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SMALL-SCALE IONOSPHERIC DYNAMO: ITS ROLE IN NEAR-EARTH SPACE PHYSICS, INTERACTION WITH CONDUCTING EARTH AND PERSPECTIVES OF ITS PRACTICAL USE

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Abstract–We consider Small-Scale Ionospheric Dynamo, its role in near-Earth space physics and interaction with conducting Earth. For complete multilayer model we calculate the electric fields, currents, the magnetic fields and nonuniformities of density charged particles excited by ionospheric winds. Perspectives of their practical use are analyzed.

1. Introduction

The customary dynamo theory [1–5], establishing relationship between the wind distribution in ionosphere and resulting system of fields and currents, deals with tide winds caused by gravitational forces and thermal effect of the Sun. The theory thus considers global-scale motion which to a certain degree of approximation may be considered the same in conjugated points of ionosphere in the northern and southern hemisphere, lying on the same force line of magnetic field H. The initial equations of consistent dynamo theory [3–5] become very complicated in the case when conditions in conjugated point are not identical, i.e. when the current along the magnetic force lines is possible. Therefore the problem of electromagnetic connection between the conjugate points remained open in the dynamo theory.

However, for the small-scale gas motion where plane geometric model is applicable this problem was first solved in the paper [7]. Such motion of gas may be caused by inhomogeneous heating of atmosphere, from day and night sides, when Besnard type cells are formed. As shown in the paper [8], an additional motion, namely rotation around the cell axis, can originate in the Besnard cells positioned on horizontal rotating plane. In the case when rotation axis and temperature gradient are normal to the basis plane, the parallel component of velocity rotor is proportional to the angular velocity. When rotation effect is strong enough, the horizontal component of velocity may be dominant relative to vertical one, as a result of which "geostrophyzation" takes place inside the cells. The same phenomenon should be observed in the gas shell of rotating sphere [9, 10]. Therefore we shall consider below the motion of neutral component of ionosphere as taking place parallel to the Earth surface.

Ascending electric currents were detected experimentally, having characteristic dimension 200 to 300 km [11]. It would be natural to relate their origination with dynamo mechanism action inside the cells of corresponding size. Electrodynamic link is established through magnetosphere plasma between ionosphere in the northern and southern hemispheres. Due to small thickness of neutral atmosphere, this connection will affect the inner layers of the bulk Earth. Thus, the corresponding theory should consider the self-consistent problem for the whole magnetic force line passing through the conducting Earth and its surrounding space.

2. Small-Scale Ionospheric Dynamo for the complete multilayer model of Near-Earth Space at high and medium latitudes in the approximation of infinitely conducting Earth

It is known that the upper layers of the Earth exhibit good conductivity, and that the radius R_E and height of the ionosphere layer are much greater than the thickness of the neutral atmosphere. It is clear from this that the electromagnetic processes that take place in the upper layers of the ionosphere and in the Earth's interior should interact via the atmosphere and should be mutually reflected.

The ionosphere is a region of inhomogeneous weakly-ionized plasma, which contains free electrons in such an amount that affects the propagation of radio waves. Contemporary science divides the ionosphere into the layers, reckoning from the Earth, D, E, F_1, F_2, having minor maxima (as compared to the principal one) in the distribution of electron concentration and different thicknesses where physicochemical and dynamic processes, specific for each region, are taking place [12–15].

All the processes that take place in the ionosphere can be subdivided into two groups: photochemical processes and transfer processes. In the F region, both groups of processes are comparable in importance, in contrast to the D and E regions, where the electron distribution is primarily determined by photochemical processes.

In the F region, historically subdivided into the F_1 and F_2 layers, in contrast to the E region, atomic ions are primary, and therefore ion-molecular reactions determine both the region's photochemistry and the rate of electron annihilation. The F_1 "bulge" in the electron distribution is an ion-formation maximum; as the height increases, however, there is an increase in the part played by the diffusion process, until eventually, in the region above the F_2-layer maximum, this process begins to determine the ion distribution even during the daytime. In the region of the F_2-layer maximum, the rate of ion and electron diffusion through neutral gas (ambipolar diffusion) is roughly equal to the rate of annihilation of ions in photochemical processes, so that transfer mechanisms affect the concentration of both ions and electrons simultaneously, i.e., there is no fundamental difference between the photochemistry of the F_1 and F_2 layers. The basic difference between the F_1 and F_2 layers involves the contribution of the ambipolar diffusion process. In the neutral atmosphere, in the case of slow processes, we can disregard not only conduction currents but also displacement currents. Consequently, the electric field potential will satisfy the Laplace equation $\Delta\psi = 0$.

It is evident from the above that the structure and properties of the ionosphere vary markedly with height; the horizontal part of the system of ionosphere currents flows mainly in the E region, where the Pedersen and Hall conductivities have their maximum values. At these heights, the contribution from electrons and ions to the total current along lines of force of the Earth's magnetic field is not the same everywhere. At the lower ends of the magnetic force tube, where their ratio is governed by the mobility, currents oriented along the magnetic field are transported chiefly by electrons. At high levels, gravitation and the pressure gradient also play role, and the relationship may be entirely different. Certain particular consequences derive from this [16].

At present it is confirmed that the Earth and its gaseous shell constitute a physical unity, at least in the sense of electrodynamics. This is a complex physical system consisting of the solid inhomogeneous part, the Earth, and the surrounding gaseous part stretching up to the upper boundary of the magnetosphere. The magnetosphere is known to be the region of the circumterrestrial space which is formed by compression by the solar wind of the Earth’s magnetic field. Therefore the Earth and its gaseous shell, up to the outer boundary of magnetosphere, are pierced by the Earth’s magnetic field and have good conductivity. In such a system, having a good conductivity, any electromagnetic disturbances should naturally be transferred from one part to another and have mutual reflection. It follows that the electrodynamic state of the gaseous shell of the Earth should be mapped inside it and, vice versa, the processes in the Earth’s interior should leave an “electrodynamic trace” in the state of its gaseous shell, i.e., it looks like there exist an unambiguous equilibrium electrodynamic state correspondence between the Earth and its gaseous shell on which disturbances are imposed and transferred from one part of the system to another. It is clear that any horizontal (perpendicular to the Earth’s magnetic field) motions in ionosphere lead, by the dynamo mechanism, to appearance of electric fields and currents flowing in electroconductive layers of the system. Thus, it is necessary to solve a self-consistent problem for the entire length of a line of force of the Earth's magnetic field which penetrates the conducting Earth and the space that surrounds it.

Note, however, that ionosphere perturbations, whose sizes are smaller than the height of the homogeneous atmosphere, damp in the gaseous shell [17] and do not affect the Earth.

In this paper, in conformity with the above, we propose a multilayer model of the near-Earth space, in which the Earth, the neutral atmosphere, and the E and F_1 and F_2 layers of the ionosphere and the magnetosphere are represented, respectively, as ideally conducting, by a neutral gas, and by layers of neutral gas with an admixture of charged particles, and by an ideally conducting plasma (see Fig.1).

The D-layer was not considered because of the low concentration of charged particles and small thickness as compared to the typical sizes of the problem (< 200 km).

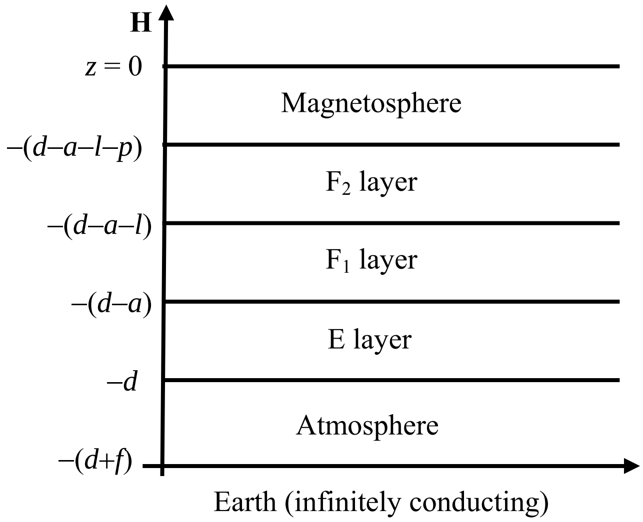

Fig. 1.

Within the framework of this model, we consider the effect of the small-scale "atmospheric dynamo" [7, 18] on the electrodynamic state of the near-Earth space, with allowance for processes of ambipolar diffusion in the upper layers of the ionosphere.

Since the characteristic dimensions of the phenomenon under consideration are small as compared to the Earth's radius, we can consider a plane problem for the following multilayer system that is symmetrical with respect to the axis $z=0$ (only the southern hemisphere is shown in the accompanying figure). A layer of infinitely conducting plasma (the magnetosphere) lies between the surfaces $z=\pm(d-a-l-p)$, and is in contact with weakly ionized gas in the layer $d-a-l-p\le d$, the planes $z=\pm d$, are boundaries between weakly ionized and neutral gas (i.e., the atmosphere). The E, F_1 and F_2 layers of the ionosphere are separated from one another by the planes $z=\pm(d-a)$, $z=\pm(d-a-l)$, respectively, while the boundaries between the atmosphere and the Earth are specified by the planes $z=\pm(d+f)$. The magnetic field **H** is perpendicular to the interfaces (for a substantiation of this model, see [18]).

Theoretical analysis of the problem of ionosphere and magnetosphere heights was performed in [19] in the quasi- and magneto-hydrodynamic approximation by considering the system of momentum-transfer equations for the three components, ions, electrons, and neutral particles, as well as ionization recombination and diffusion processes. Arbitrary small-scale motions of neutral gas in the ionosphere in southern- and northern-hemisphere conjugated points of a magnetic line are expanded into antisymmetric and symmetric. In other words, a cell involving electroconductive part of the system is considered as a liquid multilayer unipolar inductor consisting of the E, F_1, F_2 layers and magnetosphere if its outer layers rotate in the same or opposite directions. In this statement of the problem the analytical solutions of [19] for the electrodynamic quantities are given below in relation to the boundary values between the system layers [20].

It is assumed that the weakly ionized gas of the ionosphere consists of electrons, positive ions of the same kind with unit charge, and neutral molecules with perturbing horizontal constant velocity W (this type of geostrophic motion, which is known from observations, may be caused by appropriate physical factors [7]).

Then, if we assume that the condition of quasi-neutrality $N_{0i} = N_{0e}$ is observed, and the ion and electron temperatures are equal, $T_i = T_e$, then the equation of motion for ions and electrons readily fields expressions for the "background" electric field in the case of diffusion equilibrium (in the absence of a velocity for neutrals) [14]:

$$E_0 = \frac{1}{2}\frac{1}{e} g\left(m_i + m_e\right), \tag{1}$$

where g is the acceleration due to gravity; m_i and m_e are the ion and electron masses, respectively. In other words, the field that arises as a result of charge separation doubles the height scale of the neutral mixture.

