STUDY OF GROUND BAND ABOUT THE NUCLEAR STRUCTURE VARIATION OF N-Z CHART A=100–200 MASS REGION

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Abstract: The collective nuclear structure of light and medium mass atomic nuclei is studied by the energy level structures of the ground band in the first instance. We discuss the nuclear structure using the observables of collectivity and deformation i.e. the energies of 2^+ states (E_2^+) , energy ratio $R_{4/2}$ $(=E_4^+/E_2^+)$ and the ground band moment of inertia $(1/E_2^+)$ quadrant wise, which shows remarkable correlations.

Keywords: nuclear structure, ground band, energy levels, collectivity

1. Introduction

The study of collective nuclear structure with neutron number *N*, proton number *Z*, the evolution of the basic observables in nuclei, such as the energy $E(2_1^+)$, ratio $R_{4/2}$, MoI $1/E(2_1^+)$ and decay properties of the first excited states is not well understood. The purpose of this paper is to discuss correlations of these observables, and to establish their role in classifying nuclear structure. It is interesting to make a correlation between $E(2_1^+)$, $R_{4/2}$ ($=E_4^+/E_2^+$) and MoI $1/E(2_1^+)$. Using our classification of the $R_{4/2}$, we see that low values of $R_{4/2}$ near closed shell correlate with low values of B(E2) and high values of $E(2_1^+)$, begins to drop as one proceeds through a major shell, $R_{4/2}$ rises slightly to just above 2 and the B(E2) values also begin to increase. Finally, far from magic numbers where $E(2_1^+)$ drops dramatically and becomes asymptotically constant, $R_{4/2}$ approaches 3.33 and the B(E2) values increase rapidly toward their peak values. The drop in $E(2_1^+)$ and R_4 have been taken from the website of National Nuclear Data Center, Brookhaven National Laboratory, USA [1]. In the present study we discuss the evaluation of nuclear structure across *N*-*Z* chart in section 2. The result and discussion of nuclear structure using the observables of collectivity and deformation are given in section 3. Finally, in section 4 conclusions are presented.

2. N-Z Chart

In nuclear physics, the atomic number (*Z*) is the number of protons found in the nucleus of an atom. The neutron number (*N*) is the number of neutrons found in the nucleus of an atom. The mass number (*A*) is the number of protons and neutrons in the nucleus of the atom, thus A = Z + N.

Atoms having the same atomic number Z but different neutron number N are known as isotopes and the atoms having the same neutron number but different proton number are known as isotones. The evaluation of nuclear structure across N-Z chart is the concept of magic numbers 20, 28, 50, 82 and 126. These magic numbers define the major shell gap in the sequences of shell model single particle energies. Present work is the study of nuclear structure through the study of ground state band (gsb) in even-even nuclei across the (N, Z) chart. In Fig. 1, the β stability valley across the N-Z chart for N = 50–126, Z = 50–82 is illustrated for ready reference, for the nuclear region of our interest in this paper.



Fig. 1. N-Z chart of stable nuclei.

Here Fig. 1 shows β stability line of stable nuclei in the region Z = 50-82, N = 82-126. The whole region Z = 50-82, N = 82-126 is divided into four quadrants [2]. The quadrant I has *p-p* bosons, quadrant II *h-p* bosons, quadrant III has *h-h* bosons and quadrant IV has *p-h* bosons. The nuclear structure of these nuclei in different quadrants is different [2]. In quadrant I, the structure is controlled by neutron number (*N*) and in quadrant II the structure of nuclei controlled by boson number (*N_B*) and in quadrant III the structure of nuclei depends on the proton number (*Z*).

