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QUANTUM SPIN TRANSPORT IN SUPERCONDUCTING AND FERROMAGNETIC HYBRID SYSTEM

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Abstract–The quantum transport property of a mesoscopic device is investigated. A model for such mesoscopic device is proposed and it is formed of ferromagnetic/superconductor hybrid junction. An expression for the conductance was derived using Landauer-Buttiker formula. The effect of an external magnetic field was taken into consideration. The spin polarization is expressed in terms of both Andreev-reflection probabilities for spin-up and spin-down. Numerical calculations are performed for the present proposed nanoscale device. Our results showed that the proposed device operates in the mesoscopic regime as indicated from the dependence of the conductance on the temperature. Also, the results showed that two peaks appeared due to the Zeeman splitting of the quasiparticle density of states. The dependence of spin polarization on the considered parameters confirms that the spin flip of electrons when Andreev-reflection tunneling occurs through the junction.

Keywords: ferromagnetic/superconductor hybrid junction, spin, conductance, polarization

1. Introduction

Recently much of attention has been attracted to the field of spin-dependent electron transport in nanostructures. Transport and injection of spin-polarized current through nanostructures (spintronics) [1] has become an area of intense activity in the past few years. This is due to its advantages in increasing processing speed and decreasing power consumption compared with conventional semiconductor devices. Proposals for generating spin-polarized currents including spin injection by using ferromagnetic metals have been made [2].

The goal of spintronics is to understand the interaction between the particle spin and its solid-state environments and to make useful devices using the acquired knowledge. Fundamental studies of spintronics including investigations of spin transport in electronic materials, as well as understanding spin dynamics and spin relaxation, have been made [3–5]. New devices are now being designed relying on the spin [1,4,5]. Such devices should have faster switching times and lower power consumption than conventional devices, mainly because spins can be manipulated faster and at lower energy cost than charges. Spin-based devices are very important for future applications [6] especially in the field of quantum computer [7] which would represent a great breakthrough in the processing time of certain physical and mathematical problems [8]. In particular, the electron spin in quantum dots has been proposed as a building block for the implementation of quantum bits (qubits) for quantum computation [9,10]. In the present paper, the conductance and spin-polarized transport through ferromagnetic/superconductor/ferromagnetic mesoscopic device is investigated. The results will show how the conductance and polarization is sensitive to the exchange field energy in the ferromagnetic leads.

2. The Model

The present mesoscopic spintronic device is modeled as ferromagnetic/superconductor hybrid junction. This paper is devoted to derivation of an expression for the conductance (*G*) and the spin polarization (*P*) of the present mesoscopic device under the effect of magnetic field. This conductance can be obtained from the Landauer-Buttiker formula [11–13]:

$$G = \frac{e^2}{h} \int_{E_F}^{E_F+2\Delta_0} z_1 \left(1 + a_1 - b_2\right), \tag{1}$$

where a_1 and b_2 are the probabilities for Andreev-reflection and normal reflection respectively, *e* is the electronic charge, *h* is Planck's constant, Δ_0 is the superconductor energy gap at $T = 0$, E_F is the Fermi energy and z_1 is the distribution function for both spin-up and spin-down, which is expressed as [5]:

$$z_1 = n_F - \frac{\partial n_F}{\partial E} \times \delta\mu \times \sigma. \tag{2}$$

Here $\delta\mu$ refers to the shift in the chemical potential of the spin subbands, $\sigma = \pm 1$ for spin up and spin down-band respectively and n_F is the Fermi–Dirac distribution function and is given by

$$n_F = \frac{1}{\left(e^{(E - eV_b - E_F)/kT} + 1\right)} \tag{3}$$

Here *E* is the quasi particle energy measured from the Fermi energy, V_b is the bias voltage, *k* is Boltzman's constant, *T* is absolute temperature. The system we consider is described by the following Bogoliubov–de Gennes equation (BdG) [14]:

$$\begin{pmatrix} H - h(z)\sigma & \Delta(T) \\ \Delta^*(T) & -H - h(z)\sigma \end{pmatrix} \Psi_{\sigma,nl} = E\Psi_{\sigma,nl}, \tag{4}$$

where *H* is the single particle Hamiltonian with the constriction potential, *V*, and is given by

$$H = \frac{-h^2}{2m}\nabla^2 - V(r) - E_F. \tag{5}$$

We assume the effective mass *m* is the same both for ferromagnetic and superconductor [15].