Then the equations of motion, linearized with respect to perturbation of the physical parameters (with allowance for the "background' field), for ions and electrons respectively, disregarding inertial, nonlinear terms, the Coriolis force, and the frequency of collision between electrons and ions (the requisite conditions for this are readily satisfied in the ionosphere) [17, 21, 22] have the following form:

$$\begin{aligned} -\nabla \frac{\overline{P}_i}{N_{0i}} + e\left\{-\nabla\Psi + \frac{1}{c}\left[\mathbf{v}_i H\right]\right\} &= \gamma_{in}\left(v_i - W\right) + \frac{n_i}{2N_{0i}}\left(m_i + m_e\right) g, \\ -\nabla \frac{\overline{P}_e}{N_{0e}} - e\left\{-\nabla\Psi + \frac{1}{c}\left[\mathbf{v}_e H\right]\right\} &= \gamma_{en}\left(v_e - W\right) + \frac{n_e}{2N_{0e}}\left(m_i + m_e\right) g, \end{aligned} \tag{2}$$

In these equations, v_i, v_e and n_i, n_e are respectively the velocities and perturbations of the equilibrium concentrations N_{0i} and N_{0e} of ions and electrons; the perturbations of their pressures $\overline{P}_i = n_i k T_i$, $\overline{P}_e = n_e k T_e$, since processes are assumed to be isothermic; k is Boltzmann's constant, ψ is the potential of the electric field, g is the acceleration due to gravity; γ_{in}, γ_{en} are the collision frequencies of ions and electrons with neutral gas particles; and W is the velocity of neutral particles.

Processes that take place in the ionosphere (ionization, recombination, and so on), which are closely related to the wave and corpuscular emission of the Sun, are highly diverse and vary greatly with height, because of the diversity of the chemical composition of the ionosphere and of ionization agents. Therefore, system (2) should be supplemented by charged-particle continuity equations that are suitable for each region.

Photoionization is the basic factor in ion formation over the entire layer of the ionosphere. In the E region, however, where molecular ions predominate, ion loss takes place as a result of dissociative recombination. In the F region, in contrast to the E region, the primary part in physical and chemical processes is played by atomary ions, and ion loss takes place through charge transfer from primary to secondary ions, and also through particle transport [13, 14]. Consequently, the continuity equations for ions and electrons will have the following form in the E and F regions, respectively:

$$\begin{aligned} \mathrm{div} N_{0i} v_i &= J - \alpha N_i N_e = -\alpha N_{0i}\left(n_i + n_e\right), \\ \mathrm{div} N_{0e} v_e &= J - \alpha N_i N_e = -\alpha N_{0i}\left(n_i + n_e\right). \end{aligned} \tag{3}$$

and

$$\begin{aligned} \mathrm{div} N_{0i} v_i &= J - \beta N_e = -\alpha_r N_n n_e, \\ \mathrm{div} N_{0e} v_e &= J - \beta N_e = -\alpha_r N_n n_e. \end{aligned} \tag{4}$$

where J is the Chapman ion-formation function, which is evidently valid here as well, since we are considering the elementary case of photoionization of a single-component isothermic atmosphere by monochromatic radiation; α is the recombination coefficient of positive-ions with electrons; and β is the formal (since there is no sticking reaction in this region) sticking probability of electrons to neutral atoms, which depends linearly on the neutral-particle concentration, i.e., $\beta = a_r N_n$.

To these equations we should also add the Poisson equation $\Delta\psi = 4\pi e\left(n_i - n_e\right)$, since even the smallest charge separation in a quasi-neutral plasma, as a result of differences in the friction forces between charged plasma components and the neutral gas, as well as photochemical, etc., factors, can excite large electric fields [23].

Assume that the force of gravity and the temperature of all the types of particles making up the weakly ionized gas are independent of the height z; then the collision frequencies $\gamma_{i,en}$ (which are proportional to the density of neutral molecules, expressed by the barometric formula $N_n = N_{0n} \exp\left[-\left(1/H_n\right) z\right]$ and the unperturbed charged-particle density $N_{0i,e}$ will have the form

$$\gamma_{i,en} = \gamma_{0,i,e} \exp\left[-\left(1/H_n\right) z\right], \; N_{0i,e} = N_0 \exp\left[-\left(1/H_m\right) z\right].$$

Here $H_n = kT_n/m_n g$ is the height of the uniform atmosphere; T_n and m_n are the temperature and mass of neutral particles; $\gamma_{0,i,e}$ and N_0 are, respectively, the collision frequencies and charged-particle concentration at the initial height corresponding to each layer; H_m is the approximation constant for the exponential charged-particle concentration (this approximation is quite satisfactory for most problems). It should be noted here, however, that these parameters are different in the E and F layers because of the nonuniformity of the ionosphere.

Furthermore, taking into account only vertical changes in regular ionospheric parameters, we can expand the electric field potential ψ, the velocity of neutrals *W*, and the perturbations of the density n_t and n_e in a Fourier integral in the coordinates *x* and *y*, and consider the individual components:

$$\Psi = \Psi_k(z)\exp\left[i(k_1x+k_2y)\right],\quad w = w_k(z)\exp\left[i(k_1x+k_2y)\right],$$
$$n_i = n_k^i(z)\exp\left[i(k_1x+k_2y)\right],\quad n_e = n_k^e(z)\exp\left[i(k_1x+k_2y)\right],$$

If we assume that the velocity of neutrals *W* is independent of *z*, and the components $W_z = 0$, then, using the continuity equation for an incompressible fluid $\mathrm{div}W = 0$, we can break down the terms W_x, W_y, of the Fourier expansion into pairs, and solve the problem for each pair separately.

As this pair we take [7]

$$W_x = \frac{W_0}{k_1}\sin k_1x\sin k_2y,\quad W_y = \frac{W_0}{k_2}\cos k_1x\cos k_2y$$

in which case the electric potential Ψ and density perturbations n_i and n_e can be represented as follows:

$$\Psi = f_1\sin k_1x\cos k_2y,\quad n_i = f_2\sin k_1x\cos k_2y,\quad n_e = f_2\sin k_1x\cos k_2y.$$

Substituting the velocities v_i and v_e obtained from (2) into (3) and (4), we obtain equations that comprise closed systems (together with Poisson's equation) for determining the potential ψ and n_i, n_e in the E and F layers, respectively:

$$\alpha^{(1)}_{i,e,\mathrm{E,F}}(z)f''_{1zz} + \alpha^{(2)}_{i,e,\mathrm{E,F}}(z)f'_{1z} + \alpha^{(3)}_{i,e,\mathrm{E,F}}(z)f_1 + \alpha^{(4)}_{i,e,\mathrm{E,F}}(z)f''_{3zz} +$$
$$+\alpha^{(5)}_{i,e,\mathrm{E,F}}(z)f'_{2,3z} + \alpha^{(6)}_{i,e,\mathrm{E,F}}(z)f_{2,3} + A_{i,e,\mathrm{E,F}} = 0,$$
$$k_0^2f_1 - f''_{1zz} = 4\pi e(f_2 - f_3),$$
$$\alpha^{(1)}_{i,e,\mathrm{E,F}}(z) = \left(2\alpha N_0e^{z/2H_n}\right)_{\mathrm{E}} \to \frac{(a_rN_n)_{\mathrm{F}}}{N_{0i}4\pi e} + \frac{c}{H_z}\lambda_{ie},$$
$$\alpha^{(2)}_{i,e,\mathrm{E,F}}(z) = \mp\frac{\lambda_{ie}c}{H_z}\left(\frac{1}{H_n}+\frac{1}{H_m}\right),\quad \alpha^{(4)}_{i,e,\mathrm{E,F}}(z) = -\frac{kT_{i,e}}{\gamma_{i,en}N_{0i,e}}$$
$$\alpha^{(3)}_{i,e,\mathrm{E,F}}(z) = -k_0^2\left(2\alpha N_0e^{z/2H_n}\right)_{\mathrm{E}} \to \frac{(a_rN_n)_{\mathrm{F}}}{N_{0i}4\pi e} \mp \frac{c}{H_z}\frac{\lambda_{i,e}}{1+\lambda^2_{ie}},$$
$$\alpha^{(5)}_{i,e,\mathrm{E,F}}(z) = \mp\frac{1}{\gamma\lambda_{i,en}}\frac{1}{N_{0i,e}}\frac{1}{H_n}\left(kT_{i,e} + \frac{1}{2}g(m_i+m_e)\right),$$
$$\alpha^{(6)}_{i,e,\mathrm{E,F}}(z) = -\frac{1}{N_{0i,e}}\left(2\alpha N_0e^{z/2H_n}\right)_{\mathrm{E}} \to (a_rN_n)_{\mathrm{F}} - \frac{c}{H_z}\frac{1}{H_n}g\frac{m_i+m_e}{\gamma_{i,en}} + \frac{k_0^2kT_{i,e}}{\lambda^2_{ie}\gamma_{i,en}},$$
$$A_{i,e,\mathrm{E,F}} = \frac{\lambda^{-1}_{i,e}k_0^2W}{k_1k_2},\quad k_0^2 = k_1^2 + k_2^2.$$

In this unified notation for the equations, the first and second subscripts refer to ions and electrons, respectively; subscripts E and F refer to the E and F layer of the ionosphere; and $\lambda_{ie} = \left(eH_z m_{i,e} c\right)\big/\left(1/\gamma_{i,e}\right) = \omega_{i,e}/\gamma_{i,en}$ is the ratio of the Larmor rotation frequency of ions (electrons) to the frequency of their collisions with neutrals.

As can be seen from (6), the model employed takes account of the system of ionospheric winds, comprising not only the E region but also the F region of the ionosphere. Winds can play a significant role in generation of dynamo fields and currents in the F region as well [24]. However, allowance for the force of gravity of charged particles and of terms responsible for ambipolar diffusion leads to a system of three second-order equations for ψ, n_i, n_e; reduction of this system to one equation in the electric potential ψ is extremely complicated in the general case [17]. Therefore, using the condition of quasi-neutrality, we set $f_2 = f_3$ in (6), after which we obtain a two-equation system for f_1 and f_3 (which can be used, together with Poisson's equation, to determine f_2 as well [25].

