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PARAMETRIC EXCITATION OF PLASMA WAVES BY AN ELECTRON BUNCH IN THE PRESENCE OF INTENSE ELECTROMAGNETIC RADIATION

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Abstract: The excitation of the plasma wake waves by an electron bunch in the presence of an intense electromagnetic radiation with circular polarization has been studied in parametric unstable regimes. In the unperturbed state (in the absence of an electron bunch), the interaction between a pump wave and a plasma is described by the Maxwell's equations and the nonlinear relativistic hydrodynamic equations for a cold plasma. The excitation of linear waves by an electron bunch is investigated against a cold plasma background. It is shown that, in a certain range of the parameters of the bunch, pump wave, and plasma, the excitation is resonant in character and the amplitude of the excited plasma wake waves increases with distance from the electron bunch.

Keywords: Plasma accelerators, parametric excitations, electron beam, intense radiation

1. Introduction

Plasma-based methods of charged particle acceleration, which have been actively developed over the past two decades, occupy an important place among novel acceleration schemes (see, e.g., [1, 2] and the references therein). The excitation of wake waves by charged–particle bunches is one of the ways of generating strong (up to $E \sim 1$ GeV/m) electromagnetic fields in plasmas. The induced wake fields can serve not only to accelerate charged particles but also to focus electron (positron) bunches [3] with the aim of generating high density beams and ensuring high luminosity in the next generation of linear colliders.

Many papers have been devoted to the linear theory of one–dimensional wake waves [3-10]. The nonlinear theory of these waves was developed in [11-18].

This work is a continuation of [19], which was aimed at studying the effect of a circularly polarized electromagnetic wave of arbitrary intensity $A = eE_0/mc\omega_0$ (where E_0 and ω_0 are the amplitude and frequency of the electromagnetic field) on the excitation of electromagnetic wake waves by a one-dimensional relativistic electron bunch in a cold plasma, in which case the electron oscillatory velocity in the pump field can be close to the speed of light. It was shown that there are three ranges of parameter values of the pump wave, electron bunch, and plasma in which the linear equations for the induced longitudinal and transverse fields have three different solutions. The induced fields were studied only in the parameter range where the plasma is stable against the parametric instability, which was thoroughly analyzed in [20-23]. In this case, the induced fields were found to be a superposition of two harmonic oscillations with different amplitudes and frequencies [19].

The objective of the present paper is to investigate the role played by a strong electromagnetic wave with circular polarization in the excitation of one-dimensional linear wake fields in the parameter range in which the plasma is unstable against the parametric instability. As in [19], the pump wave – plasma interaction in the absence of a bunch is described by Maxwell's equations and nonlinear hydrodynamic equations in the cold plasma approximation. In this case, the plasma can be in a spatially homogeneous state [18-20]. Then, perturbation theory is applied to derive and analyze the expressions for the induced fields under the assumption that a one-dimensional bunch propagating in the plasma perturbs this state only slightly.

2. Basic Equations

Assuming that the oscillatory velocity of the plasma electrons in an external field is much higher than the electron thermal velocity and the pump frequency ω_0 is far above the electron-ion collision frequency, we start with the following basic set of equations, which includes Maxwell's equations and the relativistic hydrodynamic equation of motion for cold electron plasma:

$$\nabla\times\mathbf{B}=\frac{1}{c}\frac{\partial\mathbf{E}}{\partial t}-\frac{4\pi e}{c}n\mathbf{v}-\frac{4\pi e}{c}\mathbf{u}n_b\left(\xi\right), \tag{1}$$

$$\nabla\times\mathbf{E}=-\frac{1}{c}\frac{\partial\mathbf{B}}{\partial t},\quad \nabla\cdot\mathbf{B}=0 \tag{2}$$

$$\nabla\cdot\mathbf{E}=-4\pi e\left(n-n_0\right)-4\pi e n_b\left(\xi\right) \tag{3}$$

$$\frac{\partial\mathbf{v}}{\partial t}+\left(\mathbf{v}\cdot\nabla\right)\mathbf{v}=-\frac{e}{m}\sqrt{1-\frac{v^2}{c^2}}\left[\mathbf{E}+\frac{1}{c}\mathbf{v}\times\mathbf{B}-\frac{\mathbf{v}}{c^2}\left(\mathbf{v}\cdot\mathbf{E}\right)\right], \tag{4}$$

$$\frac{\partial n}{\partial t}+\nabla\cdot\left(n\mathbf{v}\right)=0, \tag{5}$$

where $\xi=z-ut$, n_0 is the unperturbed electron plasma density, and $n_b\left(\xi\right)$ is the density of a given one-dimensional bunch propagating with the velocity $\mathbf{u}$ (such that $\mathbf{u}=u\mathbf{e}_z$, $\left|\mathbf{e}_z\right|=1$) in a plasma. Since we are interested in relativistic bunches, we neglect oscillations of the bunch electrons in an external electromagnetic pump wave.

In the field of a circularly polarized electromagnetic wave propagating along the z-axis, the plasma can be in a spatially homogeneous state, in which the electromagnetic field and electron velocity are given by the expressions [18-20]

$$\mathbf{E}_0=E_0\left(\mathbf{e}_x\cos\psi+\mathbf{e}_y\sin\psi\right),\quad E_{0z}=0, \tag{6}$$

$$\mathbf{B}_0=\frac{k_0c}{\omega_0}E_0\left(-\mathbf{e}_x\sin\psi+\mathbf{e}_y\cos\psi\right),\quad B_{0z}=0, \tag{7}$$

$$\mathbf{v}_e=c\beta_e\left(-\mathbf{e}_x\sin\psi+\mathbf{e}_y\cos\psi\right),\quad v_{ez}=0, \tag{8}$$

where $\psi = \omega_0 t - k_0 z$, $k_0 = (\omega_0/c)\sqrt{\varepsilon(\omega_0)}$, $\varepsilon(\omega) = 1 - \omega_L^2/\omega^2$, $\omega_L^2 = \omega_p^2\sqrt{1-\beta_e^2}$, $\beta_e = v_e/c$,

$$v_e = c\frac{A}{\sqrt{1+A^2}}, \tag{9}$$

$A = eE_0/mc\omega_0$, $\omega_p^2 = 4\pi n_0 e^2/m$ is the square of the plasma frequency, and c is the speed of light in vacuum.