2.1. Energy Levels

We can learn a great deal of physics of the collective nuclear structure patterns (vibrational to rotational) of atomic nuclei, by studying the energy level structures of the ground state band in the first instance. Here one can first of all look at the energy $E(2_1^+)$, of the first excited state. As we know, in the even Z-even N nuclei, the ground state spin is $I = 0^+$ and the first excited state is

 $I = 2^+$, next being $I = 4^+$. The energy of $E(2_1^+)$ itself tells us about the deformation of the nuclear core. For a harmonic vibrator the energy of the first excited collective vibration state is

$$E(2_1^+) = \operatorname{const} \times I. \tag{1}$$

So, the energy level spectrum will be of equi-spaced levels. The constant represents the vibrational frequency $\hbar\omega$ or $hv = E(2^+_1)/2$.

If the nucleus is well deformed, the level energies are given by the rotor formula:

$$E_{(I)} = \operatorname{const} \times I(I+1)/2\Im, \qquad (2)$$

where \Im is the moment of inertia of the band.

2.2. Energy Spectrum of Ground State Band

In the rotational model the shape of the nucleus is assumed to be fixed and the nuclear system rotates like a rigid structure. The energy associated with rotation would be purely kinetic and equal to $(1/2)\Im\omega^2$. Here $\Im = \sum m_p r_p^2$ is the moment of inertia which for a rigid system is given by the relation (2). A rotational spectrum having level sequence $I^{\pi} = 0^+$, 2^+ , 4^+ , 6^+ ... is called ground state band (gsb). The energy levels of ground state band are described by the expression

$$E_{(I)} = I(I+1)/2\Im,$$
(3)

where, \Im is the moment of inertia of the band. The higher-order terms of equation (3) is given by Bohr–Mottelson [3] series

$$E_{(I)} = I(I+1)/2\Im - B[I(I+1)]^2 + C[I(I+1)]^3.$$
(4)

Usually rotational nuclei are near the middle of the shell. The first excited state is called 2^+ state and second excited state is called 4^+ state. The energy of 4^+ state is 10/3 times of energy of 2^+ state. The energy of 6^+ state is 7 times the energy of 2^+ state and the energy of 8^+ is 12 times the 2^+ state. The spectrum of 160 Dy, 166 Er, 172 Yb etc. are a good example of rotational nuclei [4,5] (see Fig. 2).



Fig.2. Energy spectrum of ¹⁶⁰Dy rotational nuclei.

2.3. Systematics of the $E(2_1^+)$ States for Nuclei around $A \approx 100$ Mass Region

The systematics of the energy of first 2^+ excited level of the even-even nuclei, as well as the energy ratio $R_{4/2} = E(4_1^+)/E(2_1^+)$, in the $A \approx 100$ mass region have characteristics shown in Figs. 3 and 4, respectively, and represent transitions from vibrational to γ -soft (or rotational) nuclear collectivity. The most dramatic changes in structure are observed for Sr and Zr isotopes, it is clear in Fig. 3 by a factor of ≈ 4.0 drop in the 2_1^+ energy as neutron number N increases from 58 to 60 for Sr and drops by a factor of ≈ 6.0 for Zr. It is also observed by a sudden rise in Zr, the energy ratio $R_{4/2}$ (Fig. 4) from 1.67 (vibrational) for $N \le 58$, to near 2.66 (γ -soft) at N = 60, and ≈ 3.15 (rotational) for N = 62. This indicate a shape change transition from N = 58 to 60.



Fig. 3. Variation of $E(2_1^+)$ with *N* for A = 100 mass region.



Fig. 4. Variation of $R_{4/2}$ with N for A = 100 mass region.

3. Results and Discussion

3.1. Shape Changes in Different Quadrants

In the study of the A = 120 - 200 mass region Casten [5] grouped these nuclei into the four state viz, A = 130, $A \ge 150$, $(Z \le 64)$, $A \ge 150$, $(Z \ge 66)$ and $A \ge 190$ on empirical consideration. The Z = 50 - 82, N = 82 - 126, major shell space is partitioned into four quadrants. The quadrant I has Z = 50 - 66, N = 82 - 104 (*p*-*p*), in quadrant II, Z = 66 - 82, N = 82 - 104 (*p*-*h*), quadrant III, Z = 66 - 82, N = 104 - 126 (*h*-*h*), and quadrant IV Z = 50 - 66, N = 104 - 126, (*p*-*h*), where, p = valence particle, proton or neutron and h = hole. The quadrant IV is empty. It was also suggested in [2] that the effect of the major shell and the particle versus hole on the nuclear structure can be better seen by dividing the major shell of Z = 50 - 82, N = 82 - 126 into 4 quadrants. Also see Fig. 1 of β -stability valley.