Making the substitution $f_1 = u(t)t^{-1}$, $f_3 = N_0 \exp\left(z/H_m\right)u_1(t)t^{-1}$, where $t = e^{-2\xi}$, $\xi = z/H_n$, we can readily eliminate $u(t)$ from (6) and obtain a fourth-order equation in $u_1(t)$ for the F_2 layer and a third-order equation for the F_1 and E layers (disregarding diffusion terms). However, on the basis of numerical estimates of real physical quantities and parameters:

$$a_1 = -\frac{1}{4}\frac{2\left(2+3H_n/H_m\right)+gH_n\left(m_i\lambda_{e0}\gamma_{e0}+m_i\lambda_{i0}\gamma_{i0}\right)}{kT_i\lambda_{e0}\gamma_{e0}+kT_e\lambda_{i0}\gamma_{i0}} \text{ in the } F_2 \text{ layer,}$$

$$a_2 = -\frac{H_n^2}{4}\frac{a_r N_{0n}\gamma_{i0}\gamma_{e0}\left(\lambda_{i0}+\lambda_{e0}\right)}{kT_i\lambda_{e0}\gamma_{e0}+kT_e\lambda_{i0}\gamma_{i0}} \text{ in the } F_2 \text{ layer,}$$

$$a_3 = a_r N_{0n} H_n \gamma_{i0}\gamma_{e0}\left(\lambda_{i0}+\lambda_{e0}\right)\big/ gm_i\left(\lambda_{e0}\gamma_{e0}+\lambda_{i0}\gamma_{i0}\right) \text{ in the } F_1 \text{ layer,}$$

$$a_4 = -8\alpha' N_0 H_n\left(\lambda_{i0}+\lambda_{e0}\right)\big/ gm_i\lambda_{e0} \text{ in the E layer,}$$

characterizing the layers in question, we can reduce the order of the equations in the F layer by one, taking into account the relation $\lambda_{i,e} \gg 1$ (when the horizontal dimensions of the problem are greater than or on the order of the height of the uniform atmosphere) (here we omit a numerical analysis of the coefficients of equations, because the expressions are unwieldy) and we can write the equations as follows (in the E layer as well, taking into account the condition $\lambda_e \gg 1$, $\lambda_i \le 1$)

$$t\frac{d^3u_1^{\mathrm{F2}}}{dt^3} + \left(a_1 - 1\right)\frac{d^2u_1^{\mathrm{F2}}}{dt^2} + a_2\frac{du_1^{\mathrm{F2}}}{dt} - 2\left(3+m\right)\frac{a_2}{4t}u_1^{\mathrm{F2}} + C_1 = 0, \tag{7}$$

$$\frac{d^3u_1^{\mathrm{F1}}}{dt^3} + a_3\frac{du_1^{\mathrm{F1}}}{dt} - \frac{3+m}{2}\frac{a_3 u_1^{\mathrm{F1}}}{t} = 0, \tag{8}$$

$$\frac{d^3u_1^{\mathrm{E}}}{dt^3}-\frac{a_4t^{1/4}-15}{4t}\frac{d^2u_1^{\mathrm{E}}}{dt^2}+\frac{30t^{-2}+a_4t^{-7/4}}{4^2}\frac{du_1^{\mathrm{E}}}{dt}-\frac{21}{4^3}a_4\lambda_{i0}^2t^{-15/4}u_1^{\mathrm{E}}=0. \tag{9}$$

The solutions of (7)-(9) in the F_2, F_1 and E layers will have, respectively, the following form:

$$u_1^{\mathrm{F2}}=3^6\sqrt{3}\,t^{13/6}\left[AJ_6\left(2t^{1/2}\sqrt{a_2}\right)+BY_6\left(2t^{1/2}\sqrt{a_2}\right)\right]+C_1L_{-1/3}+C_2L_1,$$

$$u_1^{\mathrm{F1}}=C_3+C_4\exp\left(-a_3t\right)+\frac{2}{\left(1+H_n/H_m\right)}\frac{C_5t}{a_3},\ \ u_1^{\mathrm{E}}=C_6F\left(t\right)+C_7t^{-1/4}+C_8,$$

where

$$F\left(t\right)=\frac{1}{2}t^{-1/2}-a_4t^{-1/4}\ln\left|\frac{a_4}{t^{-1/4}}\right|-\frac{1}{2}a_4^2\ln t^{-1/4}+t^{-1/2}\sum_{n=2}^{\infty}\frac{1}{1-n}\frac{1}{n}\frac{1}{\left(n+1\right)!}\left(\frac{a_4}{t^{-1/4}}\right)^{n+1}.$$

Here J_6 and Y_6 are the Bessel functions of the first and second kind, respectively; $L_{-1/3}$ and L_1 are the Lommel functions, for which we have

$$L_{-1/3}=\frac{3}{2}\Gamma\left(-\frac{1}{6}+3\right)\Gamma\left(-\frac{1}{6}-3\right)\sum_{n=0}^{\infty}\left(-a_2\right)^n t^{(n+2)}\Big/\Gamma\left(-\frac{1}{6}+3+n+1\right)\Gamma\left(-\frac{1}{6}-3+n+1\right),$$

$$L_1=3^6\Gamma\left(\frac{1}{2}+3\right)\Gamma\left(\frac{1}{2}-3\right)\sum_{n=0}^{\infty}\left(-a_2\right)^n t^{(3n+8)/3}\Big/\Gamma\left(\frac{1}{2}+3+n+1\right)\Gamma\left(\frac{1}{2}-3+n+1\right).$$

A, B, $C_{1,\dots,8}$ are arbitrary constants that are determined from the boundary conditions (continuity of the electric field potential and normal current on the interfaces).

Now, using (6), (10), and the Laplace equation, we can readily determine the electric field potential in the individual layers of the ionosphere and the atmosphere:

$$f_1^{\mathrm{F2}}=\alpha_1\left[\left(\alpha_2t^{-2}+\alpha_3t^{-1}\right)u_1^{\mathrm{F2}}+\alpha_4t^{-1}u_{1t}'^{\mathrm{F2}}+\alpha_5u_{1t}''^{\mathrm{F2}}\right]+\alpha_6,$$

$$f_1^{\mathrm{F1}}=\alpha_1\left[\left(\alpha_7t^{-2}+\alpha_8t^{-1}\right)u_1^{\mathrm{F1}}+\alpha_9t^{-1}u_{1t}'^{\mathrm{F1}}\right]+\alpha_6,$$

$$f_1^{\mathrm{E}}=\alpha_{10}t^2\left(1+\lambda_{i0}^2t^{-1}\right)\left(1+\lambda_{e0}^2t^{-1}\right)\left[\left(\alpha_7t^{-2}+\alpha_{11}t^{-7/4}\right)u_1^{\mathrm{E}}+\alpha_9t^{-1}u_{1t}'^{\mathrm{E}}\right]+\alpha_6,$$

$$\alpha_1=\frac{H_z}{c}\frac{1}{k_0^2}\frac{\lambda_{i0}\lambda_{e0}}{\lambda_{i0}^2-\lambda_{e0}^2},\ \ \alpha_2=-\frac{7}{4}\frac{1}{H_n}\frac{1}{\gamma_{i0}}\frac{1}{\gamma_{e0}}\left[gm_i\left(\omega_e+\omega_i\right)+\frac{5}{H_n}\left(kT_i\lambda_{e0}\gamma_{e0}+kT_e\lambda_{i0}\gamma_{i0}\right)\right],$$

$$\alpha_3=a_rN_{0n}\left(\lambda_{i0}+\lambda_{e0}\right)+k_0^2\left(\frac{kT_i\lambda_{e0}}{\gamma_{i0}\lambda_{i0}^2}+\frac{kT_e\lambda_{i0}}{\gamma_{e0}\lambda_{e0}^2}\right),$$

$$\alpha_4=\frac{1}{H_n}\frac{1}{\gamma_{i0}}\frac{1}{\gamma_{e0}}\left[gm_i\left(\omega_e+\omega_i\right)+\frac{2}{H_n}\left(kT_i\lambda_{e0}\gamma_{e0}+kT_e\lambda_{i0}\gamma_{i0}\right)\right],$$

$$\alpha_5=-\frac{4}{H_n^2}\frac{1}{\gamma_{i0}}\frac{1}{\gamma_{e0}}\left(kT_i\lambda_{e0}\gamma_{e0}+kT_e\lambda_{i0}\gamma_{i0}\right),\ \ \alpha_6=\frac{H_z}{c}\frac{W_0}{k_1k_2},$$

$$\alpha_7=-\frac{7}{4}\frac{1}{H_n}\frac{1}{\gamma_{i0}}\frac{1}{\gamma_{e0}}gm_i\left(\omega_e+\omega_i\right),\ \ \alpha_8=a_rN_{0n}\left(\lambda_{i0}+\lambda_{e0}\right),$$

$$\alpha_9=\frac{1}{H_n}\frac{1}{\gamma_{i0}}\frac{1}{\gamma_{e0}}gm_i\left(\omega_e+\omega_i\right),\ \ \alpha_{10}=\frac{H_z}{c}\frac{1}{k_0^2}\frac{1}{\lambda_{i0}\lambda_{e0}\left(\lambda_{i0}^2-\lambda_{e0}^2\right)},\ \ \alpha_{11}=2\alpha N_0\left(\lambda_{i0}+\lambda_{e0}\right) \tag{11}$$

and $f_1^{\text{atm}} = 2C_9 \text{sh}kz$. Here we have used the condition that the electric field potential vanish on the infinitely conducting surface of the Earth.

We would like to observe that the author of the present survey managed to solve the complete system of quasi-hydrodynamic equations for the F_2 layer of the weakly ionized ionosphere plasma. He took into account the members caused by ambipolar diffusion, gravity force of particles, and the Earth magnetic field effect. As a result, the author obtained analytic expressions for electro-dynamic quantities, provided that parameters $\lambda_i, \lambda_e \gg 1$, which is rather difficult problem [26].

After this, we can also determine the electric current $j = eN_{0i}v_i - v_e$ in the ionosphere, whose components will have the following form on the basis of Eqs. (2):

$$
\begin{aligned}
J_x &= eN_{0i}\left[-\frac{c}{H_z}\left(\alpha_1\Psi'_x + \alpha_2\Psi'_y\right) + \alpha_3\overline{p}'_{ix} + \alpha_4\overline{p}'_{iy} - \alpha_2 W_x + \alpha_1 W_y\right],\\
J_y &= eN_{0i}\left[-\frac{c}{H_z}\left(\alpha_2\Psi'_x - \alpha_1\Psi'_y\right) + \alpha_3\overline{p}'_{iy} - \alpha_4\overline{p}'_{ix} - \alpha_1 W_x + \alpha_2 W_y\right],\\
J_z &= eN_{0i}\left[-\frac{c}{H_z}\left(\lambda_i + \lambda_e\right)\Psi'_z - \frac{1}{N_{0i}}\left(\frac{1}{\lambda_{in}} - \frac{1}{\lambda_{en}}\right)\left(\overline{p}'_{iz} + \frac{n}{2}g\left(m_i + m_e\right)\right)\right],
\end{aligned}
\tag{12}
$$

where

$$
\alpha_1 = \frac{\left(\lambda_i + \lambda_e\right)\left(1 + \lambda_i\lambda_e\right)}{\left(1 + \lambda_i^2\right)\left(1 + \lambda_e^2\right)},\quad \alpha_2 = \frac{\lambda_i^2 - \lambda_e^2}{\left(1 + \lambda_i^2\right)\left(1 + \lambda_e^2\right)},
$$

$$
\alpha_3 = \frac{-\gamma_{en}^0\left(1 + \lambda_e^2\right) + \gamma_{in}^0\left(1 + \lambda_i^2\right)}{N_{0i}\gamma_{in}^0\gamma_{en}^0\left(1 + \lambda_i^2\right)\left(1 + \lambda_e^2\right)},
$$

$$
\alpha_4 = -\frac{1}{N_{0i}}\frac{1}{\gamma_{in}^0}\frac{1}{\gamma_{en}^0}\frac{\gamma_{en}^0\lambda_i\left(1 + \lambda_e^2\right) + \gamma_{in}^0\lambda_e\left(1 + \lambda_i^2\right)}{\left(1 + \lambda_i^2\right)\left(1 + \lambda_e^2\right)}.
$$

For simplicity, the indices E and F that indicate the corresponding layers have been omitted in (12).

