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REFLECTION AND TRANSMISSION BAND STRUCTURES OF A ONE-DIMENSIONAL PERIODIC SYSTEM IN THE PRESENCE OF ABSORPTION

A. Zh. Khachatrian

State Engineering University of Armenia, Yerevan, Armenia

E-mail: ashot.khachatrian@gmail.com

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Abstract–On the base of the generalized transfer matrix method the problem of electromagnetic wave propagation through an arbitrary periodic one-dimensional absorbing medium is considered. Analytical expressions for transmission and reflection amplitudes are found. The obtained results are applied to a model system represented by a one-dimensional periodic array of alternating layers of vacuum and a metal characterized by the complex frequency-dependent dielectric function. It is shown that in an occupied whole space periodic system a harmonic time wave process cannot exist. Independently of the frequency value, this process always attenuates.

Keywords: generalized transfer matrix, absorbing medium, transmission and reflection bands

1. Introduction

Photonic crystals represent a new kind of optical materials, which possess many interesting properties and render many novel applications possible as well [1–6]. The existence of photonic band gaps in photonic crystals, owing to multiple Bragg scatterings, leads to many interesting phenomena [1–9]. Most photonic crystals fabricated so far are made from two dielectric materials. Nowadays, the dielectric photonic crystals (DPC) have been widely studied and received a great deal of attention because of its perceived applications, properties and new physical phenomena.

Usually, photonic band gaps of dielectric photonic crystals are not wide. Combinations of metallic and dielectric materials may lead to more interesting properties comparing with dielectric photonic crystals. The inclusion of metal sheets into dielectric photonic crystal can increase photonic band gaps considerably [10–12]. For example, the absorption of the bulk metal can be enhanced by inserting a dielectric layer periodically to one-dimensional metal-dielectric photonic crystals. By a proper choice of the structural and material parameters, one can obtain a large absorption enhancement in the visible and the infrared ranges [12–14]. In addition, the inclusion of periodically spaced thin metallic elements is becoming quite attractive at nanodimensions, where metamaterial have already shown quite interesting phenomena. Metal-dielectric photonic crystals, as basic stacks of a single dielectric with metallic insets at very low filling fractions (less than 1%), were first studied by Kuzmiak and Maradudin [15] using dispersion relations as approximate solutions. All those single dielectric structures have no paragon with dielectric photonic crystals, where the approach based on a dispersion relation and the transfer matrix method are equivalent.

Despite the fact that an investigation of one-dimensional periodic structures consisting of absorbing layers was a subject of interest for many authors and has a great interest for many years, until now an analytical solution of this problem is unknown. The problem is that the operator of a field wave is not Hermitian, so that the flux of electromagnetic wave energy is not conserved. In the standard method of transfer matrix [16–17], the complex transmission and reflection amplitudes are derived from the elements of the transfer matrix, however the inverse relation between complex scattering characteristics and transfer matrix elements is not established. We consider the mentioned matter as a main disadvantage of the standard transfer matrix method and it is a main reason why for a multilayer system all calculations are performed by numerical methods only.

In this work, we present a general method of electromagnetic wave propagation in a one-dimensional absorbing media of an arbitrary shape. We show that for a multilayered structure the problem of determination of the scattering amplitudes is reduced to solution of a set of recurrent equations. In the case of absorbing media with a continually changing permittivity the problem is formulated as an initial condition problem for the wave equation. The developed method is applied for discussion of a photonic crystal. Namely, for transmission and reflection amplitudes of a photonic crystal analytical expressions are found and absorption band structure is investigated.

2. Generalized transfer matrix method for an arbitrary one-dimensional absorbing medium

We consider a harmonic in time electromagnetic waves ($\exp\{-i\omega t\}$) in a one-dimensional absorbing medium. It is well known that the coordinate dependence of the field electric component is described by the following equation:

$$\frac{d^2E}{dx^2}+\left(k^2-V(x)\right)E=0, \tag{1}$$

where

$$V(x)=V_1(x)+iV_2(x),\ k^2=\omega^2/c^2,\ V_1=k^2\left(1-\varepsilon'(x)\right),\ V_2=-k^2\varepsilon''(x)$$

and $\varepsilon'(x)$, $\varepsilon''(x)$ are real and imagine parts of a permittivity. Below we discuss a wave transmission problem through irregularly absorbing slab when the dielectric constant has a form of

$$\begin{cases} 1, & x<x_1, \\ \varepsilon'(x)+i\varepsilon''(x), & x_1<x<x_2, \\ 1, & x>x_2. \end{cases} \tag{2}$$

