# EQUILIBRIUM CURRENT VARIANCE IN SEMICONDUCTORS

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**Abstract–**The current equilibrium fluctuations appearing in homogeneous and non-degenerate thermal equilibrium semiconductors are considered. As independent sources of equilibrium current fluctuations an electron intraband scattering, generation-recombination transitions and the shot effect are discussed. Variances of electron current equilibrium fluctuations are calculated and analyzed. It is established that not only the electron intraband scattering (Nyquist thermal noise) but also generation-recombination transitions and the shot effect, can be equilibrium noise sources.

## 1. Introduction

In the theory of metals, semiconductors and semiconductor devices the reasons and physical mechanisms of the fluctuating current origins are widely discussed [1-6]. In the presence of external electric field, current internal fluctuations are caused by the conductivity (or carrier density and mobility) fluctuations. The following processes with random nature: intraband scattering, band-to-band or band-to-impurity level generation-recombination (g-r) transitions, shot effect, etc. [3,4,7] can be mentioned as a basic sources of the carrier density and mobility fluctuations.

Fluctuating (random) currents are observed also in the absence of an external electric field [3, 6]. Such currents are called equilibrium noise [3,4]. Thermal noise, e.g., is equilibrium noise, whose origin is generally connected with the chaotic thermal motion of free charge carriers [3, 4]. The spectral density of the current thermal noise $S_I$ does not depend on the magnitude of the average current $\overline{I}$, passing through the conductor. Therefore, thermal noise is observed as in the non-equilibrium conductor (when there is external electric field and $\overline{I} \neq 0$) as well in conductors and semiconductors in the thermal equilibrium state (when there is not external electric field and $\overline{I} = 0$). Frequency dependence of the thermal noise spectral density is described by the well-known Nyquist's formula

$$S_I(\omega) = \frac{4k_B T Y}{1 + (\omega\tau)^2}. \tag{1}$$

Here $\omega = 2\pi f$, $f$ is the frequency, $Y$ is the sample conductivity, $\tau$ is the relaxation time of the electron quasi-momentum, $T$ is the temperature, $k_B$ is the Boltzmann constant.

There are different theoretical methods for consideration of current fluctuations [3,4]. As it is shown the results of recent investigations [8,9,10], the model of carriers distribution function fluctuation is a relatively common approach for the study of current, current carrier concentration

and mobility fluctuations. It allows detecting and investigating the microscopic mechanisms of fluctuations in equilibrium as well as in non-equilibrium semiconductors. In this paper on the basis of distribution function fluctuation approach the fluctuation of currents of thermal equilibrium semiconductor are discussed. For simplicity, the conduction electron system of a non-degenerate semiconductor with the standard band structure is considered.

## 2. Main sources of current equilibrium fluctuations

As is well-known, current density in conductors can be represented through current carriers distribution function. In particular, for the conduction electrons

$$\mathbf{j}(\mathbf{r},t) = -\frac{e}{V}\sum_{\mathbf{k}}\mathbf{v}_{\mathbf{k}}f_{\mathbf{k}}(\mathbf{r},t) = -\frac{e}{4\pi^3}\int_{BZ}\mathbf{v}_{\mathbf{k}}f_{\mathbf{k}}(\mathbf{r},t)d\mathbf{k}, \tag{2}$$

where $\mathbf{j}$ is the electron current density, $V$ is the sample volume, $\mathbf{v}_{\mathbf{k}}$ is the velocity of electron in state with wave-vector $\mathbf{k}$ of conduction band of the semiconductor, $f_{\mathbf{k}}$ is the distribution function of conduction electrons, $\mathbf{r}$ is the radius vector, $t$ is the time. In Eq. (2) integration over $\mathbf{k}$ is carried out in the first Brillouin zone (BZ).