It is known that the magnetic hydrodynamics is out of limits of the usual hydrodynamics because of the "remote action" in electromagnetic phenomena. Hence, it should not be limited with consideration of the region of conducting liquid; the conditions in the remaining part of the space should also be taken into account. So, it is clear that in the magnetic hydrodynamics difficulties arise associated with the outer and boundary conditions for the variables of interest and their derivatives at the boundaries occupied by the liquid, at interfaces between the regions, and outside the conducting region. Every equation produces corresponding condition, therefore it is practically impossible to give a full guide on which boundary conditions are required for each conceivable problem of the magnetic hydrodynamics. In solution of every problem the researcher must decide on his own how many boundary conditions are needed and which necessary and sufficient boundary conditions must be imposed. According to these rules, in [7, 18, 19] the boundary conditions have been established for the physical quantities which define unambiguously the formulated problem. The details of physical considerations for justification of these conditions will not be given again here. Only the boundary conditions will be given for the corresponding physical quantities between the layers in the figure of the work [19] for the complete model of near-Earth space.

These conditions will be presented for the cases of both anti-rotation and co-rotation [7].

Anti-rotation case. If the motion of neutral gas in conjugated points of the northern and the southern hemispheres is opposite, for the electric field potential the following boundary conditions take place: $f_1^{\mathrm{F_2}} = 0$ at $z = \pm(d-a-l-p)$ on the boundary between the $\mathrm{F_2}$-layer and magnetosphere, $f_1^{\mathrm{F_2}} = f_1^{\mathrm{F_1}}$ at $z = \pm(d-a-l)$ on the boundary between the $\mathrm{F_1}$- and $\mathrm{F_2}$-layers, $f_1^{\mathrm{F_1}} = f_1^{\mathrm{E}}$ at $z = \pm(d-a)$ on the boundary between the $\mathrm{F_1}$- and E-layers, $f_1^{\mathrm{E}} = f_1^{\mathrm{a}}$ at $z = \pm d$ on the boundary between the E-layer and neutral atmosphere, $f_1^{\mathrm{a}} = 0$ at $z = \pm(d+f)$ on the boundary between the neutral atmosphere and the Earth surface. For the electric current j the following boundary conditions should be satisfied: $j_z^{\mathrm{F_2}} = j_z^{\mathrm{m}}$ at $z = \pm(d-a-l-p)$ on the boundary between the $\mathrm{F_2}$-layer and magnetosphere, and the current in the magnetosphere is directed along the Earth's magnetic field, it is constant and equal to the boundary value $j_z^{\mathrm{F_2}}$ [7], $j_z^{\mathrm{F_2}} = j_z^{\mathrm{F_1}}$ at $z = \pm(d-a-l)$ on the boundary between the $\mathrm{F_1}$- and $\mathrm{F_2}$-layers, $j_z^{\mathrm{F_1}} = j_z^{\mathrm{E}}$ at $z = \pm(d-a)$ on the boundary between the $\mathrm{F_1}$- and E-layers, $j_z^{\mathrm{E}} = j_z^{\mathrm{a}} = 0$ at $z = \pm d$ on the boundary between the E-layer and neutral atmosphere.

Co-rotation case. If the motion of neutral gas in conjugated points of the northern and the southern hemispheres is similar, the boundary conditions are the same as in the anti-rotation case, except for the condition $j_z^{\mathrm{F_2}} = 0$ at $z = \pm(d-a-l-p)$ on the boundary between the $\mathrm{F_2}$-layer and magnetosphere, i.e., there is no current in the magnetosphere.

Substituting into these relations the boundary values of physical quantities and analytical solutions for electric field potentials and currents determined in each layer in [19], the following system of equations is obtained in the case of anti-rotation:

$$
\begin{aligned}
&f_1^{\mathrm{F_2}}\Big|_{z=\pm(d-a-l-p)} = \alpha_1\left(A\overline{\beta}_1(t) + B\overline{\beta}_2(t) + C_1\overline{\beta}_3(t) + C_2\overline{\beta}_4(t)\right) + \alpha_6\Big|_{z=\pm(d-a-l-p)} = 0,\\
&f_{1\mathrm{a.d.}}^{\mathrm{F_2}}\Big|_{z=\pm(d-a-l-p)} = \alpha_1\left(A\overline{\overline{\beta}}_1(t) + B\overline{\overline{\beta}}_2(t) + C_1\overline{\overline{\beta}}_3(t) + C_2\overline{\overline{\beta}}_4(t)\right)\Big|_{z=\pm(d-a-l-p)} = 0,\\
&j_{z\mathrm{a.d.}}^{\mathrm{F_2}}\Big|_{z=\pm(d-a-l-p)} = eN_0\left\{A\overline{\overline{G}}_1(t) + B\overline{\overline{G}}_2(t) + C_1\overline{\overline{G}}_3(t) + C_2\overline{\overline{G}}_4(t)\right\}\Big|_{z=\pm(d-a-l-p)} = 0
\end{aligned}
\qquad (13)
$$

on the boundary between the $\mathrm{F_2}$-layer and magnetosphere – the regions of weakly ionized gas (where in photochemical processes ion–molecular reactions are predominant and in the distribution of charged particles the ambipolar diffusion also plays a role) and infinitely conducting plasma;

$$\begin{aligned}
&f_1^{\mathrm{F}_2}\Big|_{z=\pm(d-a-l)} = \alpha_1\left(A\overline{\beta}_1(t)+B\overline{\beta}_2(t)+C_1\overline{\beta}_3(t)+C_2\overline{\beta}_4(t)\right)+\alpha_6\Big|_{z=\pm(d-a-l)} = \\
&= f_1^{\mathrm{F}_1}\Big|_{z=\pm(d-a-l)} = \alpha_1\left(C_3\beta_5(t)+C_4\beta_6(t)+C_5\beta_7(t)\right)+\alpha_6\Big|_{z=\pm(d-a-l)} \\
&f_{1\mathrm{a.d.}}^{\mathrm{F}_2}\Big|_{z=\pm(d-a-l)} = \alpha_1\left(A\overline{\overline{\beta}}(t)+B\overline{\overline{\beta}}_2(t)+C_1\overline{\overline{\beta}}_3(t)+C_2\overline{\overline{\beta}}_4(t)\right)\Big|_{z=\pm(d-a-l)} = 0, \\
&j_z^{\mathrm{F}_2}\Big|_{z=\pm(d-a-l)} = eN_0\left[A\overline{G}_1(t)+B\overline{G}_2(t)+C_1\overline{G}_3(t)+C_2\overline{G}_4(t)\right]\Big|_{z=\pm(d-a-l)} = \\
&= j_z^{\mathrm{F}_1}\Big|_{z=\pm(d-a-l)} = eN_0\left[C_3G_5(t)+C_4G_6(t)+C_5G_7(t)\right]\Big|_{z=\pm(d-a-l)} \\
&j_{z\mathrm{a.d.}}^{\mathrm{F}_2}\Big|_{z=\pm(d-a-l)} = eN_0\left[A\overline{\overline{G}}_1(t)+B\overline{\overline{G}}_2(t)+C_1\overline{\overline{G}}_3(t)+C_2\overline{\overline{G}}_4(t)\right]\Big|_{z=\pm(d-a-l)} = 0
\end{aligned} \tag{14}$$

on the boundary between the F_1- and F_2-layers of the weakly ionized gas where photochemical processes of creation and annihilation of ions and electrons almost coincide (except for ambipolar diffusion which is absent in the F_1-layer);

$$\begin{aligned}
&f_1^{\mathrm{F}_1}\Big|_{z=\pm(d-a)} = \alpha_1\left[C_3\beta_5(t)+C_4\beta_6(t)+C_5\beta_7(t)\right]+\alpha_6\Big|_{z=\pm(d-a)} = f_1^{\mathrm{E}}\Big|_{z=\pm(d-a)} = \\
&= \alpha_{10}\beta_8(t)\left[C_6\beta_9(t)+C_7\beta_{10}(t)+C_8\beta_{11}(t)\right]+\alpha_6\Big|_{z=\pm(d-a)}, \\
&j_z^{\mathrm{F}_1}\Big|_{z=\pm(d-a)} = eN_0\left[C_3G_5(t)+C_4G_6(t)+C_5G_7(t)\right]\Big|_{z=\pm(d-a)} = j_z^{\mathrm{E}}\Big|_{z=\pm(d-a)} = \\
&= eN_0\left[C_6G_8(t)+C_7G_9(t)+C_8G_{10}(t)\right]\Big|_{z=\pm(d-a)},
\end{aligned} \tag{15}$$

on the boundary between E and F_1-regions of the weakly ionized gas where in photochemistry of charged particles processes of, respectively, dissociative recombination and ion–molecular reactions are predominant;

$$\begin{aligned}
&f_1^{\mathrm{E}}\Big|_{z=\pm d} = \alpha_{10}\beta_8(t)\left[C_6\beta_9(t)+C_7\beta_{10}(t)+C_8\beta_{11}(t)\right]+\alpha_6\Big|_{z=\pm d} = \\
&= f_1^{a}\Big|_{z=\pm d} = 2C9\,\mathrm{sh}k\left(d+f-|z|\right)\Big|_{z=\pm d}, \\
&j_z^{\mathrm{E}}\Big|_{z=\pm d} = eN_0\left[C_6G_8(t)+C_7G_9(t)+C_8G_{10}(t)\right]\Big|_{z=\pm d} = j_z^{\mathrm{a}}\Big|_{z=\pm d} = 0
\end{aligned} \tag{16}$$

on the boundary between the layers of neutral atmosphere and the layers of E-regions where, respectively, charged particles are absent and the processes of dissociative recombination are predominant;

$$f_1^{a}\Big|_{z=\pm(d+f)} = 2C9\,\mathrm{sh}k\left(d+f-|z|\right)\Big|_{z=\pm(d+f)} = 0,\quad j_z^{a}\Big|_{z=\pm(d+f)} = 0$$

on the boundary between the Earth surface and neutral atmosphere from where the electric field penetrates into interior regions of the Earth.

For the case of co-rotation the first equation in system (13) is replaced by the equation

$$j_z^{\mathrm{F2}}\Big|_{z=\pm(d-a-l-p)} = eN_0\left(AG_1(t)+BG_2(t)+C_1G_3(t)+C_2G_4(t)\right)\Big|_{z=\pm(d-a-l-p)}.$$

Here $f_1^{\mathrm{F}_2}$, j_z^{F2} and $f_{1\mathrm{a.d.}}^{\mathrm{F}_2}$, $j_{z\mathrm{a.d.}}^{\mathrm{F}_2}$ are the singled-out parts of electric fields and currents in the F2-layer, which are responsible for, respectively, photochemistry, wind motion, and ambipolar diffusion in the ionosphere, A, B, $C_{1,\dots,9}$ are arbitrary constants, $\beta_{1,\dots,11}(t)$ and $G_{1,\dots,10}(t)$ are known functions of $t=\exp\left[-2\left(d+f-|z|\right)/H_n\right]$ involved in the analytical solution of the problem [19].

