Armenian Journal of Physics, 2011, vol. 4, issue 4, pp. 193-199

$\bar{B} \to X_s\gamma\gamma$ DOUBLE DIFFERENTIAL DECAY RATE

H. M. Asatrian, A. Yeghiazaryan

Alikhanian National Scientific Laboratory, Yerevan, Armenia

Received 9 November, 2011

Abstract–We investigate the dependence of the contributions of interferences of the operators O_1, O_2 and O_7 to the double differential decay width $d\Gamma/(ds_1\, ds_2)$ for $\bar{B} \to X_S\gamma\gamma$ decay from the renormalization scale. Analytic expressions for all the contributions are presented, up to α_S order for (O_7, O_7) interference, and the tree level contributions for the interferences (O_1, O_1), (O_1, O_2), (O_1, O_7), (O_2, O_2), (O_2, O_7).

1. Introduction

Inclusive rare B-meson decays are known to be a unique source of indirect information about physics at scales of several hundred GeV. In the Standard Model (SM) all these processes proceed through loop diagrams and thus are relatively suppressed. In the extensions of the SM the contributions stemming from the diagrams with "new" particles in the loops can be comparable or even larger than the contribution from the SM. Thus getting experimental information on rare decays puts strong constraints on the extensions of the SM or can even lead to a disagreement with the SM predictions, providing evidence for some "new physics".

To make a rigorous comparison between experiment and theory, precise SM calculations for the (differential) decay rates are mandatory. While the branching ratios for $\bar{B} \to X_S\gamma$ [1] and $\bar{B} \to X_S\ell + \ell$ are known today even to next-to-next-to-leading logarithmic (NNLL) precision (for reviews, see [2,3]), other branching ratios, like the one for $\bar{B} \to X_S\gamma\gamma$, are only known to leading logarithmic (LL) precision in the SM [4-7].

The starting point of our calculation is the effective Hamiltonian, obtained by integrating out the heavy particles in the SM, leading to

$$H_{eff}\left(b \to s\gamma\right) = \frac{4G_F}{\sqrt{2}}\lambda_t \sum_{j=1}^{8} C_j\left(\mu\right) O_j\left(\mu\right)$$

where the operators O_j are defined as

$$O_1 = \left(\overline{c_{L\beta}}\gamma^\mu b_{L\alpha}\right)\left(\overline{s_{L\alpha}}\gamma_\mu c_{L\beta}\right),\ O_2 = \left(\overline{c_{L\alpha}}\gamma^\mu b_{L\alpha}\right)\left(\overline{s_{L\beta}}\gamma_\mu c_{L\beta}\right),$$

$$O_3 = \left(\overline{s_{L\alpha}}\gamma^\mu b_{L\alpha}\right)\left[\left(\overline{u_{L\beta}}\gamma_\mu u_{L\beta}\right) + \cdots + \left(\overline{b_{L\beta}}\gamma_\mu b_{L\beta}\right)\right],\ O_4 = \left(\overline{s_{L\alpha}}\gamma^\mu b_{L\beta}\right)\left[\left(\overline{u_{L\beta}}\gamma_\mu u_{L\alpha}\right) + \cdots + \left(\overline{b_{L\beta}}\gamma_\mu b_{L\alpha}\right)\right],$$

$$O_5 = \left(\overline{s_{L\alpha}}\gamma^\mu b_{L\alpha}\right)\left[\left(\overline{u_{R\beta}}\gamma_\mu u_{R\beta}\right) + \cdots + \left(\overline{b_{R\beta}}\gamma_\mu b_{R\beta}\right)\right],\ O_6 = \left(\overline{s_{L\alpha}}\gamma^\mu b_{L\beta}\right)\left[\left(\overline{u_{R\beta}}\gamma_\mu u_{R\alpha}\right) + \cdots + \left(\overline{b_{R\beta}}\gamma_\mu b_{R\alpha}\right)\right],$$

$$O_7 = \left(e/16\pi^2\right)\overline{s_\alpha}\sigma^{\mu\nu}\left[m_b\left(\mu\right)R + m_s\left(\mu\right)L\right]b_\alpha F_{\mu\nu},$$

$$O_8 = \left(g_s/16\pi^2\right)\overline{s_\alpha}\sigma^{\mu\nu}\left[m_b\left(\mu\right)R + m_s\left(\mu\right)L\right]\left(\frac{\lambda^A_{\alpha\beta}}{2}\right)b_\beta G^A_{\mu\nu},$$

where $m_b(\mu)$ is the running *b*-quark mass in the MS-scheme at the renormalization scale μ, e and g_S are the electromagnetic and strong coupling constants, $F_{\mu\nu}$ и $G^A_{\mu\nu}$ are the corresponding field strength tensors, $L=(1-\gamma_5)/2$ and $R=(1+\gamma_5)/2$ stand for left and right-handed projection operators, $\lambda_t = V_{tb}V_{ts}^*$, V_{ij} are elements of CKM matrix.