Here we are interested in the equilibrium current fluctuations in thermal equilibrium semiconductors, when external electric field is absent. The analysis of Eq. (1) [11,12] shows that equilibrium fluctuations of electron current density in a short circuit regime can be represented in the form

$$\tilde{\mathbf{j}}(\mathbf{r},t) = -\frac{e}{4\pi^3}\int_{BZ}d\mathbf{k}\mathbf{v}_{\mathbf{k}}\tilde{f}_{\mathbf{k}}^{0,a}(\mathbf{r},t), \tag{3}$$

where "~" is symbol to mark fluctuation over statistical ensemble of the components of corresponding values, $\tilde{f}_{\mathbf{k}}^{0,a}$ is the asymmetric component of distribution function fluctuations of thermal equilibrium electrons.

In the general case, fluctuations of distribution function $\tilde{f}_{\mathbf{k}}^{0}$ may be represented in the form of the sum ($\tilde{f}_{\mathbf{k}}^{0} = \tilde{f}_{\mathbf{k}}^{0,a} + \tilde{f}_{\mathbf{k}}^{0,s}$) of asymmetric $\tilde{f}_{\mathbf{k}}^{0,a}$ (i.e. $\tilde{f}_{-\mathbf{k}}^{0,a} = -\tilde{f}_{\mathbf{k}}^{0,a}$) and symmetric $\tilde{f}_{\mathbf{k}}^{0,s}$ (i.e. $\tilde{f}_{-\mathbf{k}}^{0,s} = \tilde{f}_{\mathbf{k}}^{0,s}$) components. As it is shown from Eq. (3) the equilibrium fluctuations of current density are caused by asymmetric fluctuations of distribution function. For simplifying the analysis, below we have considered only temporal fluctuations of distribution function, assuming that the value $\tilde{f}_{\mathbf{k}}^{0a}$ is spatial-coordinately independent. That is why instead of the exact, depending on spatial coordinate distribution function of electrons $\tilde{f}_{\mathbf{k}}^{0}(\mathbf{r},t)$, we shall consider their value averaged over the volume of the entire sample, i.e. the function $\tilde{f}_{\mathbf{k}}^{0}(t)$.

Let's consider processes, which can lead to the origin of distribution function fluctuations. At a thermal equilibrium state, electrons are distributed over the quantum states of the conduction band according to the Boltzmann (or Fermi-Dirac) law. Populations of electrons (i.e., the number of electrons $f_{\mathbf{k}}^0 d\mathbf{k}$) in the states $-\mathbf{k}$ and $\mathbf{k}$ of the conduction band on average are equal. Therefore, the total average quasi-momentum, and hence the average current are zero. However, the instantaneous total quasi-momentum of the electrons differs from the average values. As shown in the analysis, the main sources of random variations of electron quasi-momentum are random intraband scattering, as well as the generation-recombination transitions and the random character of electron transitions through the metal-semiconductor interface (shot effect). The random deviation of the electrons total quasi-momentum from the average value brings to the origin of the so-called equilibrium random currents.
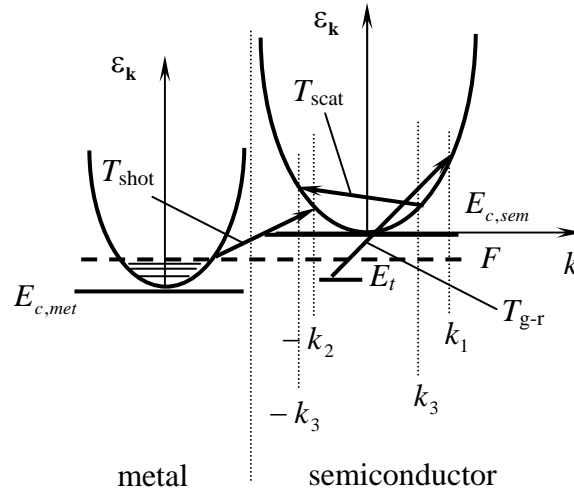


Schematic diagram of electron random transitions. $E_{c,met}$ and $E_{c,sem}$ are the bottoms of the metal and semiconductor conduction bands, correspondingly, $E_t$ is the energy level of the generation-recombination center.