In the regions outside the slab the general asymptotic solution of Eq. (1) can be written as

$$E(x)=\begin{cases} A_1\exp\{ikx\}+B_1\exp\{-ikx\}, & x<x_1, \\ A_2\exp\{ikx\}+B_2\exp\{-ikx\}, & x>x_2. \end{cases} \tag{3}$$

If $k>0$ the quantities A_1, B_2 and B_1, A_2 are the amplitudes of convergent and divergent waves. It is clear that when $k<0$ then A_1, B_2 become the amplitudes of divergent waves and B_1, A_2 become the amplitudes of convergent waves.

In accordance with the transfer matrix method (see [18]) between the amplitudes A_1, B_1 and A_2, B_2 a linear relation exists:

$$\begin{pmatrix} A_2 \\ B_2 \end{pmatrix} = \hat{M}\begin{pmatrix} A_1 \\ B_1 \end{pmatrix}, \ \hat{M} = \begin{pmatrix} \alpha & \beta \\ \gamma & \delta \end{pmatrix}, \tag{4}$$

Where α, β, γ, δ are the transfer matrix elements and

$$\alpha\delta - \gamma\beta = 1. \tag{5}$$

Let us consider the left scattering problem, when $k > 0$ and the field asymptotic behavior has the form of

$$E_{left} = \begin{cases} \exp\{ikx\} + r(k)\exp\{-ikx\}, \ x < x_1, \\ t(k)\exp\{ikx\}, \ x > x_2, \end{cases} \tag{6}$$

where $t(k)$, $r(k)$ are transmission and reflection amplitudes coefficients of the wave incident on the slab from the left. Below, mentioned transmission and reflection amplitudes will be defined in correspondence with the left scattering problem (6).

Comparing Eq. (3) and Eq. (6) it is easy to see that

$$A_1 = 1, \ B_1 = r(k) \ \text{ and } \ A_2 = t(k), \ B_2 = 0. \tag{7}$$

Substituting Eq. (7) into Eq. (4) and taking into account Eq. (5), one can get

$$\gamma = -\frac{r(k)}{t(k)} \ \text{ and } \ \delta = \frac{1}{t(k)}. \tag{8}$$

The obtained result (8) is the basic formulas of the standard transfer matrix theory, which is usually applied for calculation of transmission, reflection and absorption coefficients of a dissipative medium. In the case of nonabsorbing media ($\varepsilon''(x) = 0$) between the transfer matrix elements the following nonalgebraic relations exist: $\alpha = \delta^*$, $\beta = \gamma^*$.

Unfortunately Eq. (8) allows performing numerical calculations only and these formulas are not effective to get analytical results. Namely, until now analytical expressions for transmission and reflection amplitudes of an electromagnetic wave scattering on a periodic absorbing medium are not obtained. In the recent papers [18, 19] a so-called generalized transfer matrix method is developed, which as will be shown below, is very efficient both to perform numerical calculations and to obtain analytical results. In accordance with the main results of the above-mentioned papers, the transfer matrix for an arbitrary absorbing medium has the following form

$$\hat{M} = \begin{pmatrix} \dfrac{1}{t(-k)} & \dfrac{-r(-k)}{t(-k)} \\ \dfrac{-r(k)}{t(k)} & \dfrac{1}{t(k)} \end{pmatrix}. \tag{9}$$

As it is seen from Eq. (9) and Eq. (4) in general case the transformation between the transfer matrix elements is realized by mean of changing the sign of the quantity k. Note that for the case of nonabsorbing media this action ($k \to -k$) is equivalent to a complex conjugation action, i.e. $t^*(k) = t(-k)$, $r^*(k) = r(-k)$.

For consideration of a wave propagation problem for a multilayered system, in the papers [19, 20] the following matter is discussed: how the transfer matrix elements are changed when the slab is parallel transported on a some distance L. As it was shown

$$\alpha' = \alpha,\ \delta' = \delta \ \text{ and } \ \beta' = \beta\exp\{-i2kL\},\ \gamma' = \gamma\exp\{i2kL\},$$

where $\alpha',\ \beta',\ \gamma'$ and δ' are the transfer matrix elements of the transported slab.

