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A FEW PSF-BASED COROLLARIES OF OPTICAL SYSTEMS APODISED ASYMMETRICALLY WITH TWO-DIMENSIONAL COMPLEX PUPIL FILTERS

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Abstract–A few important corollaries of asymmetric apodized point spread function like the strehl ratio (SR), the total transmission factor (ϒ) and the half power diameter (HPD) have been investigated by introducing three level asymmetric pupil filters. For various amount of amplitude apodisation, we have observed that as semicircular ring width increases the value of above corollaries decreases and also we have evaluated half power diameter with lower values than that of airy case for higher degree of apodisation. This study aims high resolution or to improve selected aspects of imaging performance of the optical system.

Keywords**:** asymmetric apodisation, point spread function, resolution

1. Introduction

In order to improve the resolution of an optical system, one must change the pupil function with suitable apodization. Apodisation is a technique that modifies the imaging properties of an optical system such that the system impulse response does not show ringing by manipulating its entrance pupil [1]. In 1991 and 1997 Cheng and Siu [2,3], introduced asymmetric apodization and succeed in achieving low side lobes and a sharp central peak on one side termed as 'good side' and a broader central peak with enhanced side-lobes on the other side named as 'bad-side'. In fact, the good-side was obtained at the cost of the bad-side. In further continuation of their work [4] they obtained improved side-lobe suppression. Their works can be considered as a major breakthrough in apodization studies.

Lord Rayleigh [5] was first who pointed out importance of primary corollaries in diffraction pattern and derived formulas. Strehl ratio is used to determine the seriousness of aberration; it is unity for perfect optical system (Airy) without apodisation. Marechals's tolerance criterium specifies a value of 0.8 or more for strehl ratio so that system can be considered as one of 'high-quality' [6]. The total transmission factor or the past – flux ratio gives an indication of the total amount of light flux that passes into image space for given apodizer as compared to that of Airy case. It decreases with apodisation of apodizer in central part of pupil filters. K. Surendar, P.K. Mondal [7-9] studied corollaries of PSF apodised with using lanczos, Hanning amplitude filters.

The half power diameter is the diameter of the PSF curve at 50% of its peak relative intensity [10]. Dr.T.Borlinghaus [11] gave brief idea on influence of half power diameter on confocal optical section thickness. It becomes important image assessment parameter when the PSF value does not attain zero minimum. Previous corollary studies proved that HPD is a unique quality criterion of PSF, symmetrically apodised. But in our present work we are dealing with asymmetric PSF in which we found that PSF shifts into messy side as degree of apodization increases and we considered HPD as a better assess parameter to evaluate asymmetrically apodised optical systems. This is one of the important aspects of our current study. In our previous paper [12] we investigated the focusing properties of asymmetrically apodized optical systems for a slit, circular aperture and we showed how to achieve low side lobes and steep principal maximum simultaneously. In present study, we have evaluated the above mentioned corollaries for optical systems apodised with asymmetric complex pupil filters.

2. Theory

PSF based corollaries apodised by complex pupil filter consists of three zones viz. two semicircular edge rings and a central zone with an amplitude apodizer, corresponding phases are i, $-i$ and 0. β is the apodization parameter controlling the degree of non-uniformity of the transmittance $\left(1-4\beta\rho^2+4\beta\rho^4\right)$ over zone of radius $(1-b)$. The range of values it takes is $0\le\beta\le 1$. It is clear that for $\beta=0$, the transmittance of this zone is uniform. Transmittance over rest of the two zones of circular aperture of unit radius is unity. Here b is the certain width of semi-circular edge ring. The structure of complex pupil filter can be seen in Fig. 1 in detail.

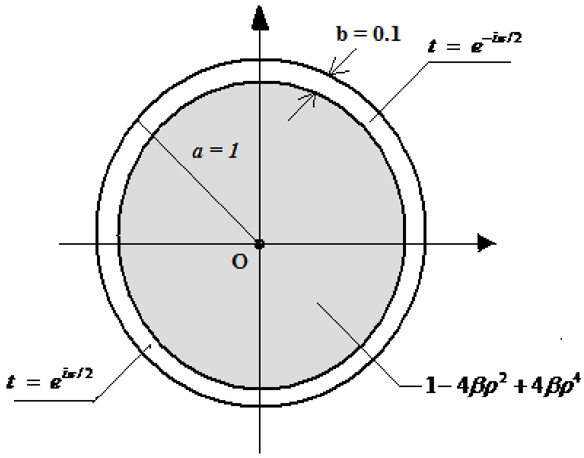

Fig. 1. Structure of three-zone complex pupil filter.