The Bogoliubov–de Gennes Eq. (4) can be written in terms of two wave functions *u* and *v* [14] as:

$$\begin{pmatrix} H - h(z)\sigma & \Delta(T) \\ \Delta^*(T) & -H - h(z)\sigma \end{pmatrix} \begin{pmatrix} u \\ \nu \end{pmatrix} = E \begin{pmatrix} u \\ \nu \end{pmatrix} \tag{6}$$

where Δ^* is the complex conjugate of the superconductor energy gap, Δ, u is the wave function of the electron occupying state of energy *E* and v is the wave function of holes in the state of energy *E* and the superconductor energy gap. The value of Δ is given by:

$$\Delta = \begin{cases} 0 & z < 0,\ z > L, \\ \Delta & 0 < z < L. \end{cases} \tag{7}$$

The temperature dependence of the superconducting energy gap is given by [16]

$$\Delta = \Delta_0 \ \tanh\left(1.47\sqrt{\frac{T_c}{T} - 1}\right) \tag{8}$$

where Δ_0 is the superconducting gap at $T = 0$ and T_c is the superconductor critical temperature. The interface between the left ferromagnetic/superconductor and superconductor/right ferromagnetic leads are located at $z = -L$ and $z = L$, respectively. The exchange field is represented by the parameter h(z) (Eq.(4)) which is given by [17]

$$h(z) = \begin{cases} h_0 & z < 0 \\ 0 & 0 < z < L \\ \pm h_0 & z > L \end{cases} \tag{9}$$

$+h_0$ for parallel alignment, $-h_0$ for antiparallel alignment and $h(z)$ is the exchange field of the ferromagnetic material. The probabilities for both Andreev-reflection, a_1 and normal reflection, b_2 in Eq. (1) can be determined by solving the Bogoliubov–de Gennes equation Eq. (4). The solution of this equation is given by using the effective mass approximation as [16]

$$\Psi_{\sigma,nl} = \left[\begin{array}{l} \begin{pmatrix} 1 \\ 0 \end{pmatrix} e^{ip^{+}_{\sigma,nl}(z)} J_n\left(\frac{2\gamma_{nl}}{w_E}\right) e^{in\Phi} \\ +\sum_{nl=1}^{M_E}\left[a_{\sigma,nl}\begin{pmatrix} 0 \\ 1 \end{pmatrix} e^{-ip^{-}_{\sigma,nl}(z)} + b_{\sigma,nl}\begin{pmatrix} 1 \\ 0 \end{pmatrix} e^{-ip^{+}_{\sigma,nl}(z)}\right] \\ J_n\left(\frac{2\gamma_{ns}}{w_1}\right) e^{in\Phi} \qquad \text{for } z < 0 \\ \sum_{nl=1}^{M_C}\left[\begin{array}{l} \alpha_{\sigma,nl}\begin{pmatrix} u \\ \nu \end{pmatrix} e^{ik^{+}_{nl}(z)} + \beta_{\sigma,nl}\begin{pmatrix} \nu \\ u \end{pmatrix} e^{-ik^{-}_{nl}(z)} \\ +\xi_{\sigma,nl}\begin{pmatrix} u \\ \nu \end{pmatrix} e^{-ik^{+}_{nl}(z)} + \eta_{\sigma,nl}\begin{pmatrix} \nu \\ u \end{pmatrix} e^{ik^{-}_{nl}(z)} \end{array}\right] \\ J_n\left(\frac{2\gamma_{ns}}{w_2}\right) e^{in\Phi} \ \text{for } 0 < z < L \\ \sum_{nl=1}^{M_E}\left[C_{\sigma,nl}\begin{pmatrix} 1 \\ 0 \end{pmatrix} e^{iq^{+}_{\sigma,nl}(z)} + d_{\sigma,nl}\begin{pmatrix} 0 \\ 1 \end{pmatrix} e^{-iq^{-}_{\sigma,nl}(z)}\right] \\ J_n\left(\frac{2\gamma_{ns}}{w_1}\right) e^{in\Phi} \qquad \text{for } z > L \end{array}\right. \tag{10}$$