In the figure the intraband scattering, generation-recombination and shot transitions are shown schematically, in the form of $T_{scat}$, $T_{g-r}$ and $T_{shot}$ arrows, respectively. The case, represented in figure, the random $T_{scat}$, $T_{g-r}$ and $T_{shot}$ transitions induce the decrease in population of $\mathbf{k}_3$ quantum state and increase of population of $-\mathbf{k}_3$, $\mathbf{k}_1$ and $-\mathbf{k}_2$ quantum states, correspondingly. Therefore, at a given moment the "number" of electrons with quasi-momenta $\hbar\mathbf{k}$ and $-\hbar\mathbf{k}$ are not equal. It is clear (see Eq. (3)) that in equilibrium semiconductors the quasi-momentum fluctuations and the random currents are connected with the asymmetric component $\tilde{f}_{\mathbf{k}}^{0,a}$.

Thus, in general, the sources of the current equilibrium fluctuations are not only the processes of electrons intraband scattering (or the randomness of electron motions), but also the generation-recombination transitions and the shot effect as well.

### 3. Equilibrium current variance

On the base of the above-presented microscopic mechanisms, one can calculate the variance of the equilibrium current $\sigma_I^2$. From Eq. (3) it follows that the $x$-component of equilibrium fluctuations of the current density can be represented as

$$\tilde{j}_x(t) = -\frac{e}{4\pi^3} \int_{BZ} d\mathbf{k} \mathbf{v}_{\mathbf{k},x} \tilde{f}_{\mathbf{k}}^{0,a}(t).$$ (4)

Then, using the relation $\tilde{I}_x = \tilde{j}_x S$, where $S$ is the area of the cross-section of the sample, for the variance of the current $x$-component $\sigma_{I_x}^2 = \overline{\tilde{I}_x^2}$, from Eq. (4) one obtains the following expression:

$$\sigma_{I_x}^2 = S^2 \iint_{BZ} d\mathbf{k} d\mathbf{k}_1 \mathbf{v}_{\mathbf{k},x} \mathbf{v}_{\mathbf{k}_1,x} \overline{\tilde{f}_{\mathbf{k}}^0 \tilde{f}_{\mathbf{k}_1}^0}.$$ (5)

where "–" is symbol to mark averaging over statistical ensemble of the components.

As is known [13, 14], if the electron concentration $n$ is a fluctuating quantity ($\tilde{n} \neq 0$, as in case of random generation-recombination transitions and the shot effect), then for the non-degenerated electron gas

$$\overline{\tilde{f}_{\mathbf{k}}^0 \tilde{f}_{\mathbf{k}_1}^0} = \frac{4\pi^3 \overline{f_{\mathbf{k}}^0}}{V} \delta(\mathbf{k} - \mathbf{k}_1).$$ (6)

Here $\delta(\mathbf{k}) = \delta(k_x)\delta(k_y)\delta(k_z)$ is the three-dimensional delta function, $\overline{f_{\mathbf{k}}^0}$ is the Boltzmann distribution function:

$$\overline{f_{\mathbf{k}}^0} = \exp\left(\frac{F - \varepsilon_{\mathbf{k}}}{k_B T}\right),$$ (7)

$F$ is the Fermi energy, $\varepsilon_{\mathbf{k}} = \hbar^2 k^2 / 2m_n$ is the electron energy in the conduction band, $m_n$ is the electron effective mass.