3. Transmission through multilayered structure

Let us consider the transfer matrix $\hat{M}_N$ of a N-layered structures as a product of transfer matrices of the system's separate layers:

$$\hat{M}_N = \begin{pmatrix} \alpha_N & \beta_N \\ \gamma_N & \delta_N \end{pmatrix} \cdots \begin{pmatrix} \alpha_n & \beta_n \\ \gamma_n & \delta_n \end{pmatrix} \cdots \begin{pmatrix} \alpha_1 & \beta_1 \\ \gamma_1 & \delta_1 \end{pmatrix} = \prod_{n=N}^{1} \begin{pmatrix} \alpha_n & \beta_n \\ \gamma_n & \delta_n \end{pmatrix}, \tag{10}$$

In accordance with Eq. (9), the transfer matrix of the system can be written as

$$\hat{M}_N = \begin{pmatrix} \dfrac{1}{T_N(-k)} & \dfrac{-R_N(-k)}{T_N(-k)} \\ \dfrac{-R_N(k)}{T_N(k)} & \dfrac{1}{T_N(k)} \end{pmatrix}, \tag{11}$$

where T_N and R_N are transmission and reflection amplitudes of the system. A transfer matrix of a single layer written with help of the scattering amplitudes can be presented as

$$\begin{pmatrix} \alpha_n & \beta_n \\ \gamma_n & \delta_n \end{pmatrix} = \begin{pmatrix} \dfrac{1}{t_n(-k)} & \dfrac{-r_n(-k)}{t_n(-k)} \\ \dfrac{-r_n(k)}{t_n(k)} & \dfrac{1}{t_n(k)} \end{pmatrix}, \tag{12}$$

where t_n and r_n are transmission and reflection amplitudes of a n-th single layer of the system.

If one considers N as a variable quantity, when on the base of Eq. (10) and Eq. (11) the problem of determination of T_N and R_N can be reduced to the solution of a set of finite-differential equations with initial conditions. Indeed, from Eq. (10) and Eq. (11) one can write down

$$\hat{M}_N = \begin{pmatrix} \alpha_N & \beta_N \\ \gamma_N & \delta_N \end{pmatrix} \hat{M}_{N-1}, \tag{13}$$

where $\hat{M}_{N-1}$ is the transfer matrix of the system consisting only of the first $N-1$ layers (without the last layer) of the system. Using Eq. (13), Eq. (12) and Eq. (10) one has

$$\begin{pmatrix} \dfrac{1}{T_N(-k)} & \dfrac{-R_N(-k)}{T_N(-k)} \\ \dfrac{-R_N(k)}{T_N(k)} & \dfrac{1}{T_N(k)} \end{pmatrix} = \begin{pmatrix} \dfrac{1}{t_N(-k)} & \dfrac{-r_N(-k)}{t_n(-k)} \\ \dfrac{-r_N(k)}{t_N(k)} & \dfrac{1}{t_N(k)} \end{pmatrix} \begin{pmatrix} \dfrac{1}{T_{N-1}(-k)} & \dfrac{-R_{N-1}(-k)}{T_N(-k)} \\ \dfrac{-R_{N-1}(k)}{T_{N-1}(k)} & \dfrac{1}{T_{N-1}(k)} \end{pmatrix}. \tag{14}$$

It is easy to see that the matrix equation (14) is equivalent to the following set of finite-differential equations:

$$\frac{1}{T_N(k)} = \frac{1}{t_N(k)} \frac{1}{T_{N-1}(k)} + \frac{r_N(k)}{t_N(k)} \frac{R_N(-k)}{T_N(-k)}, \tag{15}$$

$$\frac{R_N(-k)}{T_N(-k)} = \frac{1}{t_N(-k)} \frac{R_N(-k)}{T_N(-k)} + \frac{r_N(-k)}{t_N(-k)} \frac{1}{T_N(k)}, \tag{16}$$

with the initial condition

$$T_0(k) = 1,\ R_0(-k) = 0. \tag{17}$$

Note that the initial condition corresponds to the system without layers, i.e. to the free space motion. It is easy to see that with respect to quantities $1/T_N(k)$, $R_N(-k)/T_N(-k)$ the set of equations (15), (16) is linear. In general case of an arbitrary non-regular structure, when the layers of the system and the distances between them differ, the set of equations (15), (16) can be solved only numerically. As it will be shown below on the base of equations (15), (16) analytical expressions for transmission and reflection amplitudes can be derived.