We consider small perturbations which are driven in a plasma by an electron bunch with density n_b such that $n_b \ll n_0$. We represent all of the quantities in the form $f = f_0 + f'$, where f_0 stands for the unperturbed quantities in expressions (6)-(9), and switch from the x- and y-components of the fields and electron plasma velocities to the new variables

$$\begin{pmatrix} E^\pm \\ B^\pm \\ w^\pm \end{pmatrix} = \begin{pmatrix} E_x' \pm iE_y' \\ B_x' \pm iB_y' \\ v_x + iv_y \end{pmatrix} = \begin{pmatrix} \mathcal{E}^\pm(\xi) \\ \mathcal{B}^\pm(\xi) \\ \mathcal{V}^\pm(\xi) \end{pmatrix} e^{\pm i\psi}. \tag{10}$$

In other words, we perform the transformation to the rotating frame associated with the pump wave.

In [19] linearizing Eqs. (1)-(5) and performing the transformations (10) we obtained the following expressions for the components of the induced electromagnetic fields:

$$\begin{pmatrix} E_z'(\xi) \\ \mathcal{E}^+(\xi) \end{pmatrix} = \int_{-\infty}^{\infty} d\xi' n_b(\xi') \begin{pmatrix} G_z(\xi'-\xi) \\ G_\perp(\xi'-\xi) \end{pmatrix}. \tag{11}$$

Here, G_z and $G_\perp$ are the Green's functions for the quantities E_z' and $\mathcal{E}^+$, respectively,

$$G_z(s) = -2ie\int_{-\infty}^{\infty} \frac{dk}{k}\frac{D_1(k,\omega)}{D(k,\omega)} e^{iks}, \tag{12}$$

$$G_\perp(s) = -\frac{2e\beta_e\omega_L^2}{c}\int_{-\infty}^{\infty} dk\,(ku+\omega_0)\frac{R_{-1}(k,\omega)}{D(k,\omega)} e^{iks}, \tag{13}$$

where $\omega = ku$. According to Eq. (2), the induced magnetic field is related to the transverse electric field as

$$\mathcal{B}^\pm(\xi) = \pm\frac{c}{u}\left\{ i\mathcal{E}^\pm(\xi) + \left(\frac{\omega_0}{u} - k_0\right)\int_{-\infty}^{\xi} d\xi' \exp\left[i\frac{\omega_0}{u}(\xi'-\xi)\right]\mathcal{E}^\pm(\xi')\right\}, \tag{14}$$

$$B_z' = 0.$$

where we have introduced the notations

$$R_{\pm 1}(k,\omega) = (k \pm k_0)^2 - \frac{(\omega\pm\omega_0)^2}{c^2}\varepsilon(\omega\pm\omega_0), \tag{15}$$

$$D_1(k,\omega) = \omega^2 R_1(k,\omega)R_{-1}(k,\omega) + \frac{\beta_e^2\omega_L^2}{2}\left(k^2 - \frac{\omega^2}{c^2}\right)\left[R_1(k,\omega) + R_{-1}(k,\omega)\right], \tag{16}$$

$$D(k,\omega)=\omega^2\varepsilon(\omega)R_1(k,\omega)R_{-1}(k,\omega)+\frac{\beta_e^2\omega_L^2}{2}\left(k^2-\frac{\omega^2}{c^2}\varepsilon(\omega)\right)\left[R_1(k,\omega)+R_{-1}(k,\omega)\right]. \quad (17)$$

The transverse components of the induced electric and magnetic fields can be found from expression (10) by taking either a real or an imaginary part of the complex quantities $\mathcal{E}^+$ and $\mathcal{B}^+$. As a result, we obtain

$$E_x'(z,t)=E_r(\xi)\cos\psi-E_i(\xi)\sin\psi, \quad E_y'(z,t)=E_r(\xi)\sin\psi+E_i(\xi)\cos\psi, \quad (18)$$

where

$$E_r(\xi)=\mathrm{Re}\left[\mathcal{E}(\xi)\right], \quad E_i(\xi)=\mathrm{Im}\left[\mathcal{E}(\xi)\right]. \quad (19)$$

Hence, expression (18) for the transverse components of the induced fields describes modulational perturbations in the plasma. Note that, in the absence of a pump wave (β_e = 0), the transverse components of the perturbed quantities vanish and expressions (11) and (12) coincide with the familiar expressions for one-dimensional linear fields.

Now, we proceed to the calculation of the Green's functions defined by Eqs. (12) and (13). In these expressions the poles of the integrals are the roots of the dispersion relation $D(k, \omega) = 0$. In the general case (i.e., when the Cherenkov resonance condition $\omega = ku$ is not imposed), this dispersion relation was investigated in detail by Kalmykov and Kotsarenko [20]. In the absence of a pump wave ($\beta_e = 0$), Eq. (17) yields conventional dispersion relations for plasma waves, $\omega = \omega_p$, and for transverse (electromagnetic) waves, $\omega^2=\omega_p^2+k^2c^2$. The presence of a pump wave ($\beta_e \neq 0$) gives rise to the coupled waves in a plasma. If the pump wave is sufficiently weak ($\beta_e \ll 1$), then the growth rate of the coupled waves increases linearly with β_e. Consequently, the coupled waves are parametrically unstable down to $\beta_e = 0$.