If electron concentration is a constant (non-fluctuating) quantity ($\tilde{n} = 0$, as in case of random intraband scattering), then for the non-degenerated electron gas [13, 14]

$$\overline{\tilde{f}_{\mathbf{k}}^0 \tilde{f}_{\mathbf{k}_1}^0} = \frac{4\pi^3 \overline{f_{\mathbf{k}}^0}}{V} \delta(\mathbf{k} - \mathbf{k}_1) - \frac{\overline{f_{\mathbf{k}}^0}\ \overline{f_{\mathbf{k}_1}^0}}{\overline{n} V}.$$ (8)

Subsequently, inserting Eqs. (6) and (8) into Eq. (4) and carrying out corresponding integration it is possible to determine the variance $\sigma_{I_x}^2$. The calculations show that at random intraband scattering as well as at random generation-recombination transitions and the shot effect the same expression for the variance $\sigma_{I_x}^2$ is obtained:

$$\sigma_{I_x,scat}^2 = \sigma_{I_x,g-r}^2 = \sigma_{I_x,shot}^2 = \frac{k_B T}{m_n} \frac{e^2 S^2}{V^2} \overline{N}.$$ (9)

Here $\bar{N} = \bar{n}V$ is the electrons total number in sample.

As seen from the expression (9), the variance of the equilibrium current depends on the electrons number and the temperature via the proportionality law. Note that above mentioned mechanisms of fluctuation origin independently of one another and may act simultaneously. Therefore in the general case the random current can be represented as a sum of three components

$$\tilde{I}_x = \tilde{I}_{x,scat} + \tilde{I}_{x,g-r} + \tilde{I}_{x,shot}. \tag{10}$$

From this expression it follows that the equilibrium current variance $\sigma_{I_x}^2$ in the general case consists of intraband $\sigma_{I_x,scat}^2$, generation-recombination $\sigma_{I_x,g-r}^2$ and shot $\sigma_{I_x,shot}^2$ components

$$\sigma_{I_x}^2 = \sigma_{I_x,scat}^2 + \sigma_{I_x,g-r}^2 + \sigma_{I_x,shot}^2. \tag{11}$$

As is well-known, the variance can also be defined on the basis of the noise spectral density:

$$\sigma_{I_x}^2 = \frac{1}{2\pi} \int_0^\infty S_{I_x}(\omega)d\omega. \tag{12}$$

In order to verify the results, obtained on the basis of the distribution function fluctuations, one can calculate, e.g., the thermal noise variance, using the Nyquist's theorem Eq. (1). It is not difficult to ascertain that the results of simple calculations, made on the basis of Eqs. (1) and (12), exactly coincide with the expression (9).

## 4. Conclusion

The results, obtained in this paper, supplement the notions about the sources and properties of equilibrium current noises. The main conclusion is that various fluctuation mechanisms make contributions to equilibrium current noise. The result shows that the equilibrium noise source is not only the thermal motions of electrons (Nyquist noise), which is conditioned by intraband scattering of electrons. The generation-recombination transitions and the shot effect can also be equilibrium noise sources.

In contrast to intraband scattering processes, the generation-recombination and shot sources are more particular sources. They may appear only in the samples, where one cannot neglect the generation-recombination transitions and/or transition of carriers across the metal-semiconductor interface. In these samples, as follows from Eqs. (10) and (11), $\sigma_{I_x}^2 = 2\sigma_{I_x,scat}^2$ or $\sigma_{I_x}^2 = 3\sigma_{I_x,scat}^2$, i.e. variance of the equilibrium current may two times (if there is a generation-recombination or shot source) or three times (if simultaneously there is generation-recombination and shot sources) exceed the Nyquist noise variance $\sigma_{I_x,scat}^2$. As shows the analysis of the results of measurements, carried out by different authors, there are experimental data, where in the low-frequency range ($\omega\tau \ll 1$) the level of equilibrium noise is greater than the low-frequency level of Nyquist noise $4k_B TY$. In

particularly, in [15] the results of noise measurement in the range 1 Hz – 25 kHz are reported for $n^+nn^+$ and $n^+pn^+$ GaAs near ballistic devices. Measurements was carried in ohmic region of I-V characteristic. The $n^+nn^+$ mesastructures show very low $1/f$ noise. The thermal noise above 1 kHz is equal to Nyquist noise at the lowest currents, rising to 1.2–1.5 time above Nyquist noise for high currents.

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