4. The transfer matrix elements as functions of a slab border point

It is interesting to apply the result (15)–(17) to the problem of determination of scattering amplitudes for a single layer with an arbitrary but continuously changing from point to point permittivity. We will follow to the method developed in the papers [20-22], where the scattering amplitudes as functions of the slab borders are considered (see Fig. 1). For simplicity we will consider a slab with the initial point at $x = 0$. Let us identify a slab having width y with a N-layered structure and a slab of width $y - \Delta y$ with $N-1$-layered structure where the last N layer is absent. The last N layer of the structure will be considered as a layer with small width Δy. In accordance with the above mentioned Eq. (15), (16) one can consider

$$\frac{1}{T_N(k)} \approx \delta(y),\ \frac{R_N(-k)}{T_N(-k)} \approx -\beta(y) \text{ and } \frac{1}{T_{N-1}(k)} \approx \delta(y - \Delta y),\ \frac{R_N(-k)}{T_N(-k)} \approx -\beta(y - \Delta y), \tag{18}$$

where $\delta(y) = 1/t(k, y)$. Note that in accordance with Eq. (2) when $y = x_1$ the slab width equals zero and the magnitude of $y = x_2$ corresponds to the system whole width.

To obtain transmission and reflection amplitudes for a thin layer of a width Δy we will use the well-known formulas corresponding to the uniform slab [17]:

$$\frac{1}{t_N(k)} \approx \exp\{ik\Delta y\}\left[\cos[q(y)\Delta y] - i\frac{q^2(y)+k^2}{2kq(y)}\sin[q(y)\Delta y]\right], \tag{19}$$

$$\frac{r_N(k)}{t_N(k)} \approx i\frac{q(y)^2-k^2}{2kq(y)}\exp\{i2ky\}\sin[q(y)\Delta y], \tag{20}$$

where $q(y) = k\sqrt{\varepsilon(y)}$. Considering $q(y)\Delta y << 1$ and taking into account (1), it is easy to get

$$\frac{1}{t_N(k)} \approx 1 + \frac{iu(x)}{2k}\Delta y, \ \frac{r_N(k)}{t_N(k)} = -\frac{iu(x)}{2k}\Delta y \exp\{i2ky\}. \tag{21}$$

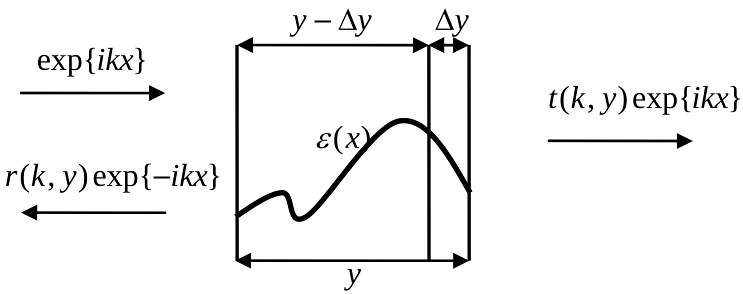

Fig. 1. Transmission and reflection amplitudes as functions of the slab right border.

Substituting Eq. (18) and Eq. (21) in the set (15), (16) one can found the following set of equations for determination of the transfer matrix elements of a slab with a continually changing permittivity:

$$\frac{d\delta(x)}{dx} = \frac{iV(x)}{2k}\delta(x) + \frac{iV(x)}{2k}\beta(x)\exp\{i2kx\}, \tag{22}$$

$$\frac{d\beta(x)}{dx} = -\frac{iV(x)}{2k}\beta(x) - \frac{iV(x)}{2k}\delta(x)\exp\{-i2kx\}, \tag{23}$$

with initial conditions

$$\delta(x_1) = 1, \ \beta(x_1) = 0. \tag{24}$$

Equations for the transfer matrix elements $\alpha(x)$, $\gamma(x)$ ($\alpha(x) = 1/t(-k,x)$, $\gamma(x) = -r(k,x)/t(k,x)$) are obtained from Eq. (22) and Eq. (23) by changing k by $-k$:

$$\frac{d\alpha(x)}{dx} = -\frac{iV(x)}{2k}\alpha(x) - \frac{iV(x)}{2k}\gamma(x)\exp\{-i2kx\}, \tag{25}$$

$$\frac{d}{dx}\gamma(x) = \frac{iV(x)}{2k}\gamma(x) + \frac{iV(x)}{2k}\alpha(x)\exp\{i2kx\}, \tag{26}$$

with initial conditions

$$\alpha(x_1) = 1, \ \gamma(x_1) = 0. \tag{27}$$

It is easy to see that the pairs of quantities $\delta(x)$, $\beta(x)$ and $\alpha(x)$, $\gamma(x)$ satisfy the same set of equations but initial conditions for them are different. Note that in case of a nonabsorbing slab