Under the Cherenkov resonance condition $\omega = ku$, Eq. (17) gives the dispersion relation

$$\left(k^2-\frac{\omega_L^2}{u^2}\right)\left[k^2-4\gamma^4\left(k_0-\beta\frac{\omega_0}{c}\right)^2\right]+\frac{\beta_e^2\omega_L^2}{u^2}\left(k^2+\gamma^2\frac{\omega_L^2}{c^2}\right)=0, \quad (20)$$

where $\beta = u/c$ and $\gamma^{-2} = 1-\beta^2$.

We introduce the dimensionless wave vector χ, which is related to k by $k = (\omega L/u)\chi$, in order to represent the

$$\chi_\pm^2=\frac{1}{2a^4}+2\beta^2\gamma^4F^2\pm\sqrt{\left(\frac{1}{2a^4}+2\beta^2\gamma^4F^2\right)^2-\beta^2\gamma^2\left(\frac{a^4-1}{a^4}+4\gamma^2F^2\right)}, \quad (21)$$

where

$$F=\sqrt{a^2\Delta^2-1}-\beta\alpha\Delta, \quad (22)$$

$\Delta = \omega/\omega_p$, and $a^2 = \sqrt{1+A^2}$. The character of the solutions of Eq. (20) (and, accordingly, the nature of the induced fields and electron plasma velocities) is governed by the sign of the expression under the square root in Eq. (21). We denote the regions where this expression is positive and negative by I and III, respectively. The boundary between these regions (at which this expression vanishes) is denoted by II. We equate the expression at hand to zero to obtain the following expression for Δ at boundary II:

$$\Delta_{\pm}^{(\mu)} = \frac{\gamma}{a}\left(\sqrt{1+\gamma^2 F_\mu^2} \pm \beta\gamma F_\mu\right), \tag{23}$$

where the index μ indicates a plus or minus sign and

$$F_{\pm} = \left[\frac{2 - 1/a^4 \pm 2\gamma\sqrt{1-1/a^4}}{4\gamma^2\left(\gamma^2-1\right)}\right]^{1/2}. \tag{24}$$

With μ = + and μ = −, expression (23) is valid for $a > 1$ and $1 < a < a_0(\gamma)$, respectively, where

$$a_0(\gamma) = \left(\frac{1+\sqrt{1-1/\gamma^2}}{2\sqrt{1-1/\gamma^2}}\right)^{1/4}. \tag{25}$$

Equations (23)-(25) were obtained under the assumption $\gamma > \gamma_1 \simeq 1.45$, where γ_1 is the real positive root of the equation $2\gamma^2(\gamma^2 - 2) = \gamma - 1$, satisfying the condition $\gamma > 1$. For a small-amplitude pump wave ($a - 1 \ll 1$), the function $\Delta_{+}^{(+)}$ coincides with $\Delta_{+}^{(-)}$ and the function $\Delta_{-}^{(+)}$ coincides with $\Delta_{-}^{(-)}$. The expressions for these functions, which will be denoted by $\Delta_{\pm}^{0}$, can be obtained from Eqs. (23) and (24):

$$\Delta_{\pm}^{(0)}(\gamma) = \gamma\sqrt{1+\frac{1}{4\left(\gamma^2-1\right)}} \pm \frac{1}{2}. \tag{26}$$

Additionally, for $a = a_0(\gamma)$, the function $\Delta_{+}^{(-)}$ coincides with $\Delta_{-}^{(-)}$. For this function, which will be denoted by $\Delta_0(\gamma)$, Eqs. (23) and (24) give

$$\Delta_0(\gamma) = \frac{\gamma}{a_0(\gamma)} = \gamma\left(\frac{2\sqrt{1-1/\gamma^2}}{1+\sqrt{1-1/\gamma^2}}\right)^{1/4}. \tag{27}$$

From the above expressions we can see that the boundary II is composed from the four curves $\Delta_{\pm}^{(+)}$ and $\Delta_{\pm}^{(-)}$, which envelop a closed region (the curves $\Delta_{+}^{(+)}$ and $\Delta_{-}^{(+)}$ close upon themselves at infinity). We denote the curves $\Delta_{-}^{(+)}$, $\Delta_{-}^{(-)}$, $\Delta_{+}^{(-)}$, and $\Delta_{+}^{(+)}$ by a', b', c', and d', respectively. In the case of relativistic bunches, two of the curves, c' and b', lie in the very narrow region $1 < a < a_0(\gamma) \simeq 1 + 1/16\gamma^2$. The pump wave amplitude is maximum at $a = a_0(\gamma)$ and $\Delta = \Delta_0(\gamma)$:

$$E_{0\max}(\gamma) = \frac{E_p}{\left(2\sqrt{1-1/\gamma^2}\right)^{1/4}\left(1+\sqrt{1-1/\gamma^2}\right)^{3/4}}, \qquad (28)$$

where $E_p = mc\omega_p/e$. For relativistic bunches, we have $E_{0\max}(\gamma) \simeq (E_p/2)(1+5/16\gamma^2)$.

Figure 1 shows regions I and III and boundary II between them for $\gamma =$ 10 and 100. The curves close upon themselves at infinity (i.e., for $E_0 \to \infty$). Figure 1 also shows a part of the dependence of Δ on E_0 at boundary II (curves a', b', c', and d') for $\gamma = 100$ and for a small-amplitude pump wave such that $0 \le E_0 \le E_{0\max} \simeq E_p/2$. For electromagnetic pump waves with intensities $I_L \le 10^{16}$ W/cm^2, parameter a is on the order of unity at a pump frequency of about $\omega_0 \simeq 3\times10^{15}$ s^{-1}. Consequently, Fig. 1 and Eqs. (23)-(27) imply that, in order for the solutions of Eq. (20) in the parameter range $\omega_0 \simeq 10^{15}$ s^{-1} and $n_0 < 10^{17}$ cm^{-3} ($\omega_p < 10^{13}$ s^{-1}) to lie at boundary II, the electron bunch should be ultrarelativistic ($\gamma \sim a\Delta > 100$). In what follows, we restrict ourselves to treating the problem in the parameter range corresponding to boundary II and to studying the induced fields for the relevant parameter values. The induced fields in region I were investigated in detail in Ref. [19]. As for the solutions in region III, expression (21) shows that the amplitude of the induced electric fields decreases exponentially when away from the bunch, so that these solutions are unimportant for the generation of high accelerating or focusing electric fields in plasmas.