where γ_{nl} is the l-th zero of the Bessel function J_n [18], M_E, M_C are the cutoff constants which truncates the number of channels in the reservoir and contact, respectively, the set of quantum numbers (n, l, σ) defines the transport channel and Φ is the phase angle of quasiparticles, W_1 and W_2 are the dimensions of the device,

$$p^{\pm}_{\sigma,nl} = \sqrt{\frac{2m^*}{h^2}}\sqrt{\left(E_F \pm E + \sigma h\right)}, \tag{11}$$

$$k^{\pm}_{nl} = \sqrt{\frac{2m^*}{h^2}}\sqrt{E_F \pm \sqrt{E^2 - \Delta^2} - \varepsilon_{nl}}, \tag{12}$$

$$q^{\pm}_{\sigma,nl} = \sqrt{\frac{2m^*}{h^2}}\sqrt{\left(E_F \pm E \pm \sigma h \pm \varepsilon_{nl}\right)}, \tag{13}$$

where m^* is the effective mass of the quasiparticle and the energy, ε_{nl} is expressed in terms of the Fermi velocity v_F, Fermi-momentum p_F and the magnetic field B as [18]:

$$\varepsilon_{nl} = -\left(S_g + k_f L \sin\theta\right)\mu_B B \pm \left[v_F^2 p_F^2 \left(1 - \sin\theta\right)^2 + \Delta^2\right]^{1/2} + eV_b, \tag{14}$$

$S = \pm 1/2$ for spin up and spin-down respectively, $-\pi/2 < \theta < \pi/2$, $v_F = \sqrt{2E_F/m^*}$ and $p_F = m^* v_F$, μ_B is the Bohr magneton, g is the Lande g-factor equal to 0.8, k_f is the Fermi wave vector [19].

Now the probabilities a_1, b_2 can be obtained by applying the matching conditions at the boundaries $z = 0$ and $z = L$ to the wave function $\Psi_{\sigma,nl}$ in Eq. (10) [20]. For simplicity let $\xi, \beta = 0$ [20]. These matching conditions are

$$\Psi^{FM_1}_{\sigma,nl}\Big|_{z=0} = \Psi^{SC}_{\sigma,nl}\Big|_{z=0} \tag{15}$$

$$\Psi^{SC}_{\sigma,nl}\Big|_{z=L} = \Psi^{FM_2}_{\sigma,nl}\Big|_{z=L}, \tag{16}$$

$$\frac{d\Psi^{Sc}_{\sigma,nl}}{dz}\bigg|_{z=0} - \frac{d\Psi^{FM_1}_{\sigma,nl}}{dz}\bigg|_{z=0} = \frac{2m^* V_b}{h^2}\Psi^{FM_1}_{\sigma,nl}\Big|_{z=0}, \tag{17}$$

$$\frac{d\Psi^{FM_2}_{\sigma,nl}}{dz}\bigg|_{z=L} - \frac{d\Psi^{SC}_{\sigma,nl}}{dz}\bigg|_{z=L} = \frac{2m^* V_b}{h^2}\Psi^{FM_2}_{\sigma,nl}\Big|_{z=L}. \tag{18}$$