($u(x)$ is a real function) the set of equations (22), (23) are the complex conjugate with respect to the set (25), (26).

So we have shown that the problem of determination of scattering amplitudes for an absorbing slab with an arbitrary continually changing $\varepsilon(x)$ is reduced to Cauchy problem for a set of linear differential equations. It is important to mention that the set of equations (22), (23) (or (25), (26)) are very useful for a numerical integration but these equations are not applicable for analytical calculations. By using equations (22), (23) and (25) it is possible to show that the problem of determination of transfer matrix elements can be formulated as an initial type of problem for the wave equation (2):

$$\delta(x)=\frac{\exp\{ik(x-x_1)\}}{2}\left[\Psi_1(x)-ik\Psi_2(x)+\frac{i}{k}\frac{d\Psi_1(x)}{dx}+\frac{d\Psi_2(x)}{dx}\right], \tag{28}$$

$$\beta(x)=\frac{\exp\{-ik(x+x_1)\}}{2}\left[\Psi_1(x)-ik\Psi_2(x)-\frac{i}{k}\frac{d\Psi_1(x)}{dx}+\frac{d\Psi_2(x)}{dx}\right], \tag{29}$$

where $\Psi_1(x)$, $\Psi_2(x)$ satisfy the wave equation (2):

$$\frac{d^2\Psi_{1,2}}{dx^2}+\left(k^2-V(x)\right)\Psi_{1,2}=0\,, \tag{30}$$

with the initial conditions

$$\Psi_1(x_1)=1,\ d\Psi_1(x_1)/dx=0\ \text{ and }\ \Psi_2(x_1)=0\,,\ d\Psi_2(x_1)/dx=1\,. \tag{31}$$

It is important to mention that due to the fact that the equation (30) contains only k^2 the expressions for $\alpha(x)$, $\gamma(x)$ can be obtained from (28), (29) by changing k by $-k$. Note that determination of the transfer matrix elements in the form of the initial problem (28)–(31) is useful both for analytical and numerical calculations. This result is a generalization of the corresponding formulas of the paper [23] obtained for media without dissipation.

5. Scattering on an ideal transfer structure

Let us consider a layered structure, when the layers are identical and equidistantly located. At this step of consideration, we do not assume that a single layer structure must be uniform or that we explicitly know the dependence of the transfer matrix elements on the frequency of an incident radiation and the parameters of a scattering layer. For the considered layered structure the permittivity is characterized by the following dependence:

$$V(x)=\sum_{n=1}^{N}V_n(x)\,, \tag{32}$$

where $V_n(x)$ relates to the *n*-th layer of the system, which is presented by means of the function $V_1(x)$, defining the optical properties of the first layer, in accordance with the following relation

$$V_n(x) = V_1(x-(n-1)L)\,, \quad n = 1, 2 \cdots N\,, \tag{33}$$

where L is the system period.

Denoting the border points of the first layer as $y_1,\ y_2$ ($y_1 < y_2$), one can write down

$$u_1(x) = \frac{\omega^2}{c^2}\left(1-\varepsilon(x)\right)\theta(x-y_1)\theta(y_2-x)\,, \tag{34}$$

where $\varepsilon(x) = \varepsilon'(x) + i\varepsilon''(x)$ is the permittivity of the first layer and $\theta(x)$ is the step function. Note that the possible value of the period L should be considered more than a width magnitude of a single layer ($L \geq (y_2 - y_1)$), otherwise an overlapping of neighbor layers will take place.

Let us consider a relation between the transfer matrix elements of the system's n-th layer and the transfer matrix elements of its first layer. In accordance with the last equation of the section 2, one can write down

$$\alpha_n = \alpha,\ \ \delta_n = \delta\,,\ \ \beta_n = \beta\exp\{-i2kL(n-1)\},\ \ \gamma_n = \gamma\exp\{i2kL(n-1)\}\,, \tag{35}$$

where for simplicity we denoted the transfer matrix elements of the first layer as $\alpha_1 = \alpha$, $\beta_1 = \beta$, $\gamma_1 = \gamma$, $\delta_1 = \delta$.