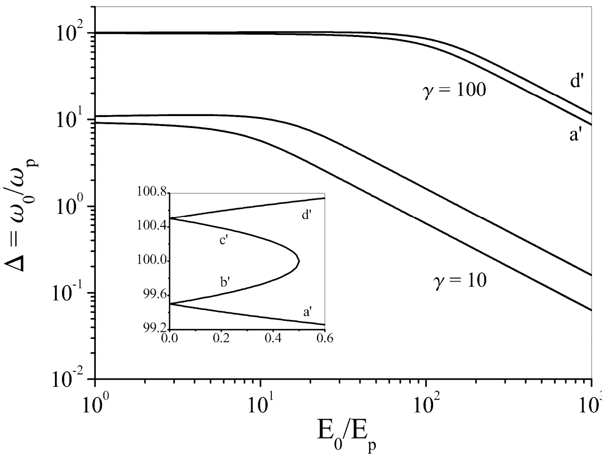

Fig. 1. The curves $\Delta = \Delta(E_0)$ (boundary II), on which the expression under the square root in formula (21) equals zero, for $\gamma = 10$ and 100. Regions I and III lie outside and inside the curves, respectively.

Now we evaluate the Green's function for the parameters corresponding to boundary II. In the general case Eq. (21) describes coupled longitudinal and transverse waves with the wavenumbers $k_+ = (\omega_L/u)\chi_+$ and $k_- = (\omega_L/u)\chi_-$ (or the frequencies $\omega_+ = \omega_L\chi_+$ and $\omega_- = \omega_L\chi_-$). At boundary II, the expression under the square root in Eq. (21) vanishes and the equality $k_+ = k_-$ (or $\omega_+ = \omega_-$) holds. Consequently, at the boundary, the wave excitation is resonant in character. Taking into account the fact that the roots of Eq. (20) are multiple and lie in the upper half-plane of the complex variable k, we integrate expressions (12) and (13) over k to obtain

$$G_z(s) = 4\pi e\theta(s)\left[\cos(\sigma_\mu k_p s) + \mathcal{L}_\mu(\sigma_\mu k_p s)\sin(\sigma_\mu k_p s)\right], \tag{29}$$

$$\begin{aligned} G_\perp(s) = \pi i e\sqrt{1-\frac{1}{a^4}}\,\frac{4Q_\pm^\mu}{\beta}\left[\theta(s)-\theta(-s)\right] - \frac{2\pi e\beta\gamma^2}{a^3\sigma_\mu^3}\sqrt{1-\frac{1}{a^4}}\theta(s) \\ \times\left\{\left[\mathcal{A}_\pm^\mu + i\mathcal{B}_\pm^\mu(k_p,s)\right]\sin(\sigma_\mu k_p s) - \left[\mathcal{A}_\pm^\mu(k_p,s) - iC_\pm^\mu\right]\cos(\sigma_\mu k_p s)\right\}, \end{aligned} \tag{30}$$

where

$$\sigma_\mu^2 = \frac{1}{a^2}\left(1+\mu\gamma\sqrt{1-\frac{1}{a^4}}\right),\quad \mathcal{L}_\mu = \frac{\sigma_\mu^2 a^2 - 1}{2a^2\sigma_\mu}, \tag{31}$$

$$\mathcal{A}_\pm^{(\mu)} = \gamma\left(\sqrt{1+\gamma^2 F_\mu^2} \mp \beta_\gamma F_\mu\right),\quad B_\pm^{(\mu)} = \pm\frac{2\beta\gamma^3}{a}F_\mu\sqrt{1+\gamma^2 F_\mu^2} - \frac{1}{2a^5}, \tag{32}$$

$$C_\pm^{(\mu)} = 2\left(\frac{B_\pm^\mu}{\sigma_\mu} + a\sigma_\mu\right),\quad Q_\pm^{(\mu)} = \frac{\beta^3\gamma^5 F_\mu\left(\beta_\gamma F_\mu \pm \sqrt{1+\gamma^2 F_\mu^2}\right)}{a^4\sigma_\mu^4}, \tag{33}$$

$k_p = \omega_p/u$, $\theta(s)$ is the Heaviside unit-step function, and the function F_μ is defined by Eq. (24). In expressions (29)-(33) with $\mu = +$ and $\mu = -$, the values of the parameters lie in the domains given by the curves $\Delta_\pm^{(+)}$ and $\Delta_\pm^{(-)}$, respectively. For $a = 1$ (i.e., in the absence of a pump wave), the function $G_\perp$ vanishes and the function G_z is given by the expression that coincides with the corresponding equations presented in [24, 25].

3. Wake Fields In The Case Of a Gaussian Bunch

In this section we investigate the fields induced by a Gaussian electron bunch. We assume that the electron density in the bunch is described by the expression

$$n_b(\xi) = \frac{N_b}{\sqrt{\pi}}\exp\left(-\frac{\xi^2}{d^2}\right), \tag{34}$$

where N_b is the mean bunch density such that $N_b \ll n_0$.