In this paper, we present the contribution of (O_7, O_7) interference up to α_s order, and the tree level contributions of the interferences (O_1, O_1), (O_1, O_2), (O_1, O_7), (O_2, O_2), (O_2, O_7) to the double differential decay width $d\Gamma/(ds_1\, ds_2)$, where $s_1 = (p_b - q_1)^2/m_b^2$, $s_2 = (p_b - q_2)^2/m_b^2$ (q_1, q_2, p_b are the momenta of photons and the b-quark), then investigate the dependence of the double decay width from the renormalization scheme. We take the contribution of only O_1, O_2 and O_7 operators, because the Wilson coefficients $C_j(\mu)$ for the remaining operators are very small compared to that 3 and, consequently, their contribution is much smaller.

The contribution (O_7, O_7) up to the α_s order is calculated in our previous paper [8], in this paper we also present the results for all the other interferences (O_1, O_1), (O_1, O_2), (O_1, O_7), (O_2, O_2), (O_2, O_7) in tree level order in analytical form. The latter were also given in [9], we calculated them using other methods – the Mellin–Barnes representation [10] and the automatized algorithm of Laporta [11,12] FIRE.

2. Details of Calculation

In the first phase of our calculation we use the algorithm FIRE (Feynman Integral REduction) [13], which is based on the "integration by parts" method [11,12] and the Gauss theorem. The algorithm reduces the large number of integrals from the matrix element of the process to several master-integrals. All the other integrals are expressed linearly through the master-integrals.

The next step is the calculation of the master-integrals. Here, to get analytic results for them, we use the Mellin–Barnes representation. The "propagators" of the form $1/(x+y)^\lambda$ ($\lambda > 0$) are given by

$$\frac{1}{(x+y)^\lambda} = \frac{1}{\Gamma(\lambda)} \int_\gamma \frac{ds}{2\pi i} \frac{x^s}{y^{\lambda+s}} \Gamma(\lambda+s)\Gamma(-s),$$

where the integration is done by the contour γ, which goes parallel to the imaginary axis in the complex plane *s*, and crosses the real axis between the points $-\lambda$ and 0. If the integral on the infinite semicircle is 0, then we can close the contour and the calculation of the integral is reduced to the calculation of sum of the residues, which are enclosed within the contour. As a result we

obtain a series by powers of x/y if $x<y$, or of y/x if $y<x$, depending on which side of the real axis had we closed the contour.

We found that there are only three master-integrals, that must be calculated to get all the contributions except the (O_7,O_7). As usual, we work in $d=4-2\varepsilon$ dimensions. In order to obtain the desired contributions in the tree-level order, we have to calculate those three integrals to the first order of series by parameter ε. One of the master-integrals has only one propagator and is calculated easily:

$$M_1=\frac{\hat{m}_c^2}{192\varepsilon\pi^2}\ \Big(12+12\varepsilon+12\varepsilon^2+\varepsilon^2\pi^2+6\varepsilon^2\log^2\left(\hat{m}_c^2\right)+6\varepsilon^2\log^2 s_1-$$
$$-12\varepsilon\log\left(1-s_1-s_2\right)-12\varepsilon^2\log\left(1-s_1-s_2\right)+6\varepsilon^2\log^2\left(1-s_1-s_2\right)-12\varepsilon\log s_2-12\varepsilon^2\log s_2+$$
$$+12\varepsilon^2\log\left(1-s_1-s_2\right)\log s_2+6\varepsilon^2\log^2 s_2+12\varepsilon\left(-1-\varepsilon+\varepsilon\log(1-s_1-s_2)+\varepsilon\log s_2\right)\log s_1+$$
$$+12\varepsilon\ \log\left(\hat{m}_c^2\right)\left(-1-\varepsilon+\varepsilon\ \log\left(s_1\right)+\varepsilon\ \log\left(1-s_1-s_2\right)+\varepsilon\ \log s_2\right)\Big),$$

where $\hat{m}_c=m_c/m_b$, m_c is the mass of the *c*-quark.