The normalized complex field amplitude $A(u,\phi)$ on the focal region is equal to the sum of three amplitudes contributing by three zones of pupil function.

The diffraction field amplitude contributing by circular aperture of radius $(1-b)$:

$$A_0(u,\phi) = \int_0^{1-b}\int_0^{2\pi}\left(1-4\beta\rho^2+4\beta\rho^4\right)\exp\left(iu\rho\cos\left(\phi-\varphi\right)\right)\rho\,d\rho\,d\varphi, \qquad (1)$$

where $u = k\sin\theta = (2\pi/\lambda)\sin\theta$. Here ρ is the normalized radial coordinate in pupil plane.

The diffraction field amplitudes contributing by left and right semicircular ring are

$$A_1(u,\phi) = i\int_{1-b}^{1}\int_{\pi/2}^{3\pi/2} \exp(iu\rho\cos(\phi-\varphi))\rho\, d\rho\, d\varphi, \tag{2}$$

$$A_2(u,\phi) = -i\int_{1-b}^{1}\int_{-\pi/2}^{\pi/2} \exp(iu\rho\cos(\phi-\varphi))\rho\, d\rho\, d\varphi. \tag{3}$$

Hence, the field amplitude can be written as

$$A(u,\phi) = A_0(u,\phi) + A_1(u,\phi) + A_2(u,\phi). \tag{4}$$

The PSF intensity $I(u) = |A(u,\phi)|^2$, which is real measurable quantity.

The strehl ratio (SR) for aberration-free apodised systems defined as ratio of central irradiance (intensity) with non-uniform pupil to that with the uniform (Airy) function. In terms of pupil function it is given as

$$SR = 4\left[\int_0^1 f(\rho)\rho d\rho\right]^2. \tag{5}$$

Pupil function considered here is an asymmetric complex pupil function given by

$$f(\rho) = \begin{Bmatrix} -i\exp\left(iu\rho\cos(\phi-\varphi)\right) & 1-b \le \rho \le 1,\ -\pi/2 \le \varphi \le \pi/2 \\ \left(1-4\beta\rho^2+4\beta\rho^4\right)\exp\left(iu\rho\cos(\phi-\varphi)\right) & 0 \le \rho \le 1-b,\ 0 \le \varphi \le 2\pi \\ i\exp\left(iu\rho\cos(\phi-\varphi)\right) & 1-b \le \rho \le 1,\ \pi/2 \le \varphi \le 3\pi/2 \end{Bmatrix}.$$

The total transmission factor ϒ is given as,

$$\Upsilon = \int_0^{\delta} |A_F(o,u)|^2\, u du \Big/ \int_0^{\delta} |A_A(o,u)|^2\, u du, \tag{6}$$

where $A(o,u)$ is the complex amplitude of diffraction field. The subscripts F and A refer to cases with and without (Airy: $\beta = 0,\ b = 0$) apodisation.

In terms of pupil function, the total transmission factor ϒ is expressed as

$$\Upsilon = 2\int_0^1 |f(\rho)|^2 \rho d\rho. \tag{7}$$

The half power diameter is found as the distance u between the two points, where the intensity becomes half of the central maximum. The direct corollary of apodised PSF can be measured from resultant intensity $I(u)$ of image plane.

3. Results and Discussions

All the PSF-based corollaries mentioned above have been obtained from Eqs. (4–7) are functions of diffraction coordinate u varying from −12 to +12 by employing twelve-point gauss quadrature numerical method of integration. An iterative method has been developed and applied to obtain corollaries of PSF apodised by complex pupil filters.

From listed values in Table 1, it is clear that the Strehl ratio and the total transmission factors decrease with the degree of apodisation. If pupil filter becomes Airy ($\beta = 0,\ b = 0$) SR and ϒ possess maximum value (equal to 1). For various amount of apodisation parameter (β), as the semicircular ring width *b* increases from 0 to 0.1ϒ and SR gradually decrease. This is clearly evident from Figs. 2 and 3 and it is important to notice that Marechals's criteria can be satisfied for values of *b* ranging between 0 to 0.06, when central circular region is unapodized. For zero degree of apodisation, ϒ obtained optimum values. When apodisation in the central circular region of the complex pupil function increases, optical side lobes suppression increases on good side of PSF. Then amount of light flux appeared in image plane decreases with degree of apodisation. This can be seen in more detail from values listed in Table 1.