3.2. Dependence of Energy of $E(2_1^+)$ in Different Quadrants

The dependence of energy of first 2^+ states of even-even nuclei with neutron number (*N*) tells about the nuclear core deformation. The energy $E(2_1^+)$ of the lowest 2^+ state of even-even nuclei for Z = 50-82, N = 82-104 are plotted against the neutron number *N*, we get data points scattered irregularly over the $E(2_1^+) - N$ plane as seen in Fig. 5, while the same data are plotted against *N* quadrant wise, we find that the data points are neatly rearrange and simplified as shown in Fig. 6–8, respectively. Figure 6 shows $E(2_1^+)$ values plotted against *N* for quadrant I, it is seen that the energy of 2^+ correlate tightly and the $E(2_1^+)$ values decrease rapidly when *N* increase 82 to 84 and then slow down for N = 84-90 and finally remain nearly unchanged when $N \ge 90$.



Fig. 5. The energy $E(2_1^+)$ of even-even nuclei for Z = 50-82, N = 82-104 are plotted against N.



Fig. 8. The plot of $E(2_1^+)$ vs. *N* in quadrant III.

In Fig. 7, i.e. quadrant II, the $E(2_1^+)$ values decrease rapidly with increasing *N* from 82 to 104 and the splitting of 2^+ increases with increasing *N*. In Fig. 8, i.e. quadrant III, the $E(2_1^+)$ values plotted against *N* on magnified scale. It is seen that the $E(2_1^+)$ values for Yb-Pt increases with increasing *N*, but for Hg it is upward parabolic.

3.3. Dependence of The Energy Ratio $R_{4/2}$ in Different Quadrants

The ratio $R_{4/2} = E(4_1^+)/E(2_1^+)$ is a good measure of collectivity [6], where E_2^+ and E_4^+ are energy value of 2⁺ and 4⁺ ground state. The nuclei are classified into different categories depending on $R_{4/2}$ values:

For rotational or U(5) nuclei $R_{4/2} = 2.0$,

For E(5) nuclei $R_{4/2} = 2.2$,

For γ -unstable or O(6) nuclei $R_{4/2} = 2.5$,

For *X*(5) nuclei $R_{4/2} = 2.9$,

For vibrational *SU*(3) nuclei $R_{4/2} = 3.33$.



Fig. 9. The plot of $R_{4/2}$ for quadrant I.

The Fig. 9 shows the variation of $R_{4/2}$ with neutron number N. In quadrant I $R_{4/2}$ illustrates the strong correlation of collectivity with neutron number. The Fig. 10 shows $R_{4/2}$ against neutron number for several Z values. One sees, overall, a clear onset of deformation as $R_{4/2}$ increases with N from ≈ 2 to ≈ 3.33 . It is difficult to easily see much more than this at a quick glance. A closer relation does show a crossing pattern where $R_{4/2}$ values for Nd to Dy rise from below (for N < 90) to above (for N > 90) those for Ba and Xe. The Fig. 10 shows variation of $R_{4/2}$ against neutron number N for Z = 66-82, i.e. quadrant II. The $R_{4/2}$ values of collective nuclei are ranging from 2.0 (vibrator) to 3.33 for good rotor. In quadrant III, the variation of ratio $R_{4/2}$ against N is different from the quadrants I and II in this quadrant $R_{4/2}$ decreases with increasing N (Fig. 11). These nuclei are in rotational region because $R_{4/2}$ decreases from 3.33 to 3.30 and the data point of Pt indicates a new signature of vibrational nature because $R_{4/2} = 2.5$.



