, $B\left(E2\right)$ value

1. Introduction

The interacting boson approximation represents a significant step towards our understanding of nuclear structure. It offers a simple Hamiltonian, capable of describing collective nuclear properties across a wide range of nuclei, and is founded on rather general algebraic group theoretical techniques which have also recently found application to problems in atomic, molecular, and high-energy physics [1,2]. The applications of this model for the deformed nuclei are currently a subject of considerable interest and controversy [3].

Interacting boson model calculations of even-even Sm isotopes using both IBM1 and IBM2 versions have been performed many times [4-8] and were used to account for energy levels and electromagnetic properties of the deformed nuclei for different regions in the nuclear chart. Special attention was paid to the effect of the partial sub-shell closure for $Z = 64$ [9,10] and its inclusion into the calculations using an effective number of bosons.

In the previous studies, the shape transition in the Sm isotopes was studied in the framework of the *s-d* IBA model using the most general Hamiltonian. It is found that only three terms are necessary to reproduce the transition and obtain an excellent fit [11]. Effective boson numbers were introduced in 1BA-1 calculations to simulate the partial sub-shell closure effects at $Z = 64$. The energy levels, wave functions and $B\left(E2\right)$ values of Sm isotopes were calculated and compared with the experimental data. It was found that agreement with the experimental data can be improved by introducing effective boson numbers in the calculations [12]. The interacting boson model had

been used to calculate the isotope shift in $^{146-154}$Sm isotopes [13]. The $E2$ transition rates in Sm isotopes with neutron number $N = 86 - 90$ were studied in the frame of the $(sd)_N^{B\ -1}f$ interacting boson model [14]. A truncation scheme of interaction boson model 2 with an *F*-spin value equal to $F_{\max}$ and $F_{\max} - 1$ was suggested [15]. Wave functions with definite *F*-spin values were constructed. The boson coefficients of fractional parentage with F spin were calculated by a group theoretical method. The model was applied to calculate the energy levels, $B(E2)$ values, and $B(M1)$ values of the Sm isotopes. It was found that the energy levels including the lowest 1^+ state can be reproduced quite well. The $B(E2)$ and $B(M1)$ values can be produced reasonably well. The low-lying decay scheme of ^{150}Sm was studied through the (n,γ) reaction. Precise energy levels as well as lifetimes measured with the gamma-ray induced Doppler technique provide important structural information of this nucleus. A comparison with the predictions of interacting boson model calculations confirm that this nucleus in near to the phase transitions between spherical and quadrupole axial symmetry shapes, lying just before the critical point on the spherical side of the shape transitions [16].

In this work, the energy levels of the even Sm isotopes were calculated by using IBA-1 and IBA-2. Sm isotopes have proton boson number $N_\pi = 6$ $(Z = 62)$ and neutron boson numbers $N_\nu = 2$ 3, 4, 5, 6 and 7 $(N = 86 - 96)$. In addition, the reduced electromagnetic transition probabilities have been also calculated $(B(E2))$.

2. Theoretical

The IBA [17,18] provides a unified description of collective nuclear states in terms of a system of interacting bosons.

2.1. The Interacting Boson Approximation Model-1 (IBA-1)

The IBA-1 Hamiltonian which was used to describe the ^{148,150,152,154,156,158}Sm nuclei in this study has the standard form as given in [17]. The calculations were done using the computer codes PHINT for energy levels and BEFM for $B(E2)$ values [19]. To obtain the values of the parameters which provide the best fit, it is necessary to calculate for each energy level the difference between the experimental and calculated values. It takes into account the sum over the squares of all these differences to find the local minimum for this summation. The least square fit procedure was used to find the best fit to the three lowest bands (ground state and γ-state bands) of the Sm isotopes under consideration. In terms of *s*- and *d*-boson operators the most general IBA Hamiltonian can be expressed as [20]

$$H_B = EPSnd + \frac{1}{2}ELL(L.L) + \frac{1}{2}QQ(Q.Q) - 5\sqrt{7}\left[\left(d^{\dagger}\tilde{d}\right)^{(3)} x\left(d^{\dagger}\tilde{d}\right)^{(3)}\right]_0^{(0)} + 15HEX\left[\left(d^{\dagger}\tilde{d}\right)^{(4)} x\left(d^{\dagger}\tilde{d}\right)^{(4)}\right]_0^{(0)}, \quad (1)$$