The quantities $\alpha_{1,\dots,9}$ enter $\beta_{1,\dots,11}(t)$ and $G_{1,\dots,10}(t)$ linearly. The notations for the physical parameters are the same as in [19]. It follows from this that $\beta_{1,\dots,11}(t)$ and $G_{1,\dots,10}(t)$ may be, with allowance for (11), represented as sums of two terms $\beta_{1,\dots,11}(t)=\overline{\beta}_{1,\dots,11}(t)+\overline{\overline{\beta}}_{1,\dots,11}(t)$ and $G_{1,\dots,11}(t)=\overline{G}_{1,\dots,11}(t)+\overline{\overline{G}}_{1,\dots,11}(t)$ where each pair $\overline{\beta}_i(t)$, $\overline{\overline{\beta}}_i(t)$ and $\overline{G}_i(t)$, $\overline{\overline{G}}_i(t)$ contain, respectively, the terms describing the photochemistry, the motion of winds, and the ambipolar diffusion in the ionospere. Just this sense $\overline{\beta}_i(t)$, $\overline{\overline{\beta}}_i(t)$ and $\overline{G}_i(t)$, $\overline{\overline{G}}_i(t)$ have in the system of equations (13), (14).

It is obvious that equations (13), (14), (15), (16), (17) constitute a complete system of linear algebraic equations with respect to the constants A, B, $C_{1,\dots,9}$ contained in the analytical solutions of the problem [19]. This means that the formulated problem [19] in the approximation of infinitely conducting Earth is defined unambiguously in interconnection of different physical processes in each layer. It should be noted here that these equations are written with allowance for the condition of vanishing of the potential of electric fields and of the normal component of currents, caused by the ambipolar diffusion, on the boundaries between the F_2-layer of the ionosphere and the magnetosphere and the F_1-layer, as well as with allowance for the linearity of the problem by means of dividing the potential of electric fields and the currents into the terms responsible for photochemistry, motion of neutral gas, and ambipolar diffusion.

The perturbation of the magnetic field h corresponding to current (12) is determined from the equation $\mathrm{rot}\mathbf{H}=\left(4\pi/c\right)j$, with the requirement that the field be continuous on all boundaries and that h vanish at infinity.

We should note that the requisite information regarding the structure of near-Earth space are given here in extremely compressed form for the following parameter values: distance along line of force of the Earth's magnetic field between conjugate points, at 65° latitudes, d = 4×10^4 km; height of the atmosphere from the Earth's surface f = 100 km; thickness of E layer a = 40 km; thickness of F_1 layer l = 60 km (occupying the region from 140 to 200 km); F_2 layer extends above 200 km to a provisional height of 400 km; and p is 200 km. The ion and neutral components in the E and F layers consist of O_2^+, O_2 and O^+, O, respectively.

It is assumed that the temperature of neutrals T_n = 300 K in the E layer and 1000 K in the F layer; the temperature of ions (electrons) in the F_2 layer $T_{i,e}$ = 1000 K. The height of the uniform atmosphere H_n = 8×10^5 cm in the E layer and H_n ~ 8×10^6 in the F layer, $H_n/H_m=1/2$; α = 10^{-7} cm^3/sec, a_r = 10^{-13} cm^3/sec. The concentration of neutral and charged particles at the initial provisional heights in the E and F layers, respectively, are N_{0n} = 10^{12} at cm^{-3}, N_0 = 10^5 and N_{0n} = 4×10^{10} at.cm^{-3}, N_0 = 4×10^5. The collision frequencies for ions and electrons with neutrals at the same heights $\gamma_{i0} = 10^4$ sec^{-1}, $\gamma_{e0} = 3\times10^4$ sec^{-1} in the E layer and $\gamma_{i0} = 2\times10^3$ sec^{-1}, $\gamma_{e0} = 10^4$ sec^{-1} in the F layer. The amplitude of the wind speed $W_0/k_0 = 200$ m/sec; the cell size l = 200 km. Furthermore, approximating the experimental curve for the charged particles concentrations (taken from [15]) by the formula $N_{0i,e} = N_0 e^{z/H_m}$, we determine the currents and the variations of the fields and densities.

Thus, it is considered a multilayer model of near-Earth space at high and medium latitudes, with allowance for the electrical conductivity of the Earth. The F region of the ionosphere is subdivided into F_1 and F_2 layers, where ambipolar diffusion plays a significant role in the dynamics of the processes involved. In the approximation of an infinitely conducting Earth, and disregarding the frequency of collisions between electrons and ions in the upper layers of the ionosphere [19], in all the region of space considered (including the region near the surface) we have calculated the electric and magnetic fields excited by ionospheric winds, the currents, and the charged particles density nonuniformities.

Subsequent study. In this formulation, the problem is of great interest for comparison of satellite and terrestrial measurements with theoretical values, and for studying the physical phenomena that take place in the Earth's interior and mantle. Since the Earth is a laminated conductor, variations of the primary terrestrial magnetic field caused by excited currents can be used for magnetotelluric sounding and investigation of the deep structure of the Earth, while ionospheric nonunifonnities play a major role in long-term prediction of space radiocommunica-tions. However, the diversity of factors at work in space, and the difficulties involved in evaluating their relative role, obliges us to confine ourselves to order-of-magnitude estimates in making quantitative calculations.

It is clear that the condition of perfect conductivity of the Earth at the boundary with the neutral atmosphere significantly simplifies the problem. However, in this case the studied electrodynamic system turns to be shielded from the external actions, which does not allow determining magnetic fields outside this system, which is sometimes necessary.

3. Small-Scale Ionospheric Dynamo for the complete multilayer model of Near-Earth Space and the internal structure of the Earth at high and medium latitudes

However, the model, where the Earth is considered, in electrodynamical sense, as a homogeneous, infinitely conducting medium, does not allow to judge, even theoretically, about a penetration of electric fields into the interior regions of Earth and the arising of currents and magnetic fields, forecasting of various natural phenomena (earthquakes etc.), as well as physical–chemical composition of the Earth’s interior.

This is the fact because surface charges appearing at the neutral atmosphere-superconducting medium boundary hinder the penetration of electric fields from atmosphere into the Earth, i.e., the electrostatic potential on the atmosphere-Earth boundary becomes constant (say, zero).

Furthermore, with taking into account these considerations, for the complex model [19] of the near-Earth space and the Earth consisting of plane layers of crust, mantle, and liquid and solid core [27] (see Fig. 2),

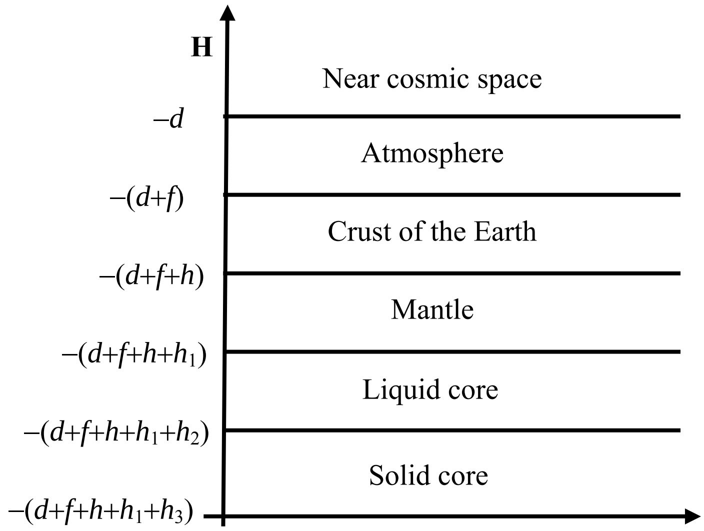

Fig. 2.

The same problem of the dynamo mechanism has been considered and it was shown that electric fields produced by ionosphere winds and having sizes larger than the height of the homogeneous atmosphere are transferred to neighboring regions almost without damping and, propagating through the atmosphere, penetrate into the Earth, which has a certain electroconductivity, and create there electric currents and magnetic fields. For each region of the near-Earth space and the internal structure of the Earth the systems of equations describing the considered processes in every layer have been solved and the general-form analytical solutions were obtained for electrodynamic quantities [19, 27].

However, the solution of the Laplace equation for the electric potential ψ in neutral atmosphere obtained as a Fourier-expansion harmonic at a closed lower boundary, i.e., when the electric potential is joining to the infinitely conducting Earth, goes to zero. The model under consideration here, according to what has been said above, allows the penetration of the field inside the Earth. If follows that the potential must be determined from the system of equations for all arbitrary constants of the obtained electrodynamic quantities. As it has been said above, the motion of neutral gas in conjugate points is expanded into a co- and antirotation. It is also known from the previous investigations that these two cases of motion impose certain conditions on the symmetry of solutions in Southern and Northern hemispheres [7]. This results in boundary conditions between the ionosphere and magnetosphere for the electric field potential, and in possibility of the electric currents appearance in ionosphere and their flowing along the Earth's magnetic lines through the magnetosphere between the two hemispheres. The symmetry of solutions of basic equations of the Earth's gaseous shell affects as well the electric phenomena inside the Earth, since they are caused by perturbations in the ionosphere. This will be mathematically obvious when the electric potential will be joint at the boundary between the neutral atmosphere and the crust of the Earth in both cases of motion (co- and antirotation).

The electric currents in terrestrial layers excited by ionospheric winds can be written in a general form

$$\mathbf{j} = \sigma_{\parallel}\mathbf{E}_{\parallel} + \sigma_p \mathbf{E}_{\perp} + \sigma_H \frac{[\mathbf{HE}]}{H}, \tag{19}$$

where $\mathbf{E}_{\parallel}$, $\mathbf{E}_{\perp}$ are the components of the electric field parallel with and perpendicular to the magnetic field, $\mathbf{H}$ is the Earth's magnetic field, and $\sigma_{\parallel}$, σ_p, σ_H are termed as the longitudinal conductivity, the Pedersen conductivity, and the Hall conductivity, respectively. This equation with the corresponding values of the conductivities is true for all layers of the Earth (including, with certain restrictions, the solid part of the core) except for the outer liquid layer of the core, which is considered as infinitely conducting and described by the Euler equation $\nabla p = (1/c)[\mathbf{jH}]$. Note that in the rectangular coordinate system employed here the *z*-axis is, as in [19], directed along the magnetic field of the Earth. In this case the Cartesian components of the electric current and the conductivity tensor are represented as follows:

$$\mathbf{j}_x = \sigma_p \mathbf{E}_x - \sigma_H \mathbf{E}_y, \mathbf{j}_y = \sigma_H \mathbf{E}_x + \sigma_p \mathbf{E}_y, \mathbf{j}_z = \sigma_{\parallel}\mathbf{E}_z, \bar{\sigma} = \begin{pmatrix} \sigma_p & -\sigma_H & 0 \\ \sigma_H & \sigma_p & 0 \\ 0 & 0 & \sigma_{\parallel} \end{pmatrix}. \tag{20}$$

Since we consider a stationary case, the closeness of the currents is expressed by a relation $\mathrm{div} j = 0$. Assuming that the conductivities vary only along the *z*-axis (or, equivalently, along the Earth's radius) and having in mind the relation $E = -\nabla\psi$, we have

$$\mathrm{div}\, j = \sigma_p \left(\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} \right) + \frac{\partial}{\partial z}\left(\sigma_{\parallel} \frac{\partial \psi}{\partial z} \right) = 0, \tag{21}$$

i.e., the Hall conductivity containing term is absent. Further, requiring the continuity of the current's normal component $\mathbf{j}_z$ and of the potential ψ on the boundaries between the layers we state boundary conditions for determination of ψ from Eq. (21).