Fig. 11. The plot of $R_{4/2}$ for quadrant III.

3.3.1. Shape Transitional Regions and Changes in Sub-Shell Structure

The shape change idea is illustrated in Fig. 12 which shows $R_{4/2}$ values for the well-studied A = 150 mass region. The Fig. 12 demonstrates the $R_{4/2}$ values for the rare-earth region plotted against proton number Z for several isotonic sequences. The results are dramatic and sudden change

occurs from concave to convex curves for N = 88 and N = 90. This forms a bubble pattern. The concave curves show $R_{4/2}$ values around 2.3 or less. These are typical of spherical nuclei with anharmonic vibrational excitation modes. The convex curves lie at or above $R_{4/2} \approx 2.8$ and peak near $R_{4/2} \approx 3.33$, typical of well-deformed nuclei. Here key point is that the difference in these curves is greatest near Z = 64 where the respective curves show minima and maxima. It is well known that $R_{4/2}$ is the smallest near magic numbers and maximizes in well-deformed nuclei. Thus, there is no escaping the conclusion that Z = 64 acts here as a magic number for N < 90, and that it disappears (in which case Z = 64 is near mid-shell in the Z = 50 - 82 major shell) for N > 90. This is the kind of rapid shape change referred to above. Here the variable Z for which a bubble pattern appears is the type of nucleon which experiences the sub-shell change. None of these structural ideas are new, and the role of sub-shell changes has been thoroughly discussed [7-10]. In a region of rapid change, therefore, it can happen that structure jumps over the transitional (critical) point, or, in other cases, such as the $A \approx 150$ region, one set of nuclei those with $N \ge 90$ almost exactly at the critical point. Indeed, it has led to recent descriptions of nuclei in first order phase transitional regions by so-called critical point symmetries [11,12] such as X(5), and the realization [13,14] that nuclei such as ¹⁵²Sm reflect the predictions of such descriptions.



Fig. 12. Variation of energy ratio $R_{4/2}$ with proton number (Z) for fixed N.

3.4. The Ground State Band Moment of Inertia

In the Bohr–Mottelson collective model [3], the nucleus is treated as a rotating vibrating nuclear core. For the axially symmetric deformed core, the rotation takes place about the short axes. Then the nucleus has a moment of inertia \Im , which is displayed in the level energy equation

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$$E_{(I)} = \hbar^2 I (I+1) / 2\Im.$$
 (5)

The moment of inertia \Im is a function of the nuclear mass and the quadrupole deformation (β) . A good deal of information on the level spectra of the atomic nuclei can be derived from the study of the MoI \mathfrak{I} . The variation of MoI with atomic mass number A was displayed in [3]. In the earlier study [15] of the SU(3) symmetry in the Interacting Boson model [6], the linear dependence of the ground state MoI on the $N_p N_n$ product of valence nucleon pairs was demonstrated. Gupta et al. [16] illustrate the β -band and γ -band moments of inertia versus the g-band moment of inertia yields a diagonal relationship and noted that even for shape transitional nuclei, the shape does not change significantly with low spin and low energy vibration yielding equal moment of inertia of the three lower bands. Recently Gupta et al. [17] presented the partial results of display of variation of MoI of rare earths with varying N, Z quadrant wise and illustrate the formation of multiplets. In the present work, we study the variation of MoI with N, Z in a different way. We limit this study to the 50 < Z < 82, 82 < Z < 126 region of even-Z even-N nuclei. The major shell space of Z = 50 - 82, N = 82 - 126 can be divided into four quadrants. The β -stability line on the N-Z nuclear chart passes across the quadrants I, II and III diagonally. The quadrant IV is vacant. Gupta et al. [2] illustrated that the nuclei in quadrant II (Z = 66 - 78, N = 82 - 104), may be grouped into F-spin multiplets [18], in quadrant I (Z = 55-60, N = 82-104) into isotonic multiplets and in quadrant III (N > 104) into isotopic multiplets, having nearly identical energy level structure. Here we illustrate these systematics by plotting the inverse of the energy of $I = 2_1$ state, i.e. $X = 1/E(2_1^+)$ which is related to the moment of inertia (3) of the nucleus $\Im = 3X$. At N = 82 closed shell, \Im (=3X) is almost zero. As the neutron pairs are added, \Im increases slowly, and at N = 88 - 90 there is a sharp increase which continues up to N = 92 (Fig. 13).