We substitute Eqs. (29) and (30) and expression (34) into Eq. (11) to obtain the following expressions for the fields induced by a moving bunch:

$$\begin{aligned} E_z'(\xi) = E_p\frac{N_b}{2n_0}\lambda\beta\left\{\left[\mathcal{F}_\mu\cos(\sigma_\mu\zeta) + \mathcal{L}_\mu\zeta\sin(\sigma_\mu\zeta)\right]\Phi_r\left(\frac{\zeta}{\lambda};\frac{\sigma_\mu\lambda}{2}\right)\right. \\ \left. +\left[\mathcal{L}_\mu\zeta\cos(\sigma_\mu\zeta) - \mathcal{F}_\mu\cos(\sigma_\mu\zeta)\right]\Phi_i\left(\frac{\zeta}{\lambda};\frac{\sigma_\mu\lambda}{2}\right)\right\}, \end{aligned} \tag{35}$$

$$\mathcal{E}^{+}(\xi)=E_p\frac{N_b}{n_0}\lambda\sqrt{1-\frac{1}{a^4}}\left\{-iQ_{\pm}^{(\mu)}\left[1-\Phi\left(\frac{\zeta}{\lambda}\right)\right]+\frac{\beta^2\gamma^2}{4\sigma_\mu^3a^3}\left[\frac{\mathcal{A}_{\pm}^{(\mu)}\sigma_\mu\lambda}{\pi^{1/2}}e^{-\zeta^2/\lambda^2}\right.\right.$$
$$+\left[\left(\mathcal{A}_{\pm}^{(\mu)}Z_\mu-i\mathcal{B}_{\pm}^{(\mu)}\zeta\right)\sin\left(\sigma_\mu\zeta\right)-\left(\sigma_\mu\mathcal{A}_{\pm}^{(\mu)}\zeta+i\mathcal{D}_{\pm}^{(\mu)}\right)\cos\left(\sigma_\mu\zeta\right)\right]\Phi_r\left(\frac{\zeta}{\lambda};\frac{\sigma_\mu\lambda}{2}\right) \tag{36}$$
$$\left.\left.+\left[\left(\sigma_\mu\mathcal{A}_{\pm}^{(\mu)}\zeta+i\mathcal{D}_{\pm}^{(\mu)}\right)\sin\left(\sigma_\mu\zeta\right)+\left(\mathcal{A}_{\pm}^{(\mu)}Z_\mu-i\mathcal{B}_{\pm}^{(\mu)}\zeta\right)\cos\left(\sigma_\mu\zeta\right)\right]\Phi_i\left(\frac{\zeta}{\lambda};\frac{\sigma_\mu\lambda}{2}\right)\right]\right\},$$

where $\zeta = kp\,\xi$, $\lambda = kp\,d$, $\Phi(x) = 1 - \mathrm{erf}(x)$, $\mathrm{erf}(x)$ is the probability integral, and

$$\mathcal{F}_\mu=1+\frac{\sigma_\mu^2a^2-1}{4a^2}\lambda^2,\quad \mathcal{Z}_\mu=1+\frac{\sigma_\mu^2\lambda^2}{2}, \tag{37}$$

$$\mathcal{D}_{\pm}^{(\mu)}=2a\sigma_\mu+\frac{1+\mathcal{Z}_\mu}{\sigma_\mu}\mathcal{B}_{\pm}^{(\mu)}, \tag{38}$$

$$\begin{pmatrix}\Phi_r(x;y)\\ \Phi_i(x;y)\end{pmatrix}=\pm\frac{2}{\sqrt{\pi}}\int_x^\infty dte^{-t^2}\begin{pmatrix}\cos(2yt)\\ \sin(2yt)\end{pmatrix}. \tag{39}$$

Note that the functions $\Phi_r(x, y)$ and $\Phi_i(x, y)$ can be expressed in terms of the probability integral $\mathrm{erf}(x + iy)$ of the complex argument.

From expressions (35) and (39), we can see that, at large distances ahead of the bunch ($\xi>\sigma_\mu k_pd^2/2$), the longitudinal electric field is exponentially small,

$$E_z^{'}(\xi)\simeq E_p\frac{N_b}{2n_0}\lambda\beta\frac{1}{\sqrt{\pi}}\frac{e^{-\zeta^2/\lambda^2}}{\zeta/\lambda} \tag{40}$$

and the transverse electric fields are unmodulated (i.e., they are essentially independent on ξ),

$$\mathcal{E}^{+}(\xi)\simeq iE_p\frac{N_b}{n_0}\lambda\sqrt{1-\frac{1}{a^4}}\left[-Q_{\pm}^{(\mu)}+\frac{\beta^2\gamma^2\mathcal{D}_{\pm}^{(\mu)}}{4\sigma_\mu^3a^3}\frac{e^{-\zeta^2/\lambda^2}}{\pi^{1/2}(\zeta/\lambda)}\right]. \tag{41}$$

Ahead of the bunch, the transverse electric fields are generated because the phase velocity $\nu_\varphi=\omega_0/k_0>c$ of the pump wave is higher than the bunch velocity for arbitrary values of the plasma and wave parameters. Consequently, part of the perturbations driven by the pump wave and bunch in the plasma can propagate at a phase velocity higher than the bunch velocity and can thereby overtake the bunch. In addition, expressions (18), (19), and (41) yield the relations $E_i(\xi)\gg E_r(\xi)\simeq 0$, $E_x^{'}\simeq -E_i(\xi)\sin\psi$, and $E_y^{'}\simeq -E_i(\xi)\cos\psi$.

As follows from expressions (18), (19), and (36), the transverse electric fields are modulated and circularly polarized. However, although the polarization vector of the transverse wave spans an entire circle, the circle's radius depends on the distance ξ from the bunch. In fact, from expressions (18) and (19), we have

$$E_x^{'2}+E_x^{'2}=E_r^2(\xi)+E_i^2(\xi)=E_{\max}^2(\xi). \tag{42}$$

In the general case, the electric field amplitude $E\max(\xi)$ is a function of ξ. Modulated transverse plasma waves are generated in the interaction between a pump wave and induced waves with frequency $\omega_L\sigma_\mu a$ and wavenumber $(\omega_L/u)\sigma_\mu a$. This interaction gives rise to oscillations with the combined frequencies $\omega_0 - \omega_L\sigma_\mu a$, $\omega_0 + \omega_L\sigma_\mu a$ and combined wavenumbers $k_0 - (\omega_L/u)\sigma_\mu a$, $k_0 + (\omega_L/u)\sigma_\mu a$; the modulated wave results from the interference between these oscillations.