Using Mathematica code together with the previous matching conditions we can evaluate the operators a and b_1. Then, using these values we calculate the following probabilities a_1 and b_2 corresponding to Andreev-reflection and normal reflection

$$a_1 = \frac{k^+_{nl}\left|J_n\left(y_2\right)\right|^2}{p^-_{\sigma,nl}\left|J_n\left(y_1\right)\right|^2}\left(a \times a^*\right), \tag{19}$$

$$b_2 = \frac{k_{nl}^{+} \left|J_n\left(y_2\right)\right|^2}{p_{\sigma,nl}^{+} \left|J_n\left(y_1\right)\right|^2}\left(b_1 \times b_1^{*}\right), \tag{20}$$

where a^* is the complex conjugate of the Andreev reflection amplitude, a, and b_1^* is the complex conjugate of the normal reflection amplitude, b_1, and the values of y_1 and y_2 for zero and first-order of the Bessel function are y_1 = 2.40483 and y_2 = 5.520082 [21]. Then, substituting Eqs. (19, 20) into Eq. (1) we get an equation for the conductance, G, which will be solved numerically. The obtained equation for conductance depends on the dimension of the device W_1, W_2, magnetic field, B, exchange field of the ferromagnetic material, h, the bias voltage, V_b, and the temperature, T. The spin polarization of the quasi particle due to Andreev-reflection at the ferromagnetic /superconductor interface can be determined through the following equation [22]:

$$P = \frac{a_{1\uparrow}(E) - a_{1\downarrow}(E)}{a_{1\uparrow}(E) + a_{1\downarrow}(E)}, \tag{21}$$

$a_{1\uparrow}$, $a_{1\downarrow}$ are given by Eq.(19) corresponding to spin-up and spin-down quasiparticles respectively

3. Results and Discussion

The present section will discuss the results of the spintronic model which is a ferromagnetic/superconducting materials hybrid structure. Spin-polarized tunneling plays an important role in the spin-dependent transport of magnetic nanostructures [23]. The spin-polarized electron injected from ferromagnetic materials into nonmagnetic one such as superconductor creates a non-equilibrium spin polarization in such nonmagnetic materials [24–26]. Also, for such present studied model, the interplay between Andreev-reflection and spin polarization plays an important role and are studied [22,27–31]. Ferromagnetism and superconductivity are two competing phenomena in condensed matter physics.

Due to the competing ordering of ferromagnetic materials and superconductors in hybrid structures, many nontrivial physical effects occur [32]. Numerical calculations are performed for the present F/S junction, in which the superconductor is Nb and the ferromagnetic leads are of any one of the ferromagnetic materials. The conductance of the junction was found to depend on the exchange field of the ferromagnetic material, h, the magnetic field, B (in the term due to Zeeman energy Eq. (14), the bias voltage V_b and the dimensions of the junction W_1 and W_2.

The features of the present results are: figure 1 shows the dependence of the conductance, G, on the temperature, T, at different parameters of B, h, V_b, W_1 and W_2. From the figure, we notice that the conductance, G decreases as the temperature, T increases. This trend confirms that the present junction is in the mesoscopic regime, that is, the tunneling process is ballistic one [33,34]. The effect of the exchange field, h, of the ferromagnetic on the spin transport through the junction is studied and the obtained results are given in figures 2, 3 which shows the dependence of the conductance, G, on the exchange field, h, at different values of bias voltage, V_b, temperature, T, magnetic field, B and dimensions W_1 and W_2.