where

$$L.L = -10\sqrt{3}\left[\left(d^{\dagger}\tilde{d}\right)^{(1)} x\left(d^{\dagger}\tilde{d}\right)^{(1)}\right]_0^{(0)} \quad (2)$$

and

$$Q.Q = \sqrt{5}\left[\left(s^{\dagger}\tilde{d} + d^{\dagger}s\right)^{(2)} + \frac{CHQ}{\sqrt{5}} x\left(d^{\dagger}\tilde{d}\right)^{(2)}\right]_0^{(0)} \chi\left[\left(s^{\dagger}\tilde{d} + d^{\dagger}s\right)^{(2)} + \frac{CHQ}{\sqrt{5}} x\left(d^{\dagger}\tilde{d}\right)^{(2)}\right]_0^{(0)}. \quad (3)$$

Values of the interaction parameters in the IBA-1 Hamiltonian (in terms of code PHINT notation EPS, ELL, QQ, OCT and HEX) which provides the best fit to the experimental data are in MeV and are given in Table 1.

Table 1. IBA-1 parameters. All parameters are in MeV except N and CHQ.

Isotope	N	EPS	ELL	QQ	CHQ	OCT	HEX
^{148}Sm	8	0.8	–0.0008	–0.0364	0.3353	0.0044	0.0074
^{150}Sm	9	0.64	0.0004	–0.0420	–0.7267	0.0039	0.0065
^{152}Sm	10	0.35	0.0013	–0.0459	–0.6149	–0.0002	0.0003
^{154}Sm	11	0.3	0.0015	–0.0045	–0.6708	–0.0018	0.003
^{156}Sm	12	0.28	0.0015	–0.0436	–0.6708	–0.0021	0.0035
^{158}Sm	13	0.26	0.0015	–0.0431	–0.7267	–0.0018	0.003

A successful nuclear model must yield a good description not only of the energy spectrum of the nucleus but also of its electromagnetic properties. The most important electromagnetic features are the $E2$ transitions. The $B(E2)$ values were calculated using the $E2$ operator. The $E2$ transition operator must be a Hermitian tensor of rank two and therefore the number of bosons must be conserved. With these constraints the general $E2$ operator can be written as

$$B\left(E2; J_i \to J_f\right) = \left[1/\left(2J_i + 1\right)\right]\left|J_f \left\|T\left(E2\right)\right\| J_i\right|^2. \quad (4)$$

2.2. The Interacting Boson Approximation Model-2 (IBA-2)

In the first version of the interacting boson model (IBA-1) [21], no distinction is made between proton and neutron variables while describing triaxiality explicitly. This can be done by introducing the cubic terms in the boson operators [22-25]. In contrast to the recent works of [26,27] that showed that triaxiality also occurs in particularly dynamic symmetries of the IBA-2 which is distinguished between protons and neutrons.

According to [28], IBA Hamiltonian takes in different forms, depending on the regions ($SU(5)$, $SU(3)$, and $O(6)$) of the traditional IBA triangle. The Hamiltonian under consideration is in the form [24,29]:

$$H = H_{sd} + \sum \theta_L \left[d^\dagger d^\dagger d^\dagger \right]^{(L)} \left[\tilde{d}\tilde{d}\tilde{d} \right]^{(L)}, \tag{5}$$

where H_{sd} is the standard Hamiltonian of the IBA [30,31]

$$H_{sd} = \epsilon_d \, \eta_d + \kappa Q.Q + \kappa' L.L + \kappa P^\dagger.P + q_3 T_3.T_3 + q_4 T_4.T_4. \tag{6}$$

Table 2. IBA-2 parameters. All parameters are in MeV except NN, CHN and CHP.