The other two master-integrals have two and three propagators, respectively, and in order to obtain analytic expressions for them, we have to use the Mellin-Barnes representation. As already is mentioned above, after that procedure we get two infinite series, one in positive powers of $\left(1-s_1-s_2\right)/4\hat{m}_c^{\,2}$ when it's value is less than 1, and the second in the negative powers of that expression, when it's value is bigger than 1. As we found out due to numerical calculations, the sum in the first case is the analytic continuation of the sum in the second case. So, the expressions given below are correct for all the values of the parameters, just for the mass of the c-quark we should make the substitution $\hat{m}_c^{\,2}\to\hat{m}_c^{\,2}-i\delta$ $(\delta>0)$:

$$M_2=\frac{1}{16\varepsilon\pi^2}\ +\ \Big(-\sqrt{-1+4\hat{m}_c^2+s_1+s_2}+s_1\sqrt{-1+4\hat{m}_c^2+s_1+s_2}\ +$$
$$+\ s_2\ \sqrt{-1+4\hat{m}_c^2+s_1+s_2}-\sqrt{1-s_1-s_2}\arcsin\sqrt{\left(1-s_1-s_2\right)\Big/\left(4\hat{m}_c^2\right)}\ +$$
$$+\ 4\hat{m}_c^2\sqrt{1-s_1-s_2}\arcsin\sqrt{\left(1-s_1-s_2\right)\Big/\left(4\hat{m}_c^2\right)}\ +$$
$$+\ s_1\sqrt{1-s_1-s_2}\arcsin\sqrt{\left(1-s_1-s_2\right)\Big/\left(4\hat{m}_c^2\right)}\ +$$
$$+\ \sqrt{1-s_1-s_2}\times s_2\arcsin\sqrt{\left(1-s_1-s_2\right)\Big/\left(4\hat{m}_c^2\right)}\Big)\Big/\left(8\pi^2\left(-1+s_1+s_2\right)\sqrt{-1+4\hat{m}_c^2+s_1+s_2}\right)\ +$$
$$+\frac{\left(-\log(\hat{m}_c^2)\ -\ \log s_2\ -\ \log(s_1(1-s_1-s_2))\right)}{16\pi^2}+M_{21},$$

$$M_3=-\left(\arcsin\sqrt{\left(1-s_1-s_2\right)/(4\hat{m}_c^2)}\right)^2\Big/\left[8\pi^2\left(1-s_1-s_2\right)\right]+M_{31},$$

where M_{21} and M_{31} are terms proportional to ε, which are not presented here because they are very large.

3. Results

The double differential decay width of the process we consider can be written in the following form:

$$\frac{d\Gamma\left(b\to X_s\gamma\gamma\right)}{ds_1\,ds_2}=\sum_{i<j}\frac{G_F^2\alpha_{em}{}^2\bar{m}_b\left(\mu\right)m_b^4}{1024\pi^5}\left|V_{tb}V_{ts}^*\right|^2 C_i^{eff}\left(\mu\right)C_j^{eff}\left(\mu\right)G_{ij},\ i,j=1,2,7,$$

where m_b and $\bar{m}_b\left(\mu\right)$ are, correspondingly, the *b*-quark mass in the pole and the running quark mass, α_{em} is the electromagnetic constant, $C_i^{eff}\left(\mu\right)$ are the Wilson coefficients.

Finally we give the results for the contributions of all interferences to the double decay width separately:

$$G_{22}=4q_u^4\frac{\left(s_1+s_2\right)\left|-1+s_1+s_2+4m_c^2\arcsin\left(\sqrt{1-s_1-s_2}\,/2m_c^2\right)\right|^2}{\left(1-s_1-s_2\right)^2},$$

$$G_{27}=16q_dq_u^2\left(1-s_1-s_2-4m_c^2\,\mathrm{Re}\left(\arcsin\left(\frac{1}{2}\sqrt{\frac{1-s_1-s_2}{m_c^2}}\right)^2\right)\right),$$

$$G_{12}=24q_u^4\frac{\left(s_1+s_2\right)\left|-1+s_1+s_2+4m_c^2\arcsin\left(\sqrt{1-s_1-s_2}\,/2m_c^2\right)\right|^2}{\left(1-s_1-s_2\right)^2},$$

