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INVESTIGATION OF ENERGY LEVEL OF EVEN-EVEN $^{104\text{-}112}$Cd ISOTOPES UNDER THE FRAMEWORK OF IBM-1

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Abstract–The interacting boson model (IBM-1) has been used in the study of the nuclear structure of even-even $^{104\text{-}112}$Cd isotopes. The values of the parameters in the IBM-1 Hamiltonian which yield the best fit to the experimental energy spectrum. It is shown that there is a good agreement between the results found and with the experimental ones.

***Keywords*:** energy level, ineracting boson model, Cd isotopes

1. Introduction

The nucleus consists of many nucleons (protons and neutrons). Each nucleon is interacting with all others and moving within a complex structure. A model of the atomic nucleus has to be able to describe nuclear properties such as spins and energies of the lowest levels, decay probabilities for the emission of gamma quanta's, probabilities (spectroscopic factors) of the transfer reaction, multipole moments and so forth. Those models outlined from which the IBM comes. The interacting boson model of Arima and Iachello [1-6] has become widely accepted as a tractable theoretical scheme of correlating, describing and predicting low-energy collective properties of complex nuclei. The basic idea of this model is based on the assumption which assumes that the low-lying collective states in even-even nuclei could be described as states of a given (fixed) number N of bosons. Each boson could occupy two levels; one with angular momentum $L=0$ (s-boson) and the other, usually with higher energy, with $L=2$ (d-boson). In the original form of the model known as the Interacting Boson Model (IBM-1), proton and neutron-boson degrees of freedom are not distinguished. Cadmium isotopes have been the subject of studies in nuclear-structure physics [7-9]. Even-even Cadmium isotopes are part of an interesting region beyond the closed proton shell at $Z=50$, while the number of neutrons in the open shell is much larger.

Our aim of the present work is to investigate even-even 104^{-112}Cd isotopes in (U5) region and calculate energy levels within framework of Interacting Boson Model.

2. Theoretical Basics of Interacting Boson Model (IBM-1)

The IBM-1 Hamiltonian can be expressed as [10]

$$H = \varepsilon \hat{n}_d + a_0 \hat{P}.\hat{P} + a_1 \hat{L}.\hat{L} + a_2 \hat{Q}.\hat{Q} + a_3 \hat{T}_3.\hat{T}_3 + a_4 \hat{T}_4.\hat{T}_4. \quad (1)$$

Here a_0, a_1, a_2, a_3 and a_4 are strength of pairing, angular momentum and multipole terms. The Hamiltonian as given in (1) tends to reduces to three limits: the vibration U(5), γ-soft O(6) and the rotational SU(3) nuclei, starting with the unitary group U(6) and finishing with group O(2) [11]. In U(5) limit, the effective parameter is ε, in the γ-soft limit, O(6), the effective parameter is the pairing a_0; and in the SU(3) limit, the effective parameter is the quadrupole a_2.

The eigenvalues for the U(5) limit is given by [12]

$$E(L, K_1, K_4, K_5) = \varepsilon \hat{n}_d + K_1 n_d (n_d + 4) + K_4 \nu(\nu + 3) + K_5 L(L+1), \quad (2)$$

where ν is the *d*-boson seniority and hence the number of *d*-bosons not paired to angular momentum zero. The parameters ε, K_1, K_4 and K_5 represent the strength of their terms. However, it turn out that for fixed boson number *N*, only one of the one-body terms and five of two-body terms are interdependent, as it can be seen by noting

$$N = n_s + n_d. \quad (3)$$

3. Results and Discussion

To produce the low-lying energy levels of a nucleus using the IBM-1 model, it is necessary to specify the symmetry shape of the nucleus, which can be predicted from the energy ratio between $R = E4_1^+ / E2_1^+$ energy levels. $R4/2$ has a limit value of 2 for the vibrational nuclei U(5), 2.5 for γ-unstable nuclei O(6) and finally 3.33 for rotational nuclei SU(3).

A computing program was written by using the MatLab 6.5 environment to calculate the energies of some states of the ground state band by applying the eigenvalue expression for the vibrational nuclei (equation 2) which is suitable for the nuclei under the investigation.