Isotope	NN	ED	RKAP	CHN	CHP	CLN	CLP
^{148}Sm	2	0.80	–0.085	–1.00	0.3	0.0	0.0
^{150}Sm	3	0.64	–0.084	–0.95	0.3	–0.8	–0.3
^{152}Sm	4	0.35	–0.083	–0.90	0.3	0.0	–0.05
^{154}Sm	5	0.30	–0.082	–0.90	0.3	0.3	0.0
^{156}Sm	6	0.28	–0.080	–0.90	0.3	0.3	0.0
^{158}Sm	7	0.26	–0.080	–0.95	0.3	0.2	0.0

In the Hamiltonian, $\epsilon_d \, \eta_d$ and $P^\dagger.P$ terms produce the characteristics of $U(5)$ and $O(6)$ structures, respectively. Therefore, the Hamiltonian is a mixture of the $U(5)$ and $SO(6)$ chains, but not diagonal in any of the IBA chains. In the IBA-2 model the "neutrons" and "protons" degrees of freedom are taken into account explicitly. Thus the Hamiltonian [32] can be written as

$$H = \epsilon_\nu \, \eta_{d\nu} + \epsilon_\pi \, \eta_{d\pi} + \kappa Q_\pi.Q_\nu + V_{\pi\pi} + V_{\nu\nu} + M_{\pi\nu}, \tag{7}$$

where $\eta_{d\nu(\pi)}$ is the neutron *d*-boson number operator:

$$\eta_{d\rho} = d^\dagger \tilde{d}, \quad \rho = \nu, \pi, \quad d_{\rho m} = (-1)^m \, d_{\rho -m}, \tag{8}$$

where $s_\rho^\dagger$, $d_{\rho m}^\dagger$ and s_ρ, $d_{\rho m}$ represent the *s*- and *d*-boson creation and annihilation operators. The rest of the operators in equation (7) are defined as

$$Q_\rho = \left(s_\rho^\dagger \tilde{d}_\rho + d_\rho^\dagger s_\rho \right) + x_\rho \left(d_\rho^\dagger \tilde{d}_\rho \right), \quad V_{\rho\rho} = \sum_{L=0,2,4} C_{L\rho} \left(\left(d_\rho^\dagger d_\rho^\dagger \right)^{(L)} \left(d_\rho^\dagger \tilde{d}_\rho \right)^{(L)} \right)^{(0)}, \ \rho = \nu, \pi \tag{9}$$

and

$$M_{\pi\nu}; \sum_{L=1,3} \xi_L \left(d_\nu^\dagger d_\pi^\dagger \right)^{(L)} \left(d_\nu^\dagger d_\pi^\dagger \right)^{(L)} + \xi_2 \left(s_\nu \tilde{d}_\pi - s_\pi \tilde{d}_\nu \right)^{(2)} . \left(s_\nu^\dagger d_\pi^\dagger - s_\pi^\dagger d_\nu^\dagger \right)^{(2)}. \tag{10}$$

In the present case, $M_{\pi\nu}$ affects only the position of the non-fully symmetric states relative to the symmetric states. For this reason $M_{\pi\nu}$ is often referred to as the Majorana force [32].

The selection rules of choice for the total angular momentum is given as

$$\left|J_i - J_f\right| \le L_\gamma \le \left|J_i + J_f\right|. \tag{11}$$

Values of the interaction parameters in the IBA-2 Hamiltonian which provides the best fit to the experimental data are given in Table 2.

3. Results and Conclusions

The $^{148-158}$Sm isotopes have proton boson number=6 (relative to $Z = 50$) and the neutron boson number varies from 2 to 7 (relative to $N = 82$), and the parameters which were used in IBA-1 and IBA-2 are shown in Tables 1 and 2, respectively. This parameter is extremely important because it is related to the nuclear shape (prolate or oblate). The behaviors of the corresponding IBA-2 parameters depend on the neutron number are shown in Fig. 1. The energy levels which are fitted by these parameters (IBA-1 and IBA-2) are shown in Fig. 2.