Using Eq. (35), the set of equations (15), (16) can be written as

$$\frac{1}{T_N(k)} = \frac{1}{t(k)}\frac{1}{T_{N-1}(k)} + \frac{r(k)}{t(k)}\exp\left\{i2kL(n-1)\right\}\frac{R_N(-k)}{T_N(-k)}\,, \tag{36}$$

$$\frac{R_N(-k)}{T_N(-k)} = \frac{1}{t(-k)}\frac{R_N(-k)}{T_N(-k)} + \frac{r(-k)}{t(-k)}\exp\left\{-i2kL(n-1)\right\}\frac{1}{T_N(k)} \tag{37}$$

with initial conditions

$$T_0(k) = 1,\ \ R_0(-k) = 0\,. \tag{38}$$

Note that here the quantities $t(k)$ and $r(k)$ are transmission and reflection amplitudes of the structure's first layer: $\alpha = 1/t(-k)$, $\gamma = -r(k)/t(k)$.

The set of equations (15), (16) can be solved by means of different methods and its solution has the form

$$\frac{1}{T_N(k)} = \exp\{ikLN\}\left(\cos(\mu NL) + \frac{1}{2}\left(\frac{\exp\{-ikL\}}{t(k)} - \frac{\exp\{ikL\}}{t(-k)}\right)\frac{\sin(\mu NL)}{\sin(\mu L)}\right), \tag{39}$$

$$\frac{R_N(k)}{T_N(k)} = \exp\{ikL(N-1)\}\frac{r(k)}{t(k)}\frac{\sin(\mu NL)}{\sin(\mu L)}\,, \tag{40}$$

where we introduced the following notation:

$$\cos\mu L = \frac{1}{2}\left(\frac{\exp\{-ikL\}}{t(k)} + \frac{\exp\{ikL\}}{t(-k)}\right). \tag{41}$$

For case of nonabsorbing media (as it was mentioned above, for that case the equality $t(-k)=t^{*}(k)$ takes place) the result (39)–(41) is reduced to the corresponding formulas of the paper [22]. Note that the quantity μ can take either real or purely imagine value since in this case the right side of the equality (41) is a real quantity: $\cos\mu L=\mathrm{Re}\left(\exp\{-ikL\}/t(k)\right)$. However, in the more general case of absorbing media μ has a complex value and as a function of k it is an even function ($\mu(k)=\mu(-k)$). The obtained formulas (39)–(41) express the explicit dependences of the system scattering amplitudes T_N, R_N on the number of layers N and the scattering amplitudes t, r of the element of the system structure. Despite the fact that Eqs. (39)–(41) are analytical expressions for their investigation there is a necessity in additional discussions.

6. Photonic crystal with a two-layered structure element

Below we will examine the obtained result (39)-(41) for a photonic crystal with a two-layered structure element. Let us suppose that the structure element consists of two homogeneous and contacting with each other layers of widths a_1, a_2, so that the period of the photonic crystal is $L=a_1+a_2$. To determine the transmission and reflection amplitudes of a two-layered structure t and r (see Eq. (39)-(41)) we will use the formulas (15), (16):

$$\frac{1}{t(k)}=\frac{1}{t^{(2)}(k)}\frac{1}{t^{(1)}(k)}+\frac{r^{(2)}(k)}{t^{(2)}(k)}\frac{r^{(1)}(-k)}{t^{(1)}(-k)}, \tag{42}$$

$$\frac{r(-k)}{t(-k)}=\frac{1}{t^{(2)}(-k)}\frac{r^{(1)}(-k)}{t^{(1)}(-k)}+\frac{r^{(2)}(-k)}{t^{(2)}(-k)}\frac{1}{t^{(1)}(k)}, \tag{43}$$

where $t^{(j)}$, $r^{(j)}$ ($j=1,2$) are the transmission and reflection amplitudes of the first and second layers of the structure elements, correspondingly:

$$\frac{1}{t^{(j)}(k)}=\exp\{ika_j\}\left[\cos[q_ja_j]-i\frac{q_j^2+k^2}{2kq_j}\sin[q_ja_j]\right],\ (j=1,2) \tag{44}$$

