# THE AC-FIELD SPECTROSCOPIC CHARACTERISTICS OF DOUBLE QUANTUM DOTS

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**Abstract:** The quantum transport characteristics for double quantum dots coupled weakly to conducting leads are studied under the influence of an AC-field of a wide range of frequencies. A numerical calculation shows the following features: (i) Two resonant peaks appear for the dependence of the normalized conductance *G* on the bias voltage,  $V_0$ . The peak heights decrease as the frequency of the imposed AC- Field increases in the case of absorption of photons. However, in case of emission of photons the peak heights increase with the frequency. (ii) The calculated value of the activation energy of the electron required for quantum transport shows a dependence on the frequency of the absorbed (or emitted) photons.

Keywords: Double quantum dots, photon-assisted tunneling, ac-field (microwave field), bias voltage

### **1. Introduction**

Interest in nanoscale quantum confined structures has been fueled by the richness of fundamental phenomena therein and the potential device applications [1-3]. In particular, ideal quantum dots can provide three-dimensional carrier confinement and resulting discrete states for electrons and holes [4]. Current can flow through a quantum dot when a discrete energy state is aligned to the Fermi energies of the leads. This current is carried by resonant elastic tunneling of electrons between the leads and the dot. An additional time-varying potential  $V_{ac}\cos\omega t$  (microwave field with amplitude  $V_{ac}$  and angular frequency  $\omega$ ) can induce *inelastic* tunnel events when electrons exchange photons of energy hf with the oscillating field. This inelastic tunneling with discrete energy exchange is known as photon-assisted tunneling (PAT). PAT has been studied before in superconductor-insulator-superconductor tunnel junctions [5], in superlattices [6], and in quantum dots [7]. The AC-field (microwave field) induces absorption and emission processes which is reflected as new features in the tunneling channels coming from the photo-side bands [8, 9]. A double quantum dot device is a tunable two-level system for electronic energy states. A DC bias voltage and an AC (microwave) voltage can be used to modulate the electron occupation of the states. This controllability allows us to study the manipulation of quantum states for future quantum logic gates [9, 10]. When the coupling between the two dots is weak, electrons are strongly localized on the individual dots. The gate voltages are used to surmount the single-electron charging energy and to align a discrete energy level in the left dot with a discrete energy level in the right dot [11]. Then, it is energetically allowed for an electron to tunnel between the dots. A current can flow when the two energy levels are aligned within the bias voltage window, defined by the electrochemical potentials of the left and right lead. Note that energy is also conserved when photons of energy *hf*, which match the energy difference between the states of the two dots, are absorbed from the microwave field. The purpose of the present paper is to investigate the effect of the frequency of the AC-field (microwave field) on the electron transport of the mesoscopic device whose dimensions are less than the mean-free path of the electron. Such device will be modeled as two coupled quantum dots based semiconductor heterostructure separated by a barrier of width c. These quantum dots are coupled weakly to two conducting leads.

## 2. The Model

The present mesoscopic device could be constructed as two series coupled quantum dots, each of diameter a and separated by an inner barrier of width c. Also, these quantum dots are separated from two leads by outer tunnel barriers from the corresponding sides, each of width b. Such model has been investigated previously by the authors [12].

Now, we shall derive an expression for the conductance G of the present mesoscopic device under the effect of an AC-field (microwave field) of wide range of frequencies. So, the tunneling will be induced by this applied AC-field (microwave field)  $V_{ac} \cos(\omega t)$ , in which  $V_{ac}$  is the amplitude of the induced microwave field. The conductance G can be obtained from the multichannel Landauer-Buttiker formula [13, 14], which is expressed as

$$G = \frac{4e^2}{h} \int_{E_F}^{E_F + n\hbar\omega} \Gamma'(E) \left(\frac{-\partial f_{FD}}{\partial E}\right) dE,$$
(1)

where *e* is the electron charge, *h* is Planck's constant, and  $\Gamma'(E)$  is the tunneling probability taking into consideration the effect of the applied AC-field with angular frequency  $\omega$ . The derivative of the Fermi-Dirac distribution function (see Eq.(1)) is given by

$$-\frac{\partial f}{\partial E} = (4k_BT)^{-1}\cosh^{-2}\left[\frac{(E+n\hbar\omega-E_F)}{2k_BT}\right].$$
(2)