Since the conductivities $\sigma_{\parallel}$, σ_p, σ_H have been assumed to vary only with *z* we can seek the solution to Eq. (5) as a Fourier expansion $\psi = f(z)e^{i(k_1 x + k_2 y)}$. Substituting this into (21) we have

$$\sigma_{\|} f''_{zz}(z) + \sigma'_{\|z} f'_z(z) - \sigma_p k^2 f(z) = 0, \tag{22}$$

where $k^2 = k_1^2 + k_2^2$. Note that k_1, k_2 are, according to problem's conception, the same as in [19] for the gaseous cover of the Earth. Making a change of variable $\xi = \int \sqrt{\sigma_p / \sigma_{\|}}\, dz$ in Eq. (21) and approximating the expression $\sigma_m = \sqrt{\sigma_p \sigma_{\|}}$ by an exponential function $\sigma_m = \sigma_0 e^{m\xi}$, we determine the function f and, hence, the electric field potential ψ to have a form

$$\psi = e^{-\frac{m}{2}\xi}(A_1 e^{t\xi} + A_2 e^{-t\xi}) \sin k_1 x \cos k_2 y\,. \tag{23}$$

Here $t = (1/2)\sqrt{m^2 + 4k^2}$ and A_1, A_2 are arbitrary constants (which are, in general, different for each layer) determined by boundary conditions.

Now, from the Poisson equation $-\Delta\psi = 4\pi\rho$, the polarization charge density $\rho = -(1/4\pi)\left(\psi''_{zz} - k^2\psi\right)$ is readily determined.

The components of electric current, with account of Eqs. (20) and (23), are

$$\begin{aligned}
\mathbf{j}_x &= \left[e^{-\frac{m}{2}\xi}(A_1 e^{t\xi} + A_2 e^{-t\xi}) \right] \cdot \left(-\sigma_p k_1 \cos k_1 x \cos k_2 y - \sigma_H k_2 \sin k_1 x \sin k_2 y \right), \\
\mathbf{j}_y &= \left[e^{-\frac{m}{2}\xi}(A_1 e^{t\xi} + A_2 e^{-t\xi}) \right] \cdot \left(-\sigma_H k_1 \cos k_1 x \cos k_2 y - \sigma_p k_2 \sin k_1 x \sin k_2 y \right), \\
\mathbf{j}_z &= -\sigma_{\|} \sqrt{\frac{\sigma_p}{\sigma_{\|}}} \left[\left(t - \frac{m}{2} \right) A_1 e^{\left(t - \frac{m}{2}\right)\xi} - \left(t + \frac{m}{2} \right) A_2 e^{-\left(t + \frac{m}{2}\right)\xi} \right] \sin k_1 x \cos k_2 y.
\end{aligned} \tag{24}$$

Under realistic conditions of existence of the terrestrial core (high temperature and high pressure) its solid part may be considered as a highly conducting homogeneous sphere consisting of iron atoms [28, 29] together with the outer liquid part of the core. Then we can, to a certain degree of accuracy, neglect the horizontal currents as flowing along the magnetic lines of the Earth and (taking into account the condition of continuity of $\mathbf{j}_z$ on the liquid core-solid core boundary) as coinciding with the normal currents in the liquid part of the core.

Magnetic fields produced by these currents can be determined from the equation $-\Delta\mathbf{h} = (4\pi/c)\,\mathrm{rot}\,\mathbf{j}$, which is obtained by taking a curl of both sides of the equation $\mathrm{rot}\,\mathbf{h} = (4\pi/c)\,\mathbf{j}$. However, this equation determines $\mathbf{h}$ to an accuracy of an arbitrary function ψ which must satisfy the Laplace equation, since $\mathrm{div}\,\mathbf{h} = 0$. Having this in mind we can choose the function ψ so as to satisfy all boundary conditions for the magnetic field $\mathbf{h}$ [7].

The disturbance of the magnetic field $\mathbf{h}$ which corresponds to the current (24) has, in general, the following form:

$$h_x = Q_1(z,\sigma_{\|},\sigma_p,\sigma_H,k_1,k_2,m,A_i)\cos k_1 x\cos k_2 y + Q_2(z,\sigma_{\|},\sigma_p,\sigma_H,k_1,k_2,m,A_i)\sin k_1 x\sin k_2 y,$$
$$h_y = Q_3(z,\sigma_{\|},\sigma_p,\sigma_H,k_1,k_2,m,A_i)\cos k_1 x\cos k_2 y + Q_4(z,\sigma_{\|},\sigma_p,\sigma_H,k_1,k_2,m,A_i)\sin k_1 x\sin k_2 y, \quad (25)$$
$$h_z = Q_5(z,\sigma_{\|},\sigma_p,\sigma_H,k_1,k_2,m,A_i)\sin k_1 x\cos k_2 y.$$

These expressions with corresponding values of the conductivities, the constant m, and arbitrary constants A_i (i = 1,2,...) refer to, respectively, layers of the crust and mantle.

The magnetic field **h** in the liquid and solid cores appears as

$$h_x = Q_6(z,\sigma_{\|},\sigma_p,\sigma_H,k_1,k_2,m,A_i)\cos k_1 x\cos k_2 y + Q_7(z,\sigma_{\|},\sigma_p,\sigma_H,k_1,k_2,m,A_i)\sin k_1 x\sin k_2 y,$$
$$h_y = Q_8(z,\sigma_{\|},\sigma_p,\sigma_H,k_1,k_2,m,A_i)\cos k_1 x\cos k_2 y + Q_9(z,\sigma_{\|},\sigma_p,\sigma_H,k_1,k_2,m,A_i)\sin k_1 x\sin k_2 y, \quad (26)$$
$$h_z = Q_{10}(z,\sigma_{\|},\sigma_p,\sigma_H,k_1,k_2,m,A_i)\sin k_1 x\cos k_2 y.$$

The expressions for Q_i are cumbersome, so we will give these expressions and their detailed analysis in a subsequent paper. The polarization charge density in the crust and in the mantle is determined from the Poisson equation:

$$\rho = -(1/4\pi)\left\{A_1 e^{(t-m/2)\xi}\left[(t-m/2)^2\sigma_p/\sigma_{\|} + (t-m/2)\left(\sqrt{\sigma_p/\sigma_{\|}}\right)'_z - k^2\right] + \right.$$
$$\left. + A_2 e^{-(t+m/2)\xi}\left[(t+m/2)^2\sigma_p/\sigma_{\|} - (t+m/2)\left(\sqrt{\sigma_p/\sigma_{\|}}\right)'_z - k^2\right]\right\}. \quad (27)$$

In [19, 27] the boundary conditions are established for physical quantities which unambiguously define the formulated problem. Here also the boundary conditions between the layers will be given for the figures of [2, 3] in the complete complex model of the near-Earth space and of the Earth's structure for the cases of both anti-rotation and co-rotation [30].

Anti-rotation case. If the motion of neutral gas in conjugated points of the northern and the southern hemispheres is opposite, for the electric field potential the following boundary conditions take place: $f_1^{F_2} = 0$ at $z = \pm(d-a-l-p)$ on the boundary between the F_2-layer and magnetosphere, $f_1^{F_2} = f_1^{F_1}$ at $z = \pm(d-a-l)$ on the boundary between the F_1- and F_2-layers, $f_1^{F_1} = f_1^{E}$ at $z = \pm(d-a-l)$ on the boundary between the F_1- and E-layers, $f_1^{E} = f_1^{a}$ at $z = \pm d$ on the boundary between the E-layer and neutral atmosphere, $f_1^{a} = f_3^{k}$ at $z = \pm(d+f)$ on the boundary between the neutral atmosphere and the Earth surface, $f_3^{k} = f_3^{m}$ at $z = \pm(d+f+h)$ at the boundary of Earth's crust and mantle, $f_3^{m} = 0$ at $z = \pm(d+f+h+h_1)$ at the boundary between the mantle and the liquid core, and $f_3^{c.l.} = f_3^{c.s.} = 0$ at $z = \pm(d+f+h+h_1+h_2)$ at the boundary between the liquid and the solid cores of the Earth. For the electric current j the following boundary conditions should be satisfied: $j_z^{F_2} = j_z^{m}$ at $z = \pm(d-a-l-p)$ on the boundary between the F_2-layer and magnetosphere, and the current in the magnetosphere is directed along the Earth's magnetic field, it is constant and equal to the boundary value $j_z^{\mathrm{F_2}}$ [7], $j_z^{\mathrm{F_2}} = j_z^{\mathrm{F_1}}$ at $z = \pm(d-a-l)$ on the boundary between the $\mathrm{F_1}$- and $\mathrm{F_2}$-layers, $j_z^{\mathrm{F_1}} = j_z^{\mathrm{E}}$ at $z = \pm(d-a)$ on the boundary between the $\mathrm{F_1}$- and E-layers, $j_z^{\mathrm{E}} = j_z^{\mathrm{a}} = 0$ at $z = \pm d$ on the boundary between the E-layer and neutral atmosphere, $j_z^{\mathrm{a}} = j_{3z}^{\mathrm{k}} = 0$ at $z = \pm(d+f)$ on the boundary between the neutral atmosphere and the surface of the Earth's layered structure, $j_{3z}^{\mathrm{k}} = j_{3z}^{\mathrm{m}}$ at $z = \pm(d+f+h)$ on the boundary between the Earth's crust and mantle, $j_{3z}^{\mathrm{m}} = j_{3z}^{\mathrm{c.l.}}$ at $z = \pm(d+f+h+h_1)$ on the boundary between the mantle and the liquid core, and $j_{3z}^{\mathrm{c.l.}} = j_{3z}^{\mathrm{c.s.}}$ at $z = \pm(d+f+h+h_1+h_2)$ on the boundary between the liquid and the solid cores of the Earth. It should be noted that the current in the liquid core as in magnetosphere is directed along the Earth's magnetic field and equals j_{3z}^{m}, the value on the boundary between the mantle and liquid core.

Co-rotation case. If the motion of neutral gas in conjugated points of the northern and the southern hemispheres is similar, the boundary conditions are the same as in the anti-rotation case, except for the condition $j_z^{\mathrm{F_2}} = 0$ at $z = \pm(d-a-l-p)$ on the boundary between the $\mathrm{F_2}$-layer and magnetosphere, i.e., there is no current in the magnetosphere.