$$G_{11}=36q_u^4\frac{\left(s_1+s_2\right)\left|-1+s_1+s_2+4m_c^2\arcsin\left(\sqrt{1-s_1-s_2}\,/2m_c^2\right)\right|^2}{\left(1-s_1-s_2\right)^2},$$

$$G_{17}=48q_dq_u^2\left(1-s_1-s_2-4m_c^2\,\mathrm{Re}\left(\arcsin\left((1/2)\sqrt{(1-s_1-s_2)/m_c^2}\right)^2\right)\right),$$

$$G_{77}=q_d^2\left(G_{77}^{(0)}+G_{77}^{(1)}\right),$$

$$G_{77}^{(0)}=r_0\left(1-s_1-s_2\right)\Big/\left[\left(1-s_1\right)^2s_1\left(1-s_2\right)^2s_2\right],$$

$$G_{77}^{(1)}=\frac{\alpha_s}{4\pi}C_F\left(\frac{-4\,r_0\left(1-s_1-s_2\right)}{\left(1-s_1\right)^2s_1\left(1-s_2\right)^2s_2}\log\frac{\mu}{m_b}+f\right),$$

where

$$r_0=\left(-48s_2^3s_1^3+96s_2^2s_1^3-56s_2s_1^3+8s_1^3+96s_2^3s_1^2-\right.$$
$$\left.-192\,s_2^2s_1^2+112s_2s_1^2-56s_2^3s_1+112s_2^2s_1-96s_2s_1+8s_1+8s_2^3+8s_2\right),$$

and the expression for f can be found in [8] (formula 5.2). $G_{77}^{(0)}$ and $G_{77}^{(1)}$ are the contributions from (O_7,O_7) of tree-level order and of α_s order respectively, $q_u=+2/3$, $q_d=-1/3$. All the infinities of the type $1/\varepsilon$ are cancelled separately.

As we see from the expression for the double decay width, it's value is dependent on the renormalization scale. In Fig.1 we give the dependence of double differential decay width $d\Gamma/(ds_1\, ds_2)$ from s_1 at fixed $s_2 = 0.2$ for three different values of renormalization scale: $\mu = m_b/2,\ m_b,\ 2m_b$.