$$\frac{r^{(j)}(k)}{t^{(j)}(k)}=i\frac{q_j^2-k^2}{2kq_j}\exp\{ikb_j\}\sin[q_ja_j],\ (j=1,2) \tag{45}$$

where $q_j=k\sqrt{\varepsilon_j}$, $k=\omega/c$ and ε_1, ε_2 are the permittivities of the layers. In Eq. (45) b_j is the coordinate of the middle point of the structure element layer. So, if the initial point of the first structure element coincides with a coordinate origin then $b_1=a_1$, $b_2=2a_1+a_2$.

Substituting Eqs. (44), (45) in Eq. (42) from Eq. (41) one can write

$$\cos\mu L=\cos q_1a_1\cos q_2a_2-\frac{q_1^2+q_2^2}{2q_1q_2}\sin q_1a_1\sin q_2a_2, \tag{46}$$

which is the well-known expression for the quasi-wave number that determines the relationship between the wave damping in a superlattice and damping for a single layer forming this structure (see, for example, [17]).

Below we will apply the result (39)–(46) for a specific structure, namely, we consider a one-dimensional photonic crystal on the base of nanocomposite: metal nanoparticle–dielectric matrix. Note that the composite media with nanoparticles of noble metals are of great practical interest in the development of various optical devices [23, 24]. It is clear that the optical properties of such media are determined by plasma oscillations in the nanoparticles and the properties of a transparent matrix.

We calculate the transmission, reflection and absorption coefficients of a one-dimensional photonic crystal consisting of nanocomposite metal nanoparticles which are randomly distributed in a transparent matrix. To determine the permittivity of the nanocomposite $\varepsilon_{mix}(\omega)$ we use the Maxwell–Garnett formula:

$$\frac{\varepsilon_{mix}(\omega)-\varepsilon_d}{\varepsilon_{mix}(\omega)+2\varepsilon_d}=f\,\frac{\varepsilon_m(\omega)-\varepsilon_d}{\varepsilon_m(\omega)+2\varepsilon_d}\,, \tag{47}$$

where f is the relative volume occupied by nanoparticles, $\varepsilon_m(\omega)$ is the permittivity of the metal material of the nanoparticle, ε_d is the dielectric constant of the matrix, and ω is the frequency of radiation.

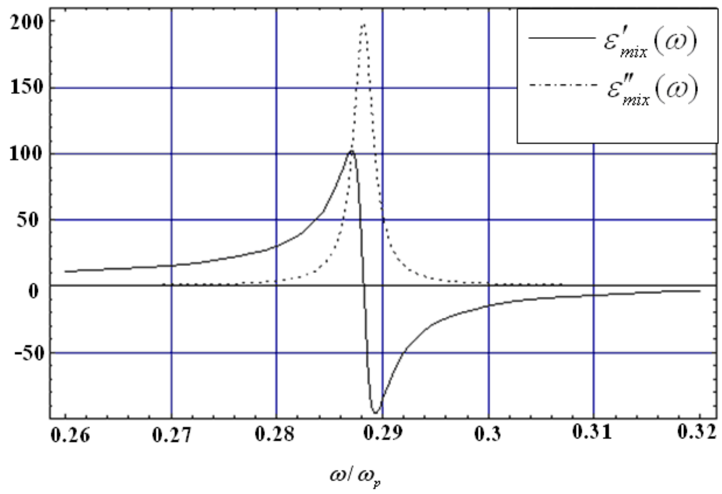

Fig. 2. Dependences of $\varepsilon'_{mix}(\omega), \varepsilon''_{mix}(\omega)$ on the dimensionless frequency.

Using Eq. (47) one can write

$$\varepsilon_{mix}(\omega)=\varepsilon_d\,\frac{\varepsilon_m(\omega)(1+2f)-2\varepsilon_d(f-1)}{\varepsilon_m(\omega)(1-f)+\varepsilon_d(2+f)}\,. \tag{48}$$

As it follows from Eq. (48), when $f=1$ (the whole volume of the matrix is occupied by metal nanoparticles) we have $\varepsilon_{mix}=\varepsilon_m$, when $f=0$ (in the volume of the matrix there are no metal nanoparticles) $\varepsilon_{mix} = \varepsilon_d$. The dielectric constant of the metal nanoparticles is determined in accordance with the Drude model:

$$\varepsilon_m(\omega) = \varepsilon_0 - \frac{\omega_p^2}{\omega(\omega + i\gamma)}, \tag{49}$$

where ε_0 is a constant, ω_p is the plasma frequency, γ the relaxation constant. In the case of silver (see [25]) $\varepsilon_0 = 5$, $\hbar\omega_p = 9$ eV, $\hbar\gamma = 0.02$ eV.