However, the seniority number in the ground state band is given by $\nu = n_d = L/2$, therefore the values of this number for 2_1^+ and 8_1^+, as example will be as following: $\nu = n_d = L/2 = 1$ for 2_1^+ state and $\nu = n_d = L/2 = 4$ for 8_1^+ state. The four unknown parameters ε, K_1, K_4 and K_5 were determined by solving Eqs. (1)–(3) for four energy states, two from the states of the ground state band, one from the beta band and the fourth from the gamma band. The evaluated parameters were used to obtain the energy spectrum in this work.

The suitable parameters for each nucleus at the evolving states are determined using Eq. (2). Table 1 shows the values of these parameters that have been used to calculate the energy of the yrast-line states for the isotopes under study. This is then compared with the experimental values [13-17]. Tables 2–6 show these results with the percentage error for each isotope.

The values of the first excited state $E2_1^+$ and the ratio $R = E4_1^+ / E2_1^+$ show that $^{104-112}$Cd(*e*–*e*) isotopes are vibrational.

Table 1. Boson number and calculated parameters in (keV) for $^{104\text{-}112}$Cd even-even isotopes

A	N	ε	K_1	K_4	K_5
104	4	495.34	50.78	−19.57	−2.16
106	5	707.99	31.61	−35.76	2.81
108	6	772.58	14.63	−39.52	9.15
110	7	878.79	−23.72	−22.63	14.79
112	8	891.99	−46.67	−14.61	16.89

Table 2. Experimental and theoretical excitation energies (in keV) of levels for ^{104}Cd nucleus

Nucleus	J^{π}_{gsb}	E^{ex}_{gsb} [13]	E^{th}_{gsb}	Δ%
^{104}Cd	0^+	0	0	0
	2^+	658(20)	658.0	0
	4^+	1492.1(4)	1361.2	8.7
	6^+	2370.2(4)	2109.5	10.9
	8^+	2903.1(4)	2903.0	0.003

Table 3. Experimental and theoretical excitation energies (in keV) of levels for ^{106}Cd nucleus

Nucleus	J^{π}_{gsb}	E^{ex}_{gsb} [14]	E^{th}_{gsb}	Δ%
^{106}Cd	0^+	0	0	0
	2^+	632.64(4)	739.8	16.9
	4^+	1493.78(5)	1493.8	0.003
	6^+	2491.66(6)	2261.9	9.2
	8^+	3044.13(7)	3044.13	0

Table 4. Experimental and theoretical excitation energies (in keV) of levels for ^{108}Cd nucleus

Nucleus	J^{π}_{gsb}	E^{ex}_{gsb} [15]	E^{th}_{gsb}	Δ%
^{108}Cd	0^+	0	0	0
	2^+	632.98(16)	742.5	17.3
	4^+	1508.46(23)	1508.5	0.002
	6^+	2994.1(2)	2397.8	19.9
	8^+	3110.49(10)	3110.6	0.003

Table 5. Experimental and theoretical excitation energies (in keV) of levels for ^{110}Cd nucleus

Nucleus	J^{π}_{gsb}	E^{ex}_{gsb} [16]	E^{th}_{gsb}	Δ%
^{110}Cd	0^+	0	0	0
	2^+	776.55(14)	758.4	2.3
	4^+	1637.9(3)	1542.4	0.05
	6^+	2230.8(3)	2352.1	−5.4
	8^+	2718.6(3)	3187.3	−17.2

Table 6. Experimental and theoretical excitation energies (in keV) of levels for ^{112}Cd nucleus

Nucleus	J^{π}_{gsb}	E^{ex}_{gsb} [17]	E^{th}_{gsb}	Δ%
^{112}Cd	0^+	0	0	0
	2^+	617.52(10)	701.5	−13.5
	4^+	1415.58(12)	1415.6	−0.001
	6^+	2168.03(15)	2142.2	1.19
	8^+	2881.26(16)	2881.3	0.001

4. Conclusion

The low-lying experimental energy levels of the ground state bands in all considered Cd-isotopes are very well reproduced in the calculated results of the IBM-1 compared to those of other models.

The analysis of the calculated results for the low-lying positive parity energy spectra obtained within the framework of interacting boson approximations shows that the Cd-isotopes under study are considered as vibrational nuclei and close to U(5) symmetry.

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