Table 1. Strehl ratio, Total transmission factor for various b and β combinations.

b	β = 0		β = 0.2		β = 0.4		β = 0.6		β = 0.8		β = 1	
	SR	ϒ	SR	ϒ	SR	ϒ	SR	ϒ	SR	ϒ	SR	ϒ
0	1	1	0.7511	0.7511	0.5378	0.5378	0.36	0.36	0.2178	0.2178	0.1111	0.1111
0.01	0.9606	0.9643	0.7173	0.7205	0.5094	0.5122	0.3371	0.3393	0.2002	0.202	0.0987	0.1001
0.02	0.9224	0.9293	0.685	0.6911	0.483	0.4882	0.3161	0.3205	0.1845	0.188	0.0881	0.0908
0.03	0.8853	0.8951	0.6543	0.663	0.4582	0.4658	0.2969	0.3034	0.1705	0.1758	0.0789	0.0831
0.04	0.8493	0.8615	0.625	0.6359	0.4351	0.4447	0.2794	0.2877	0.1581	0.1651	0.0711	0.0769
0.05	0.8145	0.8286	0.5971	0.6099	0.4134	0.4248	0.2633	0.2735	0.147	0.1558	0.0643	0.0718
0.06	0.7807	0.7964	0.5704	0.5848	0.393	0.4061	0.2486	0.2604	0.1371	0.1477	0.0585	0.0678
0.07	0.7481	0.7648	0.545	0.5606	0.374	0.3885	0.2351	0.2485	0.1283	0.1406	0.0536	0.0647
0.08	0.7164	0.7339	0.5206	0.5373	0.356	0.3718	0.2226	0.2375	0.1204	0.1344	0.0493	0.0625
0.09	0.6857	0.7037	0.4974	0.5147	0.3391	0.356	0.2111	0.2274	0.1133	0.1291	0.0457	0.0609
0.1	0.6561	0.6741	0.4751	0.493	0.3233	0.341	0.2006	0.2182	0.107	0.1245	0.0426	0.0599

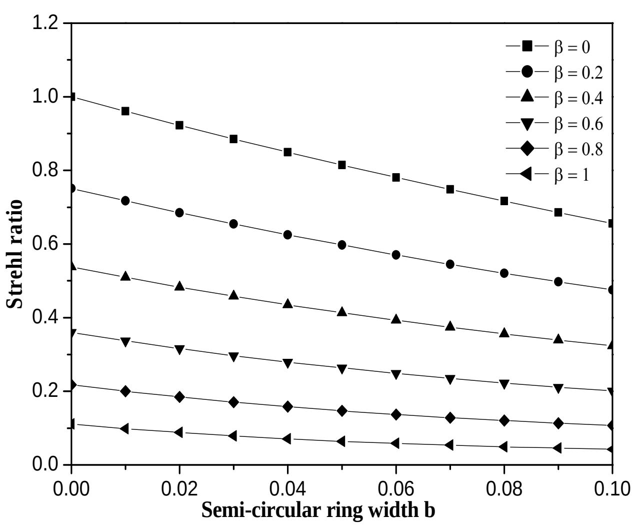

Fig. 2. Strehl ratio is function of *b* and β values.

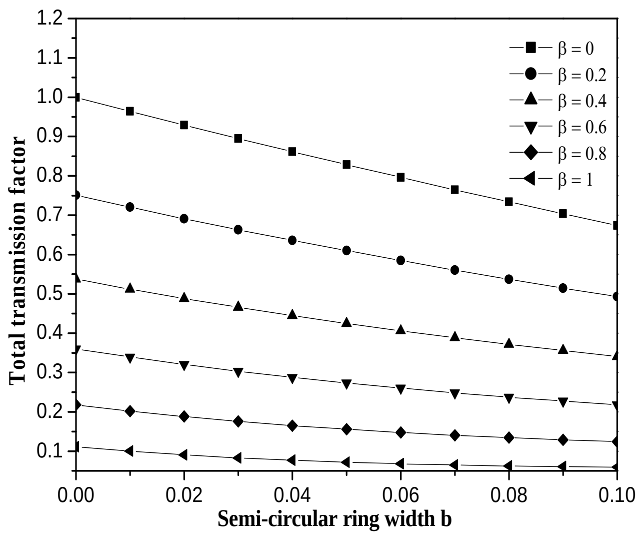

Fig. 3. Total transmission factor is function of β.