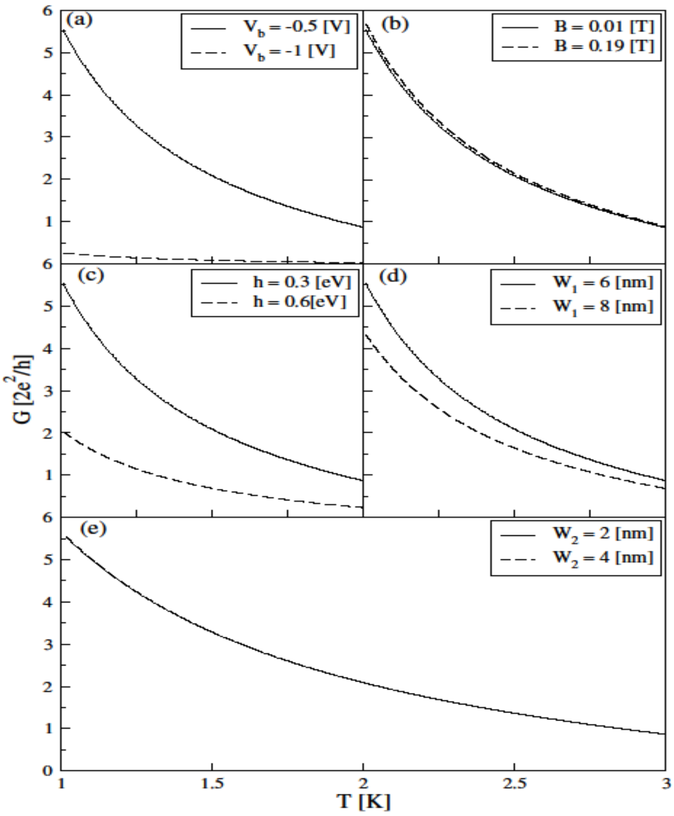

Fig. 1. The variation of the conductance (G) with the temperature (T) at different bias voltage (V_b), magnetic field (B), exchange field (h) and the dimensions of the ferromagnetic leads and superconductor QD W_1 and W_2, respectively.

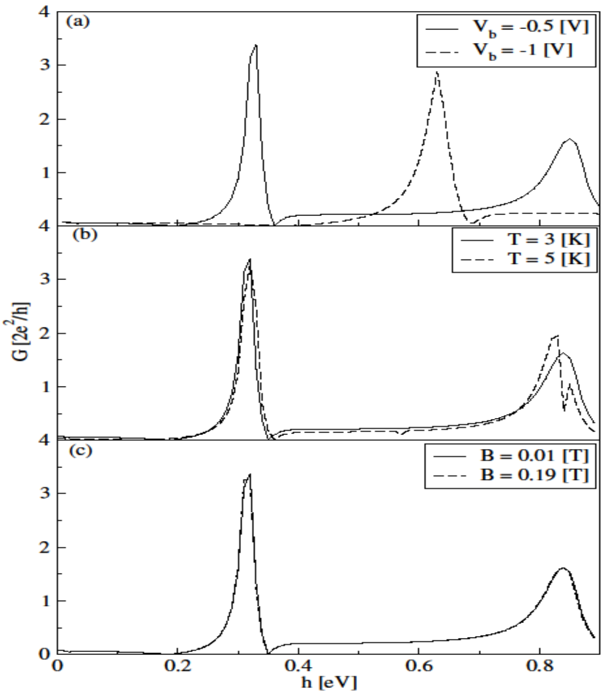

Fig. 2. The variation of the conductance (G) with the exchange field h at different parameters V_b, T and B.

The results show two prominent peaks with both different peaks heights and widths, at specific values of exchange field, *h*, of the ferromagnetic leads. Only, we noticed one peak in figure 2 when the bias voltage varies from −0.5 V to −1 V.

The range of the applied external magnetic field is in the limit of superconductivity of Nb (B_c = 0.19T). These two peaks are associated with Zeeman splitting of the quasiparticle density of states. Such present results are confirmed by other authors [35].