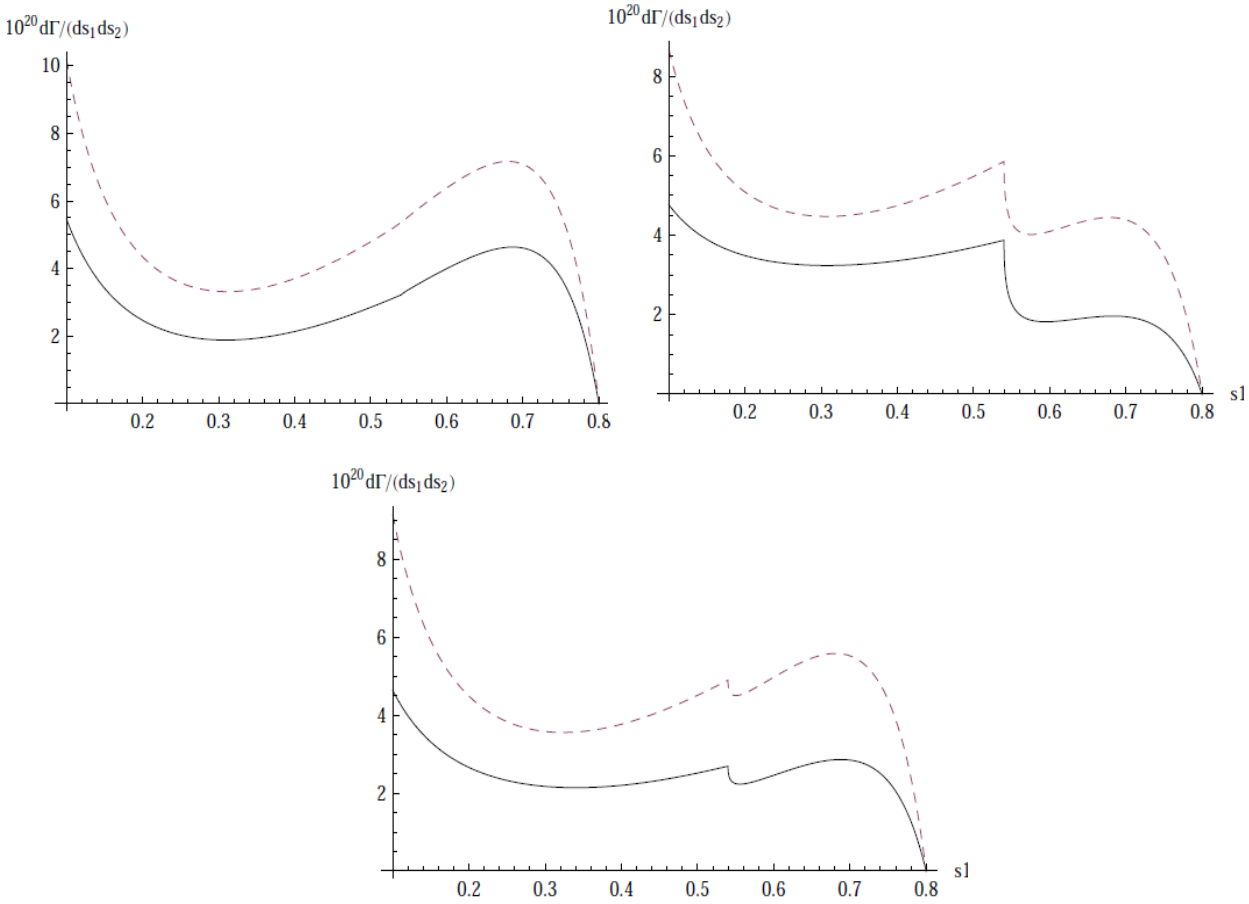

Fig. 1. Double differential decay width $d\Gamma/(ds_1\, ds_2)$ as a function of s_1 at fixed $s_2 = 0.2$, first at the matching scale $\mu = m_b/2$, second at the matching scale $\mu = m_b$, and the third at matching scale $2m_b$. The dashed line represents only the contribution of the tree level order of all interferences, the other line takes into account also the α_s-order contribution coming from $(O_7,\ O_7)$ interference.

The numerical values for the input parameters and for the Wilson coefficients at various values of the scale μ, together with the numerical values of $\alpha_s(\mu)$, are given in Table 1 and 2, respectively.

From Fig. 1 we see that even after adding to the tree level result the α_s-correction coming from (O_7, O_7) interference, the dependence of the double differential decay width from the renormalization scheme is not reduced, but remains approximately the same, which means that the α_s-corrections coming from the other interferences must be also calculated. It is expected that if we take into account all α_s-corrections, the dependence from the renormalization scale must be reduced. We are planning to calculate them in forthcoming papers.

Table 1. Values of the relevant input parameters.

Parameter	Value
m_b	4.8 GeV
G_F	$1.16637\times10^{-5}\,\mathrm{GeV}^{-2}$
$V_{tb}V_{ts}^*$	0.04
α^{-1}	137
$\alpha_s(M_z)$	0.119

Table 2. $\alpha_s(\mu)$ and the Wilson coefficients C_1, C_2 and C_7 at different values of the scale μ, $C_7^{0,\mathrm{eff}}$ and $C_7^{1,\mathrm{eff}}$ are the Wilson coefficients of the operator O_7 for the tree-level order and α_s-order, correspondingly.

	$\alpha_s(\mu)$	$C_7^{0,\mathrm{eff}}(\mu)$	$C_7^{1,\mathrm{eff}}(\mu)$	$C_1^{\mathrm{eff}}(\mu)$	$C_2^{\mathrm{eff}}(\mu)$
$m_b/2$	0.1818	–0.2796	–0.1788	–0.3558	1.1664
m_b	0.2175	–0.3142	0.4725	–0.2487	1.1074
$2m_b$	0.2714	–0.3556	1.0794	–0.1676	1.0674