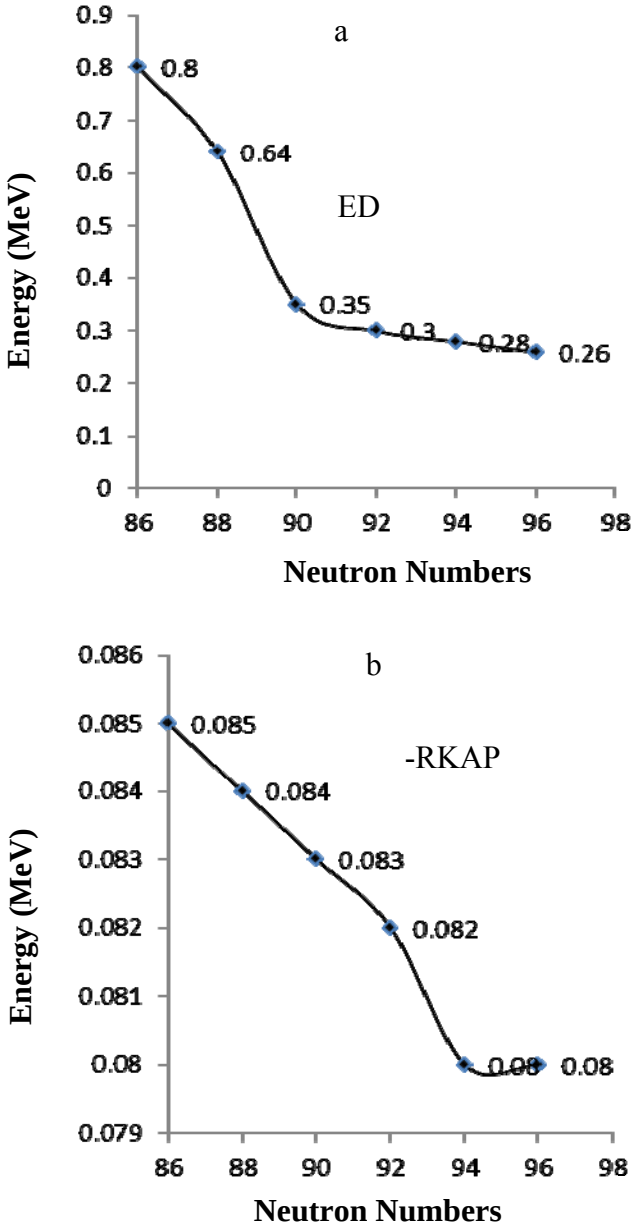

Fig. 1. Behavior of the corresponding IBA-2 parameters depending on the neutron numbers.