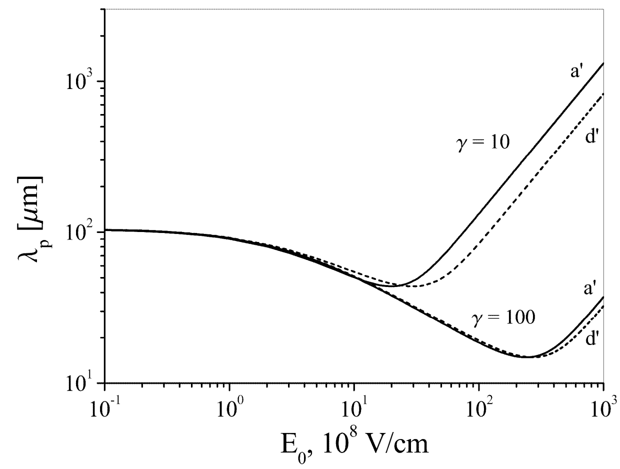

Fig. 2. Dependence of the wavelength λ_p of the induced waves on the pump field amplitude E_0 for $n_0 = 10^{17}$ cm^{-3} and $\gamma = 10$ and 100. The results obtained for curves a' and d' are shown by solid and dashed lines, respectively.

At large distances behind the bunch ($\xi < 0$, $|\xi| > \sigma_\mu k_p d^2/2$), expressions (35) and (36) become

$$E_z'(\xi) \simeq E_p \frac{N_b}{n_0}\lambda\beta e^{-\sigma_\mu^2\lambda^2/4}\left[\mathcal{F}_\mu \cos(\sigma_\mu\zeta) + \mathcal{L}_\mu \zeta \sin(\sigma_\mu\zeta)\right], \tag{43}$$

$$\begin{aligned}\mathcal{E}^+(\xi) \simeq E_p \frac{N_b}{n_0}\lambda\sqrt{1-\frac{1}{a^4}}\Bigg\{ iQ_\pm^{(\mu)} + \frac{\beta^2\gamma^2}{2\sigma_\mu^3 a^3} e^{-\sigma_\mu^2\lambda^2/4} \\ \times\left[\left(\mathcal{A}_\pm^{(\mu)} Z_\mu - i\mathcal{B}_\pm^{(\mu)}\zeta\right)\sin(\sigma_\mu\zeta) - \left(\sigma_\mu \mathcal{A}_\pm^{(\mu)}\zeta + i\mathcal{D}_\pm^{(\mu)}\right)\cos(\sigma_\mu\zeta)\right]\Bigg\}.\end{aligned} \tag{44}$$

In the above expressions, the terms that are proportional to ξ (or ζ) describe waves whose amplitude increases with distance from the bunch and whose wavelength is equal to $\lambda_p = \lambda_p^{(0)}/\sigma_\mu$, where $\lambda_p^{(0)} = 2\pi u/\omega_p$ is the wavelength in the absence of a pump wave. Figure 2 shows the wavelength computed as a function of the pump wave amplitude for $\gamma = 10$ and 100. The parameters adopted for numerical calculations correspond to two boundary curves, a' and d'. One can see that the wavelength is maximum at $E_0 \simeq 2^{-1/4}\gamma E_p$. In accordance with expression (31), the wavelength of the induced waves for the parameters corresponding to two other curves, b', and c', increases monotonically from $\lambda_p^{(0)}$ to $\lambda_p^{(0)}\sqrt{2}$ as the pump wave amplitude increases from zero to $E_{0\max}$ (see expression 28). Consequently, the pump wave can markedly change the wavelength of the excited waves; this effect is more pronounced for small values of γ (Fig. 2). Note that, in region I, the wavelength λ_p increases monotonically with the pump wave intensity [19].

Let us analyze the above expressions for the following, practically important parameter range: $n_0 \le 10^{17}\ \text{cm}^{-3}$ ($\omega_p \le 2\times 10^{13}\ \text{s}^{-1}$), $I_L \sim 10^{16}$ W/cm^2, $\omega 0 \simeq 10^{15}\ \text{s}^{-1}$, and $\gamma \gg 1$.

The coefficients in expressions (35) and (36) are comparatively easy to analyze in the two limiting cases $\Delta_{\pm}^{(-)}$ (the parts of curves c' and b' in region II) and $\Delta_{\pm}^{(+)}$ (the parts of curves d' and a' in region II). Thus, in the first case, Eqs. (31)-(33), (37) and (38) at $a \simeq a_0(\gamma)$ give

$$\sigma_- \simeq \frac{1}{2^{1/2}}, \quad \mathcal{L}_- \simeq -\frac{1}{2^{3/2}}, \quad \mathcal{F}_- \simeq 1 - \frac{\lambda^2}{8}, \tag{45}$$

$$\mathcal{A}_{\pm}^{(-)} \simeq \gamma, \quad B_{\pm}^{(-)} \simeq -\frac{1}{2}, \quad \mathcal{Q}_{\pm}^{(-)} \simeq 0, \quad \mathcal{D}_{\pm}^{(-)} \simeq -\frac{\lambda^2\sqrt{2}}{8}, \quad \mathcal{Z}_- \simeq 1 + \frac{\lambda^4}{4}. \tag{46}$$