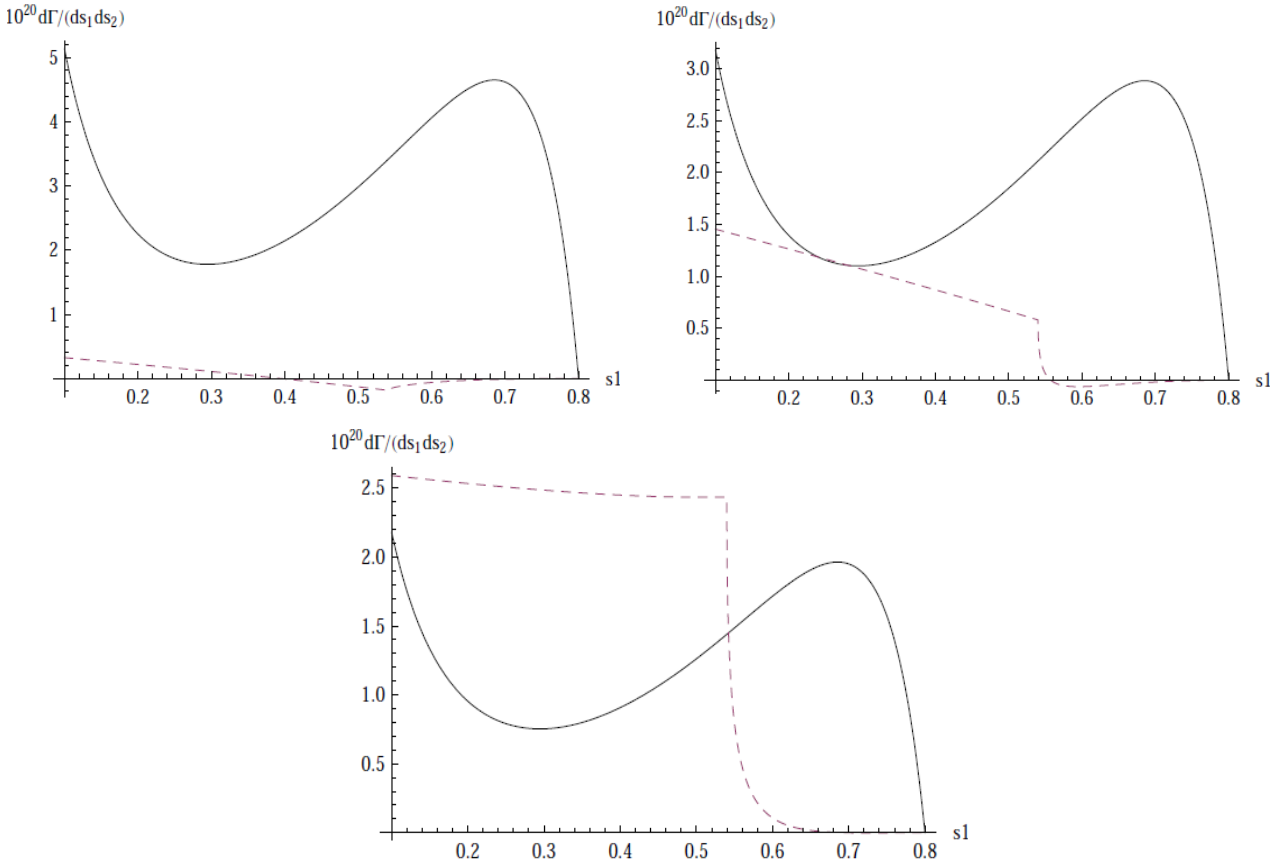

Fig. 2. Double differential decay width $d\Gamma/(ds_1\,ds_2)$ as a function of s_1 at fixed $s_2 = 0.2$, first at the matching scale $\mu = m_b/2$, second at the matching scale $\mu = m_b$ and the third at matching scale $2m_b$. The dashed line represents only the tree-level order contribution of the interferences $((O_1,O_1)$, (O_1,O_2), (O_1,O_7), (O_2,O_2), $(O_2,O_7))$, the other line takes into account only the up to α_s-order contribution of (O_7,O_7) interference.

In Fig.2 we also present plots which show the relative contributions of only (O_7,O_7) up to α_s-order and of all the rest contributions $((O_1,O_1)$, (O_1,O_2), (O_1,O_7), (O_2,O_2), $(O_2,O_7))$ of tree-level order to $d\Gamma/(ds_1\,ds_2)$.

As can be seen from Fig. 2, at the scale $m_b/2$ the contribution of all the other interferences except (O_7, O_7) is negligible in $d\Gamma/(ds_1\, ds_2)$. At scale m_b the contribution of the tree-level result gives visible effects on the α_s-order (O_7, O_7) interference, and at scale $2m_b$ becomes comparable with it.

It should be noted that he contribution of interference of the operators O_7 and O_2 was calculated also in [14].

Acknowledgments

The work was supported by the Armenian State Committee of Science under contract 11-1c014.

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