From listed values in Table 2, a quality criterion HPD is a function of semicircular ring width *b* for different values of apodization parameter β depicted in Fig. 4 for various amount of apodization parameter β. It shows that as the semicircular ring width *b* increases from 0 to 0.1 HPD also increases and then decreases with semicircular ring width. For $\beta = 0$, as *b* increases from 0 to 0.03, HPD increases from 3.24 to 3.28 units and then for higher values of b HPD decreases. For airy case HPD is 3.24 units. As degree of apodization increases for certain semicircular ring width *b*, HPD obtained lower values than that of airy case. This can be seen in more detail from listed values in Table 2. This criterion is important when image quality is limited by factors external to the optical system. For large amounts of aberrations, it is sensitive. The well-established Rayleigh criterion becomes inapplicable in dealing with non-zero minima of asymmetric PSF. In such cases the asymmetrically apodised optical system may be evaluated by its HPD.

Our further study is introducing new quality criterion named half-width at half-maximum (HWHM), to discuss distribution of half-maximum area on either side of asymmetric PSF or to evaluate asymmetrically apodised optical system in best way.

Table 2. HPD of apodised PSF for all values of β and *b*

b	β = 0	β = 0.2	β = 0.4	β = 0.6	β = 0.8	β = 1
0	3.2327	3.2357	3.2399	3.2460	3.2558	3.2739
0.01	3.2603	3.2668	3.2754	3.2873	3.3046	3.3313
0.02	3.2778	3.2835	3.2898	3.2954	3.2959	3.2708
0.03	3.2839	3.2843	3.2809	3.2678	3.2290	3.1172
0.04	3.2781	3.2689	3.2494	3.2087	3.1216	2.9371
0.05	3.2609	3.2385	3.1991	3.1279	2.9987	2.7745
0.06	3.2332	3.1958	3.1354	3.0370	2.8791	2.6416
0.07	3.1967	3.1439	3.0645	2.9454	2.7721	2.5363
0.08	3.1534	3.0865	2.9916	2.8589	2.6801	2.4533
0.09	3.1056	3.0268	2.9206	2.7805	2.6024	2.3874
0.1	3.0553	2.9671	2.8536	2.7109	2.5373	2.3350

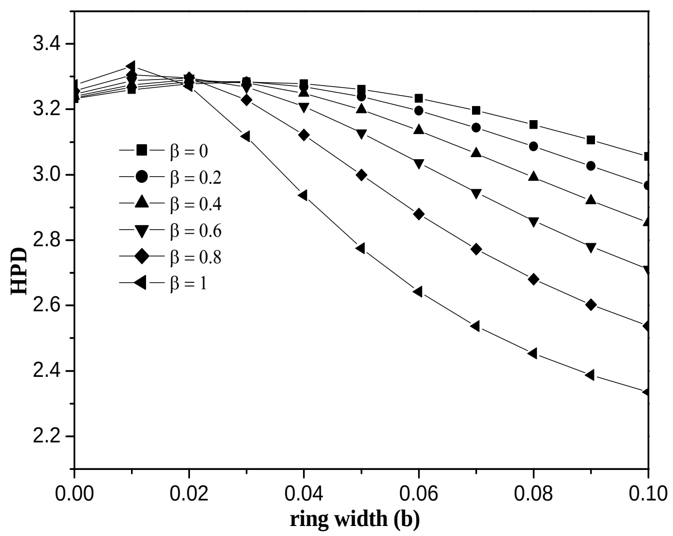

Fig. 4. HPD (Half Power Diameter) as function complex pupil filter of ring width (b) for different values of β.

4. Conclusions

Finally we may conclude that our investigations aimed high resolution by PSF-based corollaries has been evaluated by considering three-level complex pupil filters. By performing asymmetric apodization we obtained HPD lower than that of airy case. When central circular region is unapodized, for marechal criteria can be satisfied low values of b varying from 0 to 0.06 are to be considered. In this way we succeeded to improve imaging performance of optical system using asymmetric thee level complex pupil filter.

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