For $\Delta = \Delta_{\pm}^{(+)}$ and $\gamma\sqrt{1-1/a^4} \gg 1$, instead of Eqs. (45) and (46), we obtain

$$\sigma_+^2 \simeq \frac{\gamma}{a^2}\sqrt{1-\frac{1}{a^4}} \gg 1, \quad \mathcal{L}_+ \simeq \frac{\sigma_+}{2}, \quad \mathcal{F}_+ \simeq 1 + \frac{\lambda^2\sigma_+^2}{4}, \tag{47}$$

$$\mathcal{A}_{\pm}^{(+)} \simeq \gamma, \quad \mathcal{B}_{\pm}^{(+)} \simeq \pm\gamma\sigma_+, \quad \mathcal{Q}_{\pm}^{(+)} \simeq \pm\frac{1}{2}\left(\frac{\gamma^2}{1-1/a^4}\right)^{3/4}, \tag{48}$$

$$\mathcal{D}_{\pm}^{(-)} \simeq \pm 2\gamma\left(1 + \frac{\lambda^2\sigma_+^2}{4}\right), \quad \mathcal{Z}_- \simeq 1 + \frac{\lambda^2\sigma_+^2}{2}.$$

A comparison of Eqs. (45) and (46) with (47) and (48) shows that the coefficients that describe the induced fields are much larger at $\Delta = \Delta_{\pm}^{(+)}$ (the parameters correspond to the curve a' or the curve d'). In what follows, we restrict ourselves to considering curves a' and d'.

Ahead of the bunch, the transverse electric field at $a > 1 + 1/4\gamma^2$ is equal in order of magnitude to $|\mathcal{E}^+| \simeq E_p(N_b/n_0)\lambda(1-1/a^4)^{-1/4}\gamma^{3/2}/2.$ Behind the bunch, the amplitude of the longitudinal electric field increases with the bunch width (the quantity d) and reaches its maximum at $\lambda_{\max} = k_p d_{\max} \simeq \sqrt{2}/\sigma_+$. At sufficiently large distances from the bunch ($|\xi| > \sigma_\mu k_p d^2/2$) and for a longitudinal field close to its maximum value, Eq. (47) gives $\mathcal{L}_+ \gg \mathcal{F}_+$. Consequently, the longitudinal electric field behind the bunch is largely determined by the second term in expression (43). At large distances behind the bunch, the amplitude of the transverse electric field is also maximum at $\lambda = \lambda_{\max}$; near this maximum, the transverse field is mainly determined by the second and third terms in the square brackets in expression (44). In addition, expressions (43), (44), (47) and (48) show that the induced field increases with the pump wave intensity. Consequently, for fixed but sufficiently large values of ζ, the induced field is maximum for sufficiently narrow bunches ($\lambda \ll 1$).

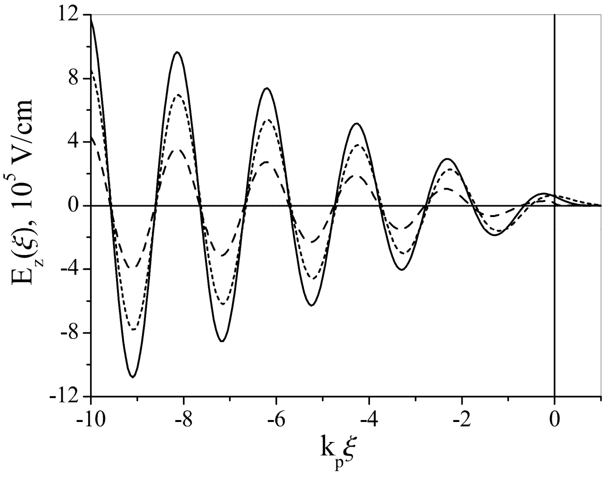

Fig. 3. Dependence of the amplitude of the induced longitudinal electric field on ξ at boundary II for$n_0 = 10^{17}$ cm^{-3}, $n_b = 10^{14}$ cm^{-3}, $\omega_0 = 1.82\times10^{15}$ s^{-1}, $E_0 = 3.04\times10^9$ V/cm, γ = 100, and $k_p^{-1} = 16.8$ μm. The dotted, solid, and dashed curves were calculated for $k_pd = 0.7$, $k_pd = k_pd_{\max} = 0.432$, and $k_pd = 0.1$, respectively.

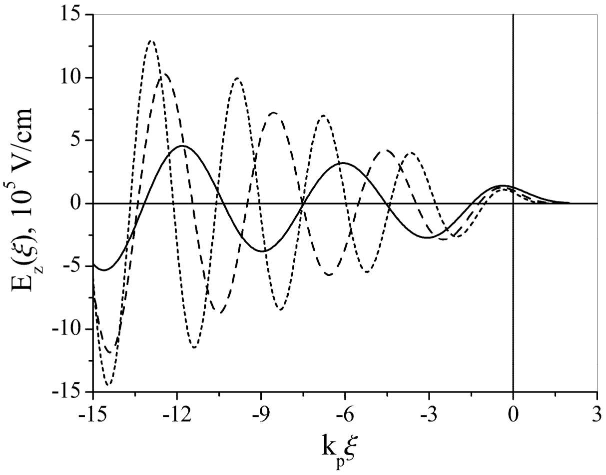

Fig. 4. Dependence of the amplitude of the induced longitudinal electric field on ξ at boundary II for $d = d_{\max}$. The solid curve corresponds to $E_0 = 10^8$ V/cm (Δ = 100.64, $k_pd_{\max} \simeq 1.228$), the dashed curve corresponds to $E_0 = 5\times10^8$ V/cm (Δ = 100, $k_pd_{\max} \simeq 0.87$), and the dotted curve corresponds to $E_0 = 10^9$ V/cm (Δ = 101.35, $k_pd_{\max} \simeq 0.687$). The remaining parameter values are the same as in Fig. 3.