Substituting into these relations the boundary values of physical quantities and analytical solutions for electric field potentials and currents determined in each layer in [19, 27], the following system of equations is obtained in the case of anti-rotation [30]:

$$\begin{aligned}
&f_1^{\mathrm{F_2}}\Big|_{z=\pm(d-a-l-p)} = \alpha_1\left(A\overline{\beta}_1(t) + \overline{B}\beta_2(t) + C_1\overline{\beta}_3(t) + C_2\overline{\beta}_4(t)\right) + \alpha_6\Big|_{z=\pm(d-a-l-p)} = 0,\\
&f_{1\mathrm{a.d.}}^{\mathrm{F_2}}\Big|_{z=\pm(d-a-l-p)} = \alpha_1\left(A\overline{\overline{\beta}}_1(t) + B\overline{\overline{\beta}}_2(t) + C_1\overline{\overline{\beta}}_3(t) + C_2\overline{\overline{\beta}}_4(t)\right)\Big|_{z=\pm(d-a-l-p)} = 0, \quad ,\\
&j_{z\mathrm{a.d.}}^{\mathrm{F_2}}\Big|_{z=\pm(d-a-l-p)} = eN_0\left\{A\overline{\overline{G}}_1(t) + B\overline{\overline{G}}_2(t) + C_1\overline{\overline{G}}_3(t) + C_2\overline{\overline{G}}_4(t)\right\}\Big|_{z=\pm(d-a-l-p)} = 0.
\end{aligned} \tag{28}$$

on the boundary between the $\mathrm{F_2}$-layer and magnetosphere – the regions of weakly ionized gas (where in photochemical processes ion–molecular reactions are predominant and in the distribution of charged particles the ambipolar diffusion also plays a role) and infinitely conducting plasma;

$$\begin{aligned}
&f_1^{\mathrm{F_2}}\Big|_{z=\pm(d-a-l)} = \alpha_1\left(A\overline{\beta}_1(t) + B\overline{\beta}_2(t) + C_1\overline{\beta}_3(t) + C_2\overline{\beta}_4(t)\right) + \alpha_6\Big|_{z=\pm(d-a-l)} =\\
&= f_1^{\mathrm{F_1}}\Big|_{z=\pm(d-a-l)} = \alpha_1\left(C_3\beta_5(t) + C_4\beta_6(t) + C_5\beta_7(t)\right)\Big|_{z=\pm(d-a-l)},\\
&f_{1\mathrm{a.d.}}^{\mathrm{F_2}}\Big|_{z=\pm(d-a-l)} = \alpha_1\left(A\overline{\overline{\beta}}_1\left(B\overline{\overline{\beta}}_2(t) + C_1\overline{\overline{\beta}}_3(t) + C_2\overline{\overline{\beta}}_4(t)\right)\right) + \alpha_6\Big|_{z=\pm(d-a-l)} = 0,\\
&j_z^{\mathrm{F_2}}\Big|_{z=\pm(d-a-l)} = eN_0\left[A\overline{G}_1(t) + B\overline{G}_2(t) + C_1\overline{G}_3(t) + C_2\overline{G}_4(t)\right]\Big|_{z=\pm(d-a-l)} =\\
&= j_z^{\mathrm{F_1}}\Big|_{z=\pm(d-a-l)} = eN_0\left[C_3G_5(t) + C_4G_6(t) + C_5G_7(t)\right]\Big|_{z=\pm(d-a-l)},\\
&j_{z\mathrm{a.d.}}^{\mathrm{F_2}}\Big|_{z=\pm(d-a-l)} = eN_0\left[A\overline{\overline{G}}_1(t) + B\overline{\overline{G}}_2(t) + C_1\overline{\overline{G}}_3(t) + C_2\overline{\overline{G}}_4(t)\right]\Big|_{z=\pm(d-a-l)} = 0,
\end{aligned} \tag{29}$$

on the boundary between the F_1- and F_2-layers of the weakly ionized gas where photochemical processes of creation and annihilation of ions and electrons almost coincide (except for ambipolar diffusion which is absent in the F_1-layer);

$$\begin{aligned} & f_1^{F_1}\Big|_{z=\pm(d-a)} = \alpha_1\left[C_3\beta_5(t)+C_4\beta_6(t)+C_5\beta_7(t)\right]+\alpha_6\Big|_{z=\pm(d-a)} = f_1^{E}\Big|_{z=\pm(d-a)} = \\ & = \alpha_{10}\beta_8(t)\left[C_6\beta_9(t)+C_7\beta_{10}(t)+C_8\beta_{11}(t)\right]+\alpha_6\Big|_{z=\pm(d-a)}, \\ & j_z^{F_1}\Big|_{z=\pm(d-a)} = eN_0\left[C_3G_5(t)+C_4G_6(t)+C_5G_7(t)\right]\Big|_{z=\pm(d-a)} = j_z^{E}\Big|_{z=\pm(d-a)} = \\ & = eN_0\left[C_6G_8(t)+C_7G_9(t)+C_8G_{10}(t)\right]\Big|_{z=\pm(d-a)} \end{aligned} \tag{30}$$

on the boundary between E and F_1-regions of the weakly ionized gas where in photochemistry of charged particles processes of, respectively, dissociative recombination and ion–molecular reactions are predominant;

$$\begin{aligned} & f_1^{E}\Big|_{z=\pm d} = \alpha_{10}\beta_8(t)\left[C_6\beta_9(t)+C_7\beta_{10}(t)+C_8\beta_{11}(t)\right]+\alpha_6\Big|_{z=\pm d} = f_1^{a}\Big|_{z=\pm d} = \\ & = \left\{C_9\exp\left[k\left(d+f-|z|\right)\right]+C_{10}\exp\left[-k\left(d+f-|z|\right)\right]\right\}\Big|_{z=\pm d}, \\ & j_z^{E}\Big|_{z=\pm d} = eN_0\left[C_6G_8(t)+C_7G_9(t)+C_8G_{10}(t)\right]\Big|_{z=\pm d} = j_z^{a}\Big|_{z=\pm d} = 0 \end{aligned} \tag{31}$$

on the boundary between the layers of neutral atmosphere and the layers of E-regions where, respectively, charged particles are absent and the processes of dissociative recombination are predominant;

$$\begin{aligned} & f_1^{a}\Big|_{z=\pm(d+f)} = \left\{C_9\exp\left[k\left(d+f-|z|\right)\right]+C_{10}\exp\left[-k\left(d+f-|z|\right)\right]\right\}\Big|_{z=\pm(d+f)} = f_1^{k}\Big|_{z=\pm(d+f)} = \\ & = \left\{A_1^{k}\exp\left(t_0-\frac{m_0}{2}\right)\xi+A_2^{k}\exp\left[-\left(t_0+\frac{m_0}{2}\right)\xi\right]\right\}\Bigg|_{z=\pm(d+f)}, \\ & j_z^{k}\Big|_{z=\pm(d+f)} = -eN_0\sigma_{\parallel}\sqrt{\sigma_P/\sigma_{\parallel}}\times \\ & \times\left\{A_1^{k}\left(t_0-\frac{m_0}{2}\right)\exp\left(t_0-\frac{m_0}{2}\right)\xi-A_2^{k}\left(t_0+\frac{m_0}{2}\right)\exp\left[-\left(t_0+\frac{m_0}{2}\right)\xi\right]\right\}\Bigg|_{z=\pm(d+f)} = 0 \end{aligned} \tag{32}$$

on the boundary between the Earth surface and neutral atmosphere from where the electric field penetrates into interior regions of the Earth.

Before proceeding to the boundary relations between the layers of the Earth structure it should be noted that the *z*-axis of the coordinate system of [19] is continued to the center of the Earth and h, h_1, h_2, h_3; c, m, l, s are, respectively, thicknesses and superscripts of constants and functions in the layers of the crust, mantle, liquid and solid cores of the Earth.

In the inner structure of the Earth the following equations should be satisfied:

$$\begin{aligned} & f_1^{c}\Big|_{z=\pm(d+f+h)} = \left\{A_1^{c}\exp\left(t_0-m_0/2\right)\xi+A_2^{c}\exp\left[-\left(t_0+m_0/2\right)\xi\right]\right\}\Big|_{z=\pm(d+f+h)} \\ & = f_1^{m}\Big|_{z=\pm(d+f+h)} = \left\{A_1^{m}\exp\left(t_0-m_0/2\right)\xi+A_2^{m}\exp\left[-\left(t_0+m_0/2\right)\xi\right]\right\}\Big|_{z=\pm(d+f+h)}, \end{aligned} \tag{33a}$$

$$j_z^{\mathrm{c}}\Big|_{z=\pm(d+f+h)} = -eN_0\sigma_{\|}\sqrt{\sigma_{\mathrm{P}}/\sigma_{\|}}\times$$
$$\times\left\{A_1^{\mathrm{c}}\left(t_0-m_0/2\right)\exp\left(t_0-m_0/2\right)\xi - A_2^{\mathrm{c}}\left(t_0+m_0/2\right)\exp\left[-\left(t_0+m_0/2\right)\xi\right]\right\}\Big|_{z=\pm(d+f+h)} =$$
$$= j_z^{\mathrm{m}}\Big|_{z=\pm(d+f+h)} = -eN_0\sigma_{\|}\sqrt{\sigma_{\mathrm{P}}/\sigma_{\|}}\times \tag{33b}$$
$$\times\left\{A_1^{\mathrm{m}}\left(t_0-\frac{m_0}{2}\right)\exp\left(t_0-\frac{m_0}{2}\right)\xi - A_2^{\mathrm{m}}\left(t_0+\frac{m_0}{2}\right)\exp\left[-\left(t_0+\frac{m_0}{2}\right)\xi\right]\right\}\Big|_{z=\pm(d+f+h)}$$

on the boundary between the crust and mantle of the Earth;

$$f_1^{\mathrm{m}}\Big|_{z=\pm(d+f+h+h_1)} = \left\{A_1^{\mathrm{m}}\exp\left(t_0-\frac{m_0}{2}\right)\xi + A_2^{\mathrm{m}}\exp\left[-\left(t_0+\frac{m_0}{2}\right)\xi\right]\right\}\Big|_{z=\pm(d+f+h+h_1)} = f_1^{\mathrm{c.l.}} = 0,$$
$$j_z^{\mathrm{m}}\Big|_{z=\pm(d+f+h+h_1)} = f_z^{\mathrm{c.l.}}, \tag{34}$$

on the boundary between the mantle and the liquid core of the Earth;

$$f_1^{\mathrm{c.l.}}\Big|_{z=\pm(d+f+h+h_1+h_2)} = f_1^{\mathrm{c.s.}} = 0,$$
$$j_z^{\mathrm{c.l.}}\Big|_{z=\pm(d+f+h+h_1+h_2)} = j_z^{\mathrm{c.s.}} \tag{35}$$

on the boundary between the liquid and solid cores of the Earth.