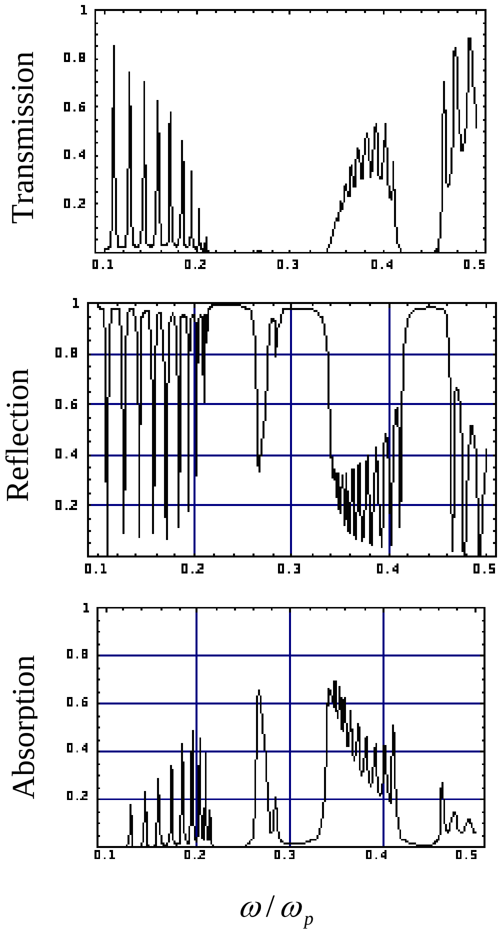

Fig. 3. Dependences of transmission, reflection and absorption coefficients on the dimensionless frequency ω/ω_p.

Further, in all numerical calculations we consider a nanocomposite based on silver. In the Fig. 2 we presented the dependences of $\varepsilon'_{mix}(\omega)$, $\varepsilon''_{mix}(\omega)$ on the dimensionless frequency ω/ω_p. As it can be seen from the figure, the both curves have a resonant character. In Fig. 3 we plotted the dependences of transmission, reflection and absorption coefficients on the dimensionless frequency ω/ω_p. The calculation was made for a photonic crystal consisting of 16 cells, each of which contains two layers – a nanocomposite layer of width a_2 with permittivity $\varepsilon_{mix}(\omega)$ and a vacuum layer of width a_2. The filling factor f was taken as $f = 0.2$. For performing of corresponding calculations the sizes of the layers were considered in units of the plasma wavelength ($\lambda_p = 2\pi c/\omega_p$). The width of the layers were chosen to be equal to each other and the period of the structure to be equal to λ_p ($L = a_1 + a_2$, $a_1 = a_2$ and $L = \lambda_p$).

As it can be seen from the upper curve of Fig. 3, for the region of frequencies being under consideration there are two bands of non-transparency and with increasing of a frequency value the bandwidth decreases. We also note that there is no frequency value where the transmission coefficient takes a value being equal to unity. On this base one can conclude that when a periodic absorbing media occupies the whole space, then in that system any wave process is doomed to attenuation. As it follows from the middle and lower curves of Fig. 3, the reason of existing a wave non-transparency band may serve as a reflection and absorption.

7. Conclusion

In this paper on the base of the generalized transfer matrix method the problem of electromagnetic wave propagation through an arbitrary periodic one-dimensional absorbing media was considered. It is shown that for an arbitrary layered structure the problem of determination of scattering amplitudes is reduced to solution of some set of recurrent equations. This result was applied to a photonic crystal with an arbitrary form of a structure element. Analytical expressions for the transmission and reflection amplitudes as functions of the number of photonic crystal cells are found.

The obtained expressions were considered for a model system representing a one-dimensional periodic array of alternating layers of vacuum and a metal characterized by the complex frequency-dependent dielectric function. It was shown that in an occupied whole space periodic system a harmonic time wave process cannot exist. It means that any wave process always attenuates.

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