Now, we consider the dependence of the induced fields on the bunch energy (or equivalently on the relativistic factor γ). According to Eqs. (43) and (44), the amplitude of the longitudinal wave depends weakly on the bunch energy, while the amplitude of the transverse wave increases with γ approximately as $\gamma^{3/2}$ (see expression (44)) and is much larger than the longitudinal field amplitude. Consequently, for $\gamma \gg 1$, the induced wave is nearly transverse.

The characteristic features mentioned above are clearly seen from Figs. 3-5, which display the induced field amplitudes calculated from Eqs. (31)-(33) and (35)-(39) for the parameter values corresponding to curve d'. Note that, in Fig. 4, the dependence of the longitudinal field on the distance from the bunch was calculated for different intensities of the pump wave under the condition $E_0 < 2^{-1/4}\gamma E_p$. From this figure, we can see that the wavelength of the excited waves decreases with increasing E_0.

It should be noted that the amplitude of the transverse wake field increases with distance from the bunch only when the frequency and amplitude of the pump wave, the plasma density, and the bunch energy (the relativistic factor γ) all satisfy relation (23), which determines the boundary II. Although these parameters are independent of each other, the situation in which they exactly correspond to boundary II is very difficult to realize in practice. Assuming that E_0, ω_0, and n_0 are fixed, we briefly discuss the question about the width $\Delta\gamma$ of the region II; i.e., we determine the maximum possible deviation of the relativistic factor from boundary II for which the amplitude of the wake waves still increases. We consider two bunch energies, γ and γ_0 ($|\gamma-\gamma_0| \ll \gamma_0$), assuming that the quantity γ lies in region I and that the quantity γ_0, together with E_0, ω_0 and n_0, satisfies relation (23), or, in other words, the values of these four parameters exactly correspond to boundary II. In Ref. [19] was shown that the values of γ lying in region I imply the excitation of two wake waves whose amplitudes and wavenumbers $\chi_\pm$ are independent of the ξ coordinate [see expression (21)].

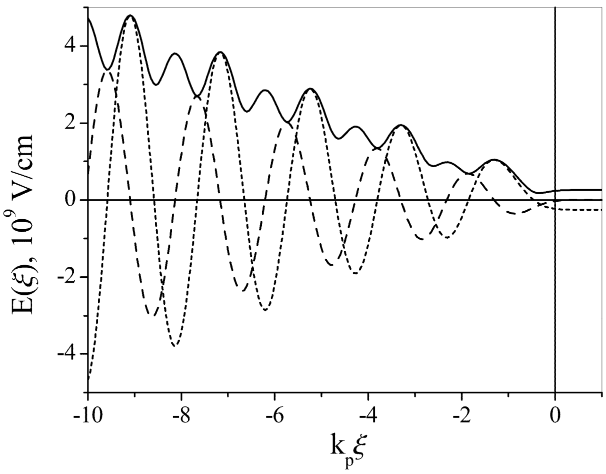

Fig. 5. Dependence of the amplitudes of the induced transverse electric field components E_r (dashed curve), E_i (dotted curve) and the amplitude $E_{\max} = \left(E_r^2 + E_i^2\right)^{1/2}$ (solid curve) on ξ at boundary II for $k_p d = k_p d_{\max} = 0.432$. The remaining parameter values are the same as in Fig. 3

Moreover, we have $\chi_+ \to \chi_- \to a\sigma_\pm$ as $\gamma \to \gamma_0$, in which case the corresponding expressions derived in [19] yield Eqs. (29) and (30) for the Green's function. Consequently, the effect of the increase of the amplitude of the excited wake waves will also take place for region I under the conditions

$$k_p l < k_p L < \frac{a}{\Delta\chi}, \tag{49}$$

where l is the distance from the bunch, L is the longitudinal plasma dimension, and

$\Delta\chi = (\chi_+ - \chi_-)/2 \ll \chi_\pm$. The second of the inequalities (49) implies that the wake wave amplitude will increase up to the plasma boundary. At $k_pL = a/\Delta\chi$, conditions (49) yield the following estimate for the maximum relative width of region II:

$$S = \frac{\gamma - \gamma_0}{\gamma_0} = \frac{0.7a^2}{\gamma_0 (k_p L)^2} \frac{1}{\sqrt{\gamma_0 \beta_e}}. \tag{50}$$

Note that expression (50) is valid in the range $\gamma_0\beta_e > 1$. For the above values of the parameters and for $k_pL \sim 10 - 10^3$ ($L \sim 1 - 10^2$ cm), the relative width of the region II is about $S < 0.1$ %.

4. Conclusion

We have investigated the excitation of linear wake waves by a one–dimensional electron bunch propagating in plasma in the presence of an intense electromagnetic pump wave with circular polarization. We have obtained and analyzed expressions describing the induced electromagnetic fields. It has been shown that there exist three ranges of the values of the plasma, bunch, and pump wave parameters in which the derived equations have qualitatively different solutions. In the most interesting case of resonant excitation (the values of the parameters correspond to the curves a' and d' of the boundary II), the amplitude of the transverse waves increases with increasing electron bunch energy and pump wave intensity. The amplitude of the longitudinal electric field depends weakly on the relativistic factor γ of the bunch electrons and increases with the pump wave intensity. The excitation of wake waves is most efficient in the case of narrow bunches such that $d \simeq d_{\max} \simeq \lambda p/\pi\sqrt{2}$. For a sufficiently intense pump wave, the wavelength λ_p increases significantly as the bunch energy decreases.

In conclusion note that although much attention has been devoted to one-dimensional wake fields (see, e.g., [1-18] and the literature cited therein), a more realistic three-dimensional case is of greater importance from the standpoint of practical applications. It can be expected that the characteristic features of the wake fields that have been revealed in this paper will also persist in the three-dimensional case, in which, however, new features may arise, stemming from the dependence on the radial coordinate.

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