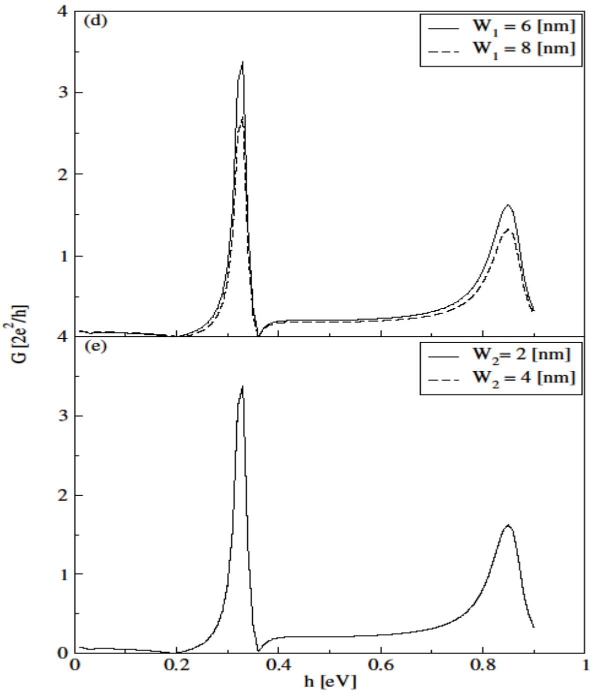

Fig. 3. The variation of the conductance (*G*) with the exchange field h at different parameters W_1 and W_2.

The present results might be explained as follows: magnetic field, *B*, is applied parallel to the plane of the tunnel junction when the thickness of the superconductor island is small as compared to the penetration depth of the magnetic field; the field penetrates the superconductor uniformly. Due to the interaction between the electron's magnetic moment and the field, the electron energy depends on the spin direction. When the field points in upward direction the energy of spin-up electrons is lowered by the following product: its magnetic moment multiplied by magnetic field [22,36] and the energy of the spin-down electrons is raised by the aforementioned product. This energy difference is known as Zeeman splitting. Such present results are confirmed by the authors [22,36].

The variation of the conductance, *G*, with the bias voltage, V_b, at different parameters *h*, *T*, *B*, W_1 and W_2 are shown in figures 4, 5. We notice those two peaks of different heights and widths as in the case of figures 2, 3 and figure 4 shows a peculiar behavior as the peaks spacing, when the exchange field, h, is doubled, are more displaced far enough from each other. For example, when *h* = 0.6eV, the first peak appears approximately V_b = –0.98 V. While when *h* = 0.3eV the first peak appears at approximately $V_b = -0.49$V. This doubling shift in the peak position may be due to the interplay of spin polarization due to Andreev-reflection of the F/S interface and Zeeman splitting of the quasi-particle of states. When a magnetic field is applied parallel to the superconductor island plane the quasiparticle density of states (DOS) in the superconductor is split due to the Zeeman interaction into the spin-up and spin-down populations. Due to unequal DOS at the Fermi energy in the ferromagnetic materials the tunneling conductance into superconductor becomes asymmetric. These results are found to be concordant with those in the literature [22,31,36,37]. So, we can show that Zeeman splitting can be used to resolve spin-up and spin-down Andreev-reflections with a different threshold voltage. A coupling between Andreev-reflection and Zeeman splitting dominate in the present spin transport mechanism.

Spintronics is a research field where two fundamental branches of physics, i.e, magnetism and electronics, are combined [30], and it is usually based on the opportunity of ferromagnetic materials to provide spin-polarized currents [38, 39].

The effectiveness of spintronics depends on the extent to which a current is spin-polarized, which turns out to depend on the degree of polarization of the ferromagnetic materials. The performance of any spintronic device, in fact, improves as the polarization attains a high value. The availability of highly spin-polarized sources is thus of crucial importance from both fundamental and technological side. According to the above argument, we perform a numerical calculation for the polarization and its variation with the bias voltage V_b at different values of T, h, B, W_1 and W_2 (Figures 6, 7). As seen from the figures, in general, the parameter polarization varies between maximum (positive value) and minimum (negative values) values at creation values of the bias voltage V_b.

The peak heights in the positive and negative directions are different for different values T, B, h, W_1 and W_2. Such trend of the dependence of P on V_b had shown previously by the authors [22,40]. These results are due to the flip of the electron spin when Andreev-reflection tunneling occurs through the junction. That is a spin-up electron incoming from the ferromagnetic leads is reflected as a hole in the spin-down band while a spin zero Cooper pair is transferred into the superconductor. The sign and magnitude of polarization can be tuned by varying the bias voltage and Zeeman energy.