Fig. 13. Plot of $1/E_2$ vs. *N* for quadrant I.



Fig. 14. Plot of $1/E_2$ vs. *N* for quadrant II.



Fig. 15. Plot of $1/E_2$ vs. N_B for quadrant II.



Fig. 16. Plot of $1/E_2$ vs. *N* for quadrant III.

Thereafter, there is almost saturation in the values. Here we have linked \Im (or 3X) values of same Z. It is also evident here that the data at each N just lie almost together over each other (see Fig. 13). That is for differing Z, the values of \Im for each N are nearly the same. Thus the \Im versus N curves vividly illustrate the occurrences of the isotonic multiplets in quadrant I as noted earlier in [2]. Also the shape phase transition at N = 88 - 90 for Nd, Sm, Gd, Dy is transparently exhibited. The shape transition in Ce is less sharp and in Ba is the minimum. In Fig. 14, the \Im (or $X = 1/E_2$) values for the Dy-Hf nuclei are depicted. Again, the data of same Z are linked. Here the pattern of the J values is very different. The MoI value \Im ($X = 1/E_2$) is decreasing with increasing Z (Dy to Hf). The total boson number N_B also determines the nuclear structure, as observed in [18]. The *F*-spin quantum number is related to N_B by $F = N_B/2$. A plot of \Im versus N_B (Fig. 15) shows that for each N_B the value of \Im is almost constant in quadrant II. The data for each N_B almost overlap each other. This illustrates the formation of F-spin multiplets here in quadrant II [15, 18].

Next we consider the values of \Im (or $X = 1/E_2$) in Dy-Pt (and Hg) with N > 104 in quadrant III (figure 16). Here the $\Im = 1/E_2^+$ pattern is different. In Yb, Hf there is fair constancy, i.e isotopic multiplets are formed. In W, Os first there is some rise up to N = 108, then slow fall. So, the constancy is somewhat weaker. As is well known, the Os nuclei here tend to be γ -soft. In Pt the same MoI value persists for N = 104 - 108. After there is continuous fall. In Hg the value of \Im is constant. Over all in quadrant III, the MoI is decreasing with increasing Z.

4. Conclusion

In conclusion, we find that the quadrant wise presentation of observables of collectivity and deformation, i.e. the energies of 2^+ states (E_2^+) , energy ratio $R_{4/2}(=E_4^+/E_2^+)$ and the ground state band moment of inertia $(1/E_2^+)$. In quadrants I and II, the energy of 2_1^+ level decreases with increasing collectivity and deformation, while $R_{4/2}$ increases from ≈ 2.0 (for harmonic vibrators) to ≈ 3.33 (for good rotor). In quadrant III the variation of energy of 2_1^+ and $R_{4/2}$ is different from quadrants I and II. This indicates that in this region the nuclear structure depends on Z. The quadrant wise presentation of moment of inertia (MoI), vividly displays the formation of isotonic multiplets in quadrant I, *F*-spin multiplets in quadrant II and the isotopic multiplets in quadrant III. Also in each case the role of *N*, *Z* is well evident. This agrees with known variation of nuclear deformation in the rare earths in this region of nuclear chart. In quadrant II, the line of beta stability runs almost diagonally, i.e. parallel to the constant total boson number N_B (proton hole bosons, and neutron particle bosons) lines running diagonally in this quadrant.

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