In case of co-rotation the first equation in (1) $f_1^{\mathrm{F}_2}\Big|_{z=\pm(d-a-l-p)} = 0$ should be replaced by the equation

$$j_z^{\mathrm{F}_2}\Big|_{z=\pm(d-a-l-p)} = eN_0\left[AG_1(t)+BG_2(t)+C_1G_3(t)+C_2G_4(t)\right]\Big|_{z=\pm(d-a-l-p)} = 0\,. \tag{36}$$

Here $f_1^{\mathrm{F}_2}$, $j_z^{\mathrm{F}_2}$ and $f_{1\mathrm{a.d.}}^{\mathrm{F}_2}$, $j_{z\mathrm{a.d.}}^{\mathrm{F}_2}$ are the extracted parts of electric fields and currents in the F_2-layer, which are responsible for, respectively, photochemistry, winds, and ambipolar diffusion in ionosphere, A, B, $C_{1,\ldots,10}$ are arbitrary constants which are determined from the boundary conditions, $\alpha_{1,\ldots,9}$ are constants determining the processes of ambipolar diffusion and consisting of regular physical parameters of the structure of near-Earth space. They enter the known functions $\beta_{1,\ldots,11}(t)$, $G_{1,\ldots11}(t)$ $(t = e^{-2(d+f-|z|)}/H_n)$ entering linearly and additively the analytical solutions of the problem [2]. Hence, $\beta_{1,\ldots,11}(t)$ and $G_{1,\ldots11}(t)$ may be represented as the sums $\beta_i(t)=\bar{\beta}_i(t)+\bar{\bar{\beta}}_i(t)$ and $G_i(t)=\bar{G}_i(t)+\bar{\bar{G}}_i(t)$, where each pair $\bar{\beta}_i(t)$, $\bar{\bar{\beta}}_i(t)$ and $\bar{G}_i(t)$, $\bar{\bar{G}}_i(t)$ contain, respectively, the terms describing photochemistry and winds in ionosphere and ambipolar diffusion. This is just the meaning of $\bar{\beta}_i(t)$, $\bar{\bar{\beta}}_i(t)$ and $\bar{G}_i(t)$, $\bar{\bar{G}}_i(t)$ in the systems of equations (28) and (29).

In equations (32), (33a,b), and (34) A_1^{k}, A_2^{k} and A_1^{m}, A_2^{m} are 4 arbitrary constants (generally different for different layers) determined from the boundary conditions, $\sigma_{\|}$ and σ_p are the longitudinal and Pedersen conductivities (also different for different layers), $t_0 = (1/2)\sqrt{m_0^2+4k^2}$, m_0

is the constant of approximation of the expression $\sigma_{m_0} = \sqrt{\sigma_{\|}\sigma_p}$ by an exponent, $\sigma_{m_0} = \sigma_0 e^{m_0}\xi$, $\xi = \int \sqrt{\sigma_{\|}\sigma_{\mathrm{P}}}dz$, and $k^2 = k_1^2 + k_2^2$ with k_1 and k_2 being the same as in [19].

It is easy to see that equations (28)–(35) constitute a complete system of linear non-uniform algebraic equations with respect to the constants $A, B, C_{1,\ldots,10}, A_{1,2}^{\mathrm{k}}, A_{1,2}^{\mathrm{m}}$ entering the analytical solutions of the problem [19, 27]. This means that the problem of the mechanism of "atmospheric dynamo" for a multilayer complex model of the near-Earth space and conducting Earth represented in the form of a multilayer unipolar inductor where every layer has specific physical, chemical, and dynamic properties, has an unambiguous solution. In other words, penetration of electric fields from the gaseous shell into the inner regions of the Earth is unambiguously possible. Electric fields excited by ionosphere winds, whose sizes are larger than the height of the homogeneous atmosphere, penetrating into the conducting layers of the Earth produce there electric currents and magnetic fields.

Equations (29)–(35) for the constants $A, B, C_{1,\ldots,10}, A_{1,2}^{\mathrm{k}}, A_{1,2}^{\mathrm{m}}$ are solved at following realistic values of relevant parameters: the distance, along the Earth's magnetic field line, between the conjugated points of the northern and southern hemispheres in latitudes 65° is d = 40,000 km; the height of the neutral atmosphere from the Earth's surface $f = 100$ km; thickness of the E-layer a = 40 km; thickness of the F_1-layer l = 60 km (the region from 140 to 200 km). At altitudes from 200 km up to conventionally accepted 400 km there is the F_2-layer, so its thickness is p = 200 km. The ionic and neutral components in the E- and F-layers consist, respectively, of O_2^+, O_2 and O^+,O. Temperature of neutrals T_n in the E- and F-layers are taken to be, respectively, 300 K and 1000 K and temperature of ions / electrons in the F_2-layer is $T_{i,e}$ = 1000 K. The height of the homogeneous atmosphere $H_n = 8\times10^5$ cm in the E-layer and $H_n = 8\times10^6$ cm in F-layers. The coefficient of recombination of positive ions and electrons in the E-layer is $\alpha = 10^{-7}$ cm^3/s; the coefficient of annihilation of charged particles in the F_1- and F_2-layers is taken $a_r = 10^{-13}$ cm^3/s and the Chapman distribution of the ionization rates is $q_0 = 100$ cm^{-3}/s. The concentrations of neutral and charged particles at the initial conventional heights is accepted to equal $N_{0n} = 10^{12}$ atoms/cm^3 and $N_0 = 10^5$ at/cm^3 in the E-layer and $N_{0n} = 4\times10^{10}$ atoms/cm^3 and $N_0 = 4\times10^5$ at/cm^3 in the F-layers. The frequencies of collisions of ions and electrons with neutral particles at the same heights are $\gamma_{i0} = 2\times10^3$ s^{-1}, $\gamma_{e0} = 10^4$ s^{-1} in the E-layer and $\gamma_{i0} = 10^4$ s^{-1}, $\gamma_{e0} = 3\times10^4$ s^{-1} in the F-layers. The amplitude of the speed of wind is $w_0/k_{1,2} = 200$ m/s, and the size of a cell $2\pi/k_{1,2} = 200$ km.

The concentration of charged particles is approximated by the formula $N_{0i,e} = N_0 \exp(z/H_m)$ and the magnetic field strength at the Earth surface is taken to be H = 0.5 G.

It is furthermore accepted that for radii of the Earth R_E = 6370 km, of its solid core R_S = 1250 km, and of its liquid core R_L = 3450 km, the crust, the mantle and the liquid and solid cores have the corresponding thicknesses: h = 40 km, h_1 = 2880 km, h_2 = 2200 km, and h_3 = 1250 km. Magnetic fields produced by the electric currents of the whole system are determined by the equation $\Delta\mathbf{h} = (4\pi/c)\,\mathrm{rot}\, j$.

Electric fields induced in conjugate points of the crust by a magnetic line going inside the Earth in the Northern and Southern hemispheres, propagating into inner layers, will produce electric currents and magnetic fields. Moreover, the electric currents between the conjugate points of the crust must be closed along the magnetic lines through the inner core and the horizontal crust-layer, since the conductivity $\sigma_{\parallel}$ along magnetic lines is essentially higher than that in perpendicular direction, i. e., σ_p and σ_H. The latters have their highest value in upper layers of the Earth.

So, it is shown above that the electric processes in near-Earth cosmic space interact with the electroconducting Earth and affect its internal electrodynamical state. Particularly, we have studied the problem of the electric field and current generation in the interior of the Earth under influence of the dynamo of ionospheric small-scale processes. In this case we have calculated the electric fields, currents, and the magnetic fields induced, as well as the polarization charge density in each of the layers in dependence on the longitudinal conductivity and on the Pedersen and Hall conductivities. The excited magnetic fields contribute to the magnetic variations of the field of the Earth and can be used to examine the natural resources (geological investigations), since the methods of separation of different-type disturbances are well developed [31]

As is seen from the expressions for the magnetic field **h**, they contain the conductivities which characterize the electrical properties of corresponding layers. Assuming that we know (with a certain reserve) the conductivities of the core and mantle, i.e., of the iron and silicates [29, 31] which are the main components of these layers, we obtain then the magnetic field disturbance in dependence on the crust conductivity.

This, in its turn, will allow drawing important physical conclusions – it is possible, by measuring the fields, to determine the characteristics of the studied object producing these fields, i.e., to solve the inverse problem of geophysics.

In conclusion, it was succeeded in the present work to obtain analytical solution, within the framework of the proposed mathematical model of the considered problem, which is unique and valid for the overall volume of the system. The problem is, however, non-correct according to Hadamard, as are the majority of geophysical problems. Because of instability of the inverse problem it is possible, with the approximate values of the input data, to obtain a solution strongly differing from the true one. But how should be solved such unstable problems. This question is answered in the mathematical theory of regularization, the basic concept of which is that of the conditionally-correct (or Tikhonov-correct) formulation of problem. The results of this theory are applied widely in all the fields of contemporary physics where solution of inverse problems is needed [32].

So, with use of the regularization theory, it is possible to solve the inverse problem proposed here: to study the physical–chemical composition of the Earth's crust from the results of the direct problem. Since, as mentioned above, the geophysical methods are indirect methods to study the structure and composition of the Earth. For this purpose, the obtained formulas must be applied to gauge magnetometers in testing ranges and to process the results of magnetic measurements. But this is the problem of exploration geophysics which requires a separate study.

In addition, magnetic variations of the field of the Earth can give information on its seismic state, since the problem under discussion is stationary, i.e., it is reversible and the ionosphere disturbances can be considered as a consequence of the disturbances in the bowels of the Earth.

There are many works concerning questions of electromagnetic probing of the Earth. Among these works [32–35] may be cited. A number of authors performed, based on the theory of skin-effect, investigations of the electroconductivity of the Earth at various frequencies and penetration depths of electromagnetic waves. The proposed multilayer model allows involving in the investigation of natural resources wider (corresponding to sizes of ionosphere winds) regions of the surface and deep-lying layers of the Earth.

However, the proposed model requires a further study and improvement with allowance for physico-chemical properties of each of the strata and for physical conditions of their existence. This will lead to more accurate practically important results.

4. Conclusions

We would like to observe that the author of the present survey managed to solve the complete system of quasi-hydrodynamic equations for the F_2 layer of the weakly ionized ionosphere plasma. He took into account the members caused by ambipolar diffusion, gravity force of particles, and the Earth magnetic field effect. As a result, the author obtained analytic expressions for electro-dynamic quantities, provided that parameters $\lambda_i, \lambda_e \gg 1,$ which is rather difficult problem [26].

1. While the physical and chemical processes running in the gaseous shell of the Earth are well known, in the same way as numerical values of the real physical parameters used in our suggested model, one cannot say the same thing about the internal structure of the Earth hidden for observations. Therefore initially the Earth is considered as infinitely conducting and electrodynamics of the near space is studied in this approximation, determined by ionosphere winds. We have done it in the first manuscript [19]. Namely, the *direct* problem of geophysics was

solved: numerical values of the fields, currents and charge-density perturbations were found, being excited by ionosphere winds, provided that real parameters of the Earth's gas shell are known. The obtained results, as mentioned in the manuscript, may be utilized for interpretation of radar and satellite measurement data for electromagnetic fields and currents in ionosphere and magnetosphere of the Earth.

2. The second work [27], by its formulation, actually represents an *inverse* problem of geophysics for completely different model. Namely, one has to determine the characteristics of the Earth-core electrical conductivity on basis of complete multi-layer model of the Earth's gaseous shell and its internal structure, as well as analytic expressions obtained by the author. However, the inverse geophysical problems are usually incorrect, in the sense of *J. Hadamard.* Therefore the analysis should be performed by means of mathematical regularization theory, namely under the "conditionally correct" statement of the problem, due to *A. N. Tikhonov.* However, this is actually a problem of exploration geophysics.

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