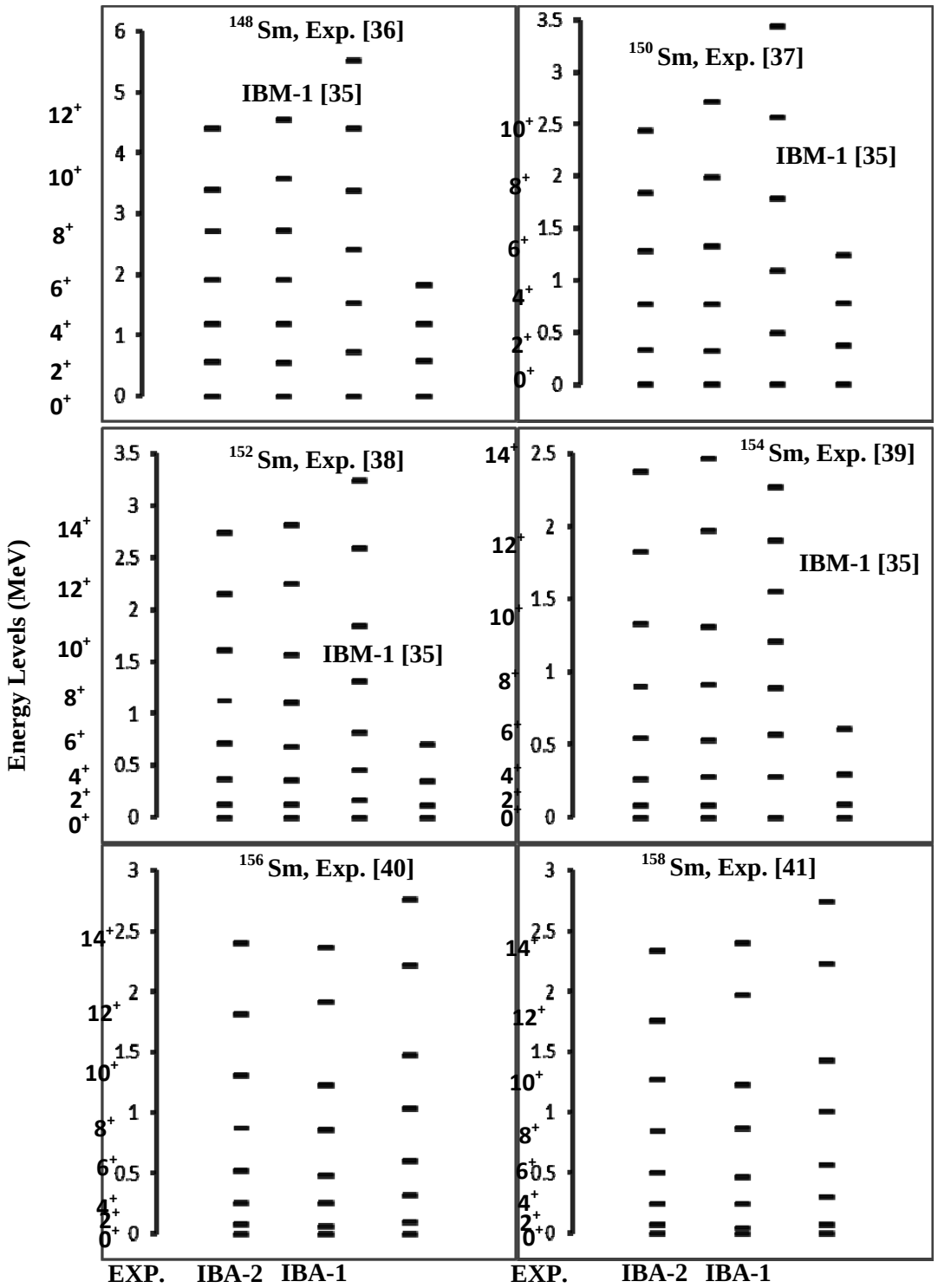

Fig. 2. Calculated and experimental energy levels (ground band) of the Sm isotopes.

Table 3 shows the comparison of estimated and experimental energy levels for $^{148-158}$Sm. As seen, the agreement between experiment and theory is quite good and the general features are reproduced well. In Fig. 3, comparison between the measured and the calculated energies of 5 levels of $_{62}$Sm are shown for even numbers of neutrons. This figure shows that the calculated energies (solid line) are in agreement with experimental data (dot symbol) over all neutron numbers. Figure 4 shows comparison between $EJ_1^{\pi}/E2_1^{+}$ ($J^{\pi}=4^{+}$ ($J^{\pi}=4^{+}$, 6^{+} and 8^{+}) measured (dot symbol) and

calculated (solid lines) energies of $_{62}$Sm for even numbers of neutrons. The compression between the calculations and the experimental data at the low neutron number is in disagreement.

Table 3. The comparison of estimated and experimental energy levels for $^{148\text{-}158}$Sm.

Isotope	Spin Parity, I^{π}	IBA-2, MeV	Exp., MeV
^{148}Sm	0_1^+	0.000	0.0 [36]
	2_1^+	0.549	0.550
	4_1^+	1.190	1.180
	6_1^+	1.914	1.905
	8_1^+	2.716	2.714
	10_1^+	3.587	3.398
	12_1^+	4.541	4.401
	0_2^+	1.093	
	2_2^+	1.194	
	4_2^+	1.918	
	6_2^+	2.719	
	8_2^+	3.590	
^{150}Sm	0_1^+	0.000	0.0 [37]
	2_1^+	0.326	0.333
	4_1^+	0.774	0.773
	6_1^+	1.325	1.278
	8_1^+	1.984	1.836
	10_1^+	2.714	2.433
	0_2^+	0.831	
	2_2^+	0.821	
	4_2^+	1.371	
	6_2^+	2.021	
	8_2^+	2.740	
^{152}Sm	0_1^+	0.000	0.0 [38]
	2_1^+	0.123	0.121
	4_1^+	0.362	0.366
	6_1^+	0.673	0.706
	8_1^+	1.104	1.125
	10_1^+	1.564	1.609
	12_1^+	2.250	2.148
	14_1^+	2.817	2.736
	0_2^+	0.728	0.684
	2_2^+	0.493	0.810
	4_2^+	0.790	1.022
	6_2^+	1.193	1.310
	8_2^+	1.633	1.666

Table 3. (Continued).