As a whole, the present results can be explained as the following: The spin-polarized transport depends on the relative orientation of magnetization in the two ferromagnetic leads. The spin polarization of the tunneled electron through the junction gives rise to a non-equilibrium spin density in the superconductor and also due to Zeeman splitting of the quasiparticle density of states

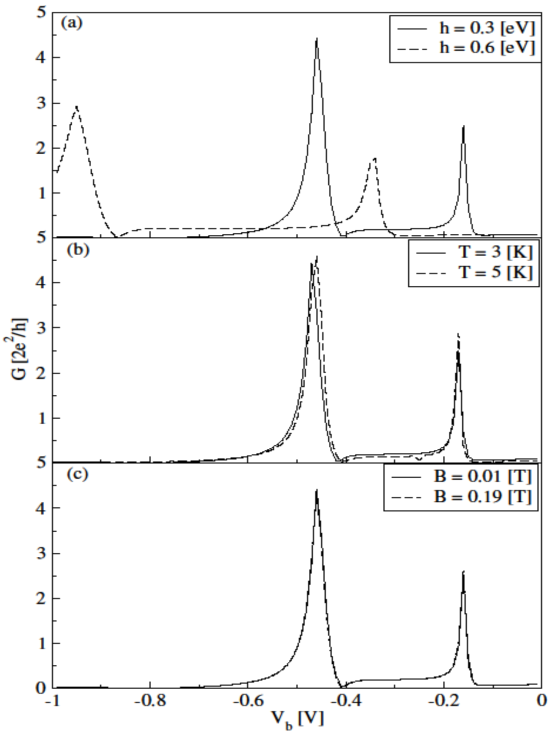

Fig. 4. The variation of the conductance (G) with the bias voltage V_b at different parameters h, T and B.

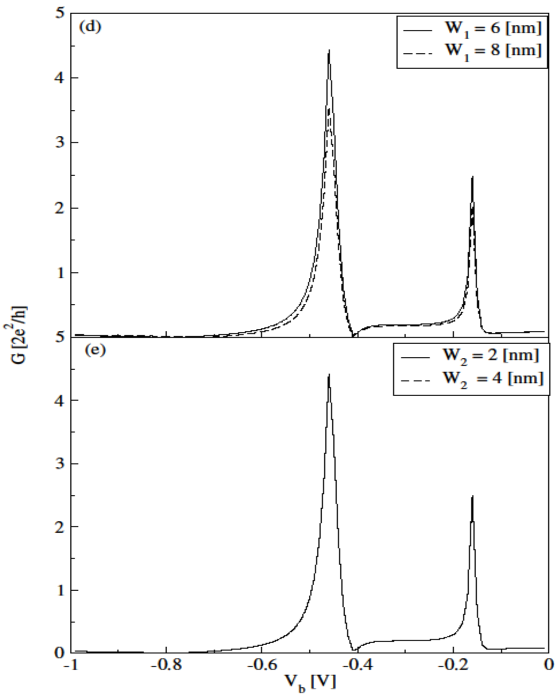

Fig. 5. The variation of the conductance (G) with the bias voltage V_b at different parameters W_1 and W_2.