Isotope	Spin Parity, I^{π}	IBA-2, MeV	Exp., MeV
^{154}Sm	0_1^+	0.000	0.0 [39]
	2_1^+	0.081	0.081
	4_1^+	0.277	0.266
	6_1^+	0.531	0.544
	8_1^+	0.912	0.902
	10_1^+	1.309	1.333
	12_1^+	1.969	1.825
	14_1^+	2.467	2.373
	0_2^+	0.679	1.099
	2_2^+	0.474	1.177
	4_2^+	0.712	1.337
	6_2^+	1.047	1.677
^{156}Sm	0_1^+	0.000	0.0 [40]
	2_1^+	0.062	0.075
	4_1^+	0.250	0.249
	6_1^+	0.482	0.517
	8_1^+	0.860	0.871
	10_1^+	1.226	1.307
	12_1^+	1.915	1.819
	14_1^+	2.369	2.400
	0_2^+	0.681	
	2_2^+	0.482	
	4_2^+	0.696	
	6_2^+	1.010	
^{158}Sm	0_1^+	0.000	0.0 [41]
	2_1^+	0.045	0.072
	4_1^+	0.241	0.240
	6_1^+	0.466	0.498
	8_1^+	0.867	0.844
	10_1^+	1.224	1.266
	12_1^+	1.971	1.765
	14_1^+	2.400	2.334
	0_2^+	0.749	
	2_2^+	0.540	
	4_2^+	0.744	
	6_2^+	1.045	

The calculations of the electromagnetic transitions provide a good test of the nuclear structural model wave functions [33]. The proton and the neutron effective charges were determined by normalizing to the experimental values for $B\left(E2;\, 2_1^+ \to 0_1^+\right)$, $B\left(E2;\, 0_2^+ \to 2_1^+\right)$,

$B\left(E2;\, 4_1^+ \to 2_1^+\right)$, $B\left(E2;\, 2_2^+ \to 2_1^+\right)$, and $B\left(E2;\, 2_2^+ \to 0_1^+\right)$. The effective charges e_π and e_ν are needed for the electric quadrupole transition operator. The $E2$ transition operator employed in this study is defined as [33,34]

$$T\left(E2\right) = e_\pi Q_\pi + e_\nu Q_\nu, \tag{12}$$

where Q_π and Q_ν are quadrupole operators. Boson effective charges have been fitted to determine the best computational B(E2) transition values. After determining the values of boson effective charges, values of $B\left(E2;\, 2_1^+ \to 0_1^+\right)$, $B\left(E2;\, 0_2^+ \to 2_1^+\right)$, $B\left(E2;\, 4_1^+ \to 2_1^+\right)$, $B\left(E2;\, 2_2^+ \to 2_1^+\right)$, and $B\left(E2;\, 2_2^+ \to 0_1^+\right)$ have been calculated using the code PCIBAEM. The calculated $B\left(E2\right)$ values for ^{148,150,152,154}Sm isotopes are compared with experimental data as shown in Table 4.

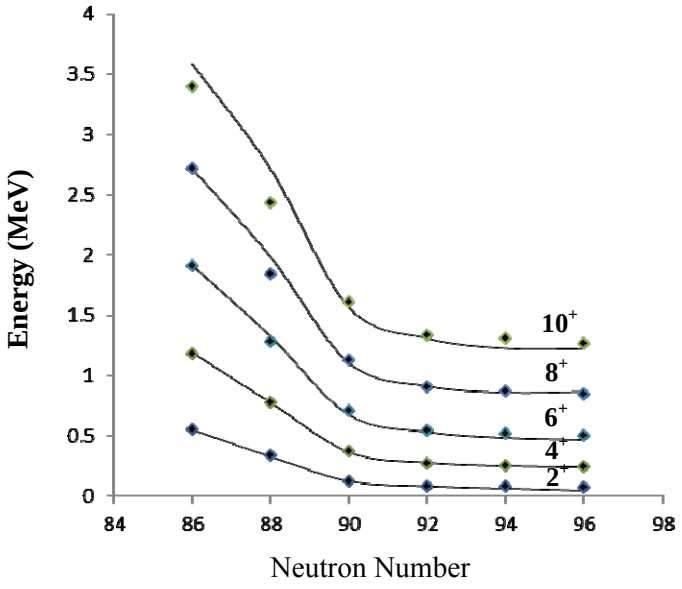

Fig. 3. Comparison between measured [36-41] and calculated energies of 5 levels of $_{62}$Sm with even number of neutrons.

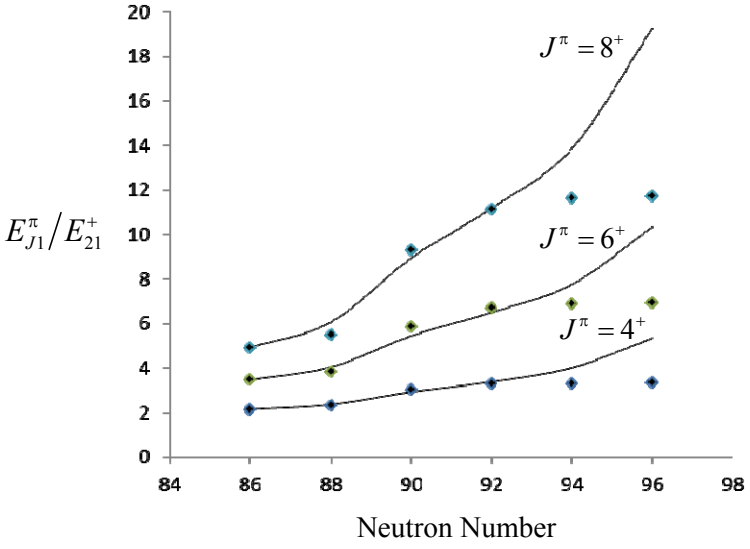

Fig. 4. Comparison between E_{J1}^{π}/E_{21}^{+} ($J^\pi = 4^+, 6^+,$ and 8^+) measured (dot symbol) [36-41] and calculated (solid lines) energies of $_{62}$Sm with even number of neutrons.

Table 4. The calculated and experimental B(E2) values for $^{148\text{-}154}$Sm. The experimental data are adopted from [15].

Isotope	Transition	B(E2) $(eb)^2$	
		Exp.	IBA-2
^{148}Sm	$2_1^+ \to 0_1^+$	1.52×10^{-1}	0.1495
	$4_1^+ \to 2_1^+$	2.46×10^{-1}	0.2633
	$2_2^+ \to 2_1^+$	3.53×10^{-2}	0.0222
	$2_2^+ \to 0_1^+$	7.91×10^{-3}	0.0067
^{150}Sm	$2_1^+ \to 0_1^+$	2.64×10^{-1}	0.2812
	$0_2^+ \to 2_1^+$	2.58×10^{-1}	0.3330
	$4_1^+ \to 2_1^+$	5.31×10^{-1}	0.6003
	$2_2^+ \to 4_1^+$	5.23×10^{-1}	0.3751
	$2_2^+ \to 0_1^+$	3.88×10^{-3}	0.0003
	$2_2^+ \to 0_2^+$	5.23×10^{-1}	0.4855
	$2_3^+ \to 4_1^+$	4.40×10^{-2}	0.00298
^{152}Sm	$2_1^+ \to 0_1^+$	6.89×10^{-1}	0.8067
	$4_1^+ \to 2_1^+$	1.00	1.2059
	$0_2^+ \to 2_1^+$	1.57×10^{-1}	0.0473
	$2_2^+ \to 4_1^+$	9.60×10^{-2}	0.0921
	$2_2^+ \to 0_1^+$	4.81×10^{-3}	0.0009
	$4_2^+ \to 2_2^+$	1.46	1.3500
	$4_2^+ \to 2_1^+$	4.82×10^{-3}	0.0244
	$2_3^+ \to 4_1^+$	3.78×10^{-3}	0.0065
^{154}Sm	$2_1^+ \to 0_1^+$	8.56×10^{-1}	0.8996
	$4_1^+ \to 2_1^+$	1.20	1.2500
	$2_2^+ \to 4_1^+$	2.11×10^{-2}	0.0600
	$2_2^+ \to 0_1^+$	4.67×10^{-3}	0.0007
	$2_3^+ \to 4_1^+$	7.35×10^{-3}	0.0013
	$2_3^+ \to 0_1^+$	1.42×10^{-2}	0.0086

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