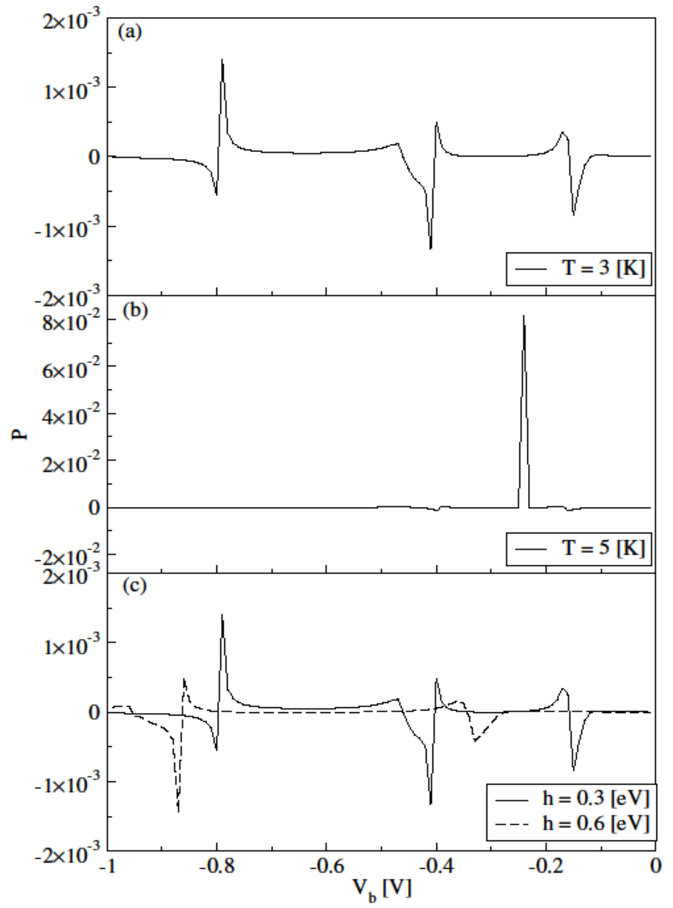

Fig. 6. The variation of the polarization P with the bias voltage V_b at different parameters T and h.

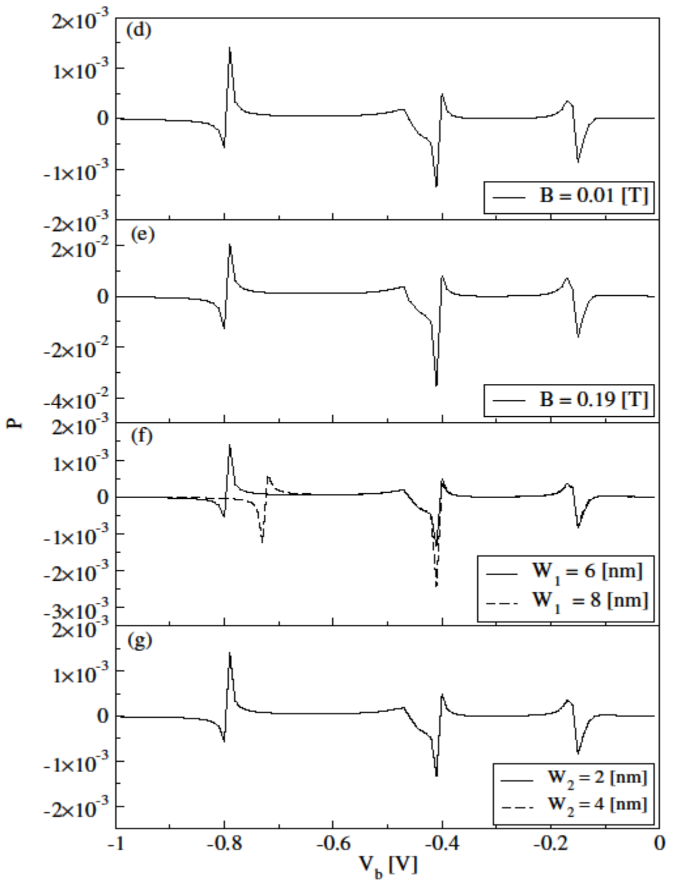

Fig. 7. The variation of the polarization P with the bias voltage V_b at different parameters B, W_1 and W_2.

4. Conclusion

The spin-dependent transport through F/S/F junction is investigated. We have deduced an expression for the conductance of the junction and also the polarization by solving Bogoliubov de-Gennes equation taking into account Andreev-reflection of spin polarized quasiparticles at the interface. The obtained expressions for both conductance and polarization depend on the dimensions of the junction, the exchange field of the ferromagnetic leads, the bias voltage, the temperature and the magnetic field. Numerical calculations have been performed using Mathematica and FORTRAN software. Our results show a quite fair agreement with those in the literature. The present investigation is important for the field of spintronics, nanoelectronics and quantum information processing.

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