In Eq.(2),  $k_B$  is the Boltzmann constant,  $E_F$  is the Fermi energy and T is the absolute temperature. When applying a time varying potential (microwave field),  $V_{ac} \cos(\omega t)$ , with angular frequency,  $\omega$ , and an amplitude,  $V_{ac}$ , inelastic tunnel events could be induced. The tunneling probability,  $\Gamma(E)$ , without the effect of the interacting photons with the tunneled electrons must be modified as [5-9]

$$\Gamma'(E) = \sum_{n=-\infty}^{\infty} J_n^2 \left( \frac{eV_{ac}}{\hbar\omega} \right) \Gamma(E + n\hbar\omega),$$
(3)

where  $J_n^2\left(\frac{eV_{ac}}{\hbar\omega}\right)$  is the square of the n-th order of the Bessel function of the first kind. This

describes the tunneling probability when:

- (a) Electron absorbs a number of photons:  $n = +1, +2, +3, \dots$
- (b) Electron emits a number of photons:  $n = -1, -2, -3, \dots$

Thus, the effect of the interaction between a single electron states with classical oscillating field is that the energy state *E* is split in a set of states,  $E + n\hbar\omega$ .

In Eq.(3) the tunneling probability,  $\Gamma(E)$ , without AC-field (microwave field) irradiation has been derived previously by the authors [12] and their formula is given by

$$\Gamma(E) = \frac{1}{1 + A^2 B^2},\tag{4}$$

where

$$A = \frac{\left(V_{0} + V_{b} + \frac{e^{2}N^{2}}{4C}\right)\sinh\left(\kappa b\right)}{2\left(E\left(V_{0} + V_{b} + \frac{e^{2}N^{2}}{4C} - E\right)\right)^{\frac{1}{2}}}$$
(5)

and

$$B = D_1 D_2 - \frac{\sinh(\kappa(2b-c))}{\sinh(\kappa b)},\tag{6}$$

in which the expressions for  $D_1$  and  $D_2$  are

$$D_{1} = 2\cosh(\kappa b)\cos(k a) - \frac{\left(2E - V_{0} - V_{b} - \frac{e^{2}N^{2}}{4C}\right)\sinh(\kappa b)\sin(k a)}{\left(E\left(V_{0} + V_{b} + \frac{e^{2}N^{2}}{4C} - E\right)\right)^{\frac{1}{2}}}$$
(7)

and

$$D_{2} = 2\cosh(\kappa c)\cos(k a) - \frac{\left(2E - V_{0} - V_{b} - \frac{e^{2}N^{2}}{4C}\right)\sinh(\kappa c)\sin(k a)}{\left(E\left(V_{0} + V_{b} + \frac{e^{2}N^{2}}{4C} - E\right)\right)^{\frac{1}{2}}},$$
(8)

where  $V_0$  is the bias voltage,  $V_b$  is the barrier height, N is the number of tunneled electrons and C is the capacitance of each quantum dot. The wave vector  $\kappa$  is given by

$$\kappa = \left[ 2m^* \left( V_0 + V_b + \frac{e^2 N^2}{4C} \right) / \hbar^2 \right]^{\frac{1}{2}}.$$
(9)

Also, the wave vector k is expressed as:

$$k = \sqrt{2m^* E/\hbar^2} \tag{10}$$

Now, substituting Eq. (3) for the tunneling probability,  $\Gamma'(E)$ , into Eq.(1), taking into consideration Eqs.(4-10), we get the full expression for the conductance *G*. The obtained equation for the conductance will be solved numerically using Mathematica software.

#### Numerical calculation and Discussion

The quantum dots are semiconductor based heterostructure GaAs–AlGaAs which has a very high mobility at low temperature [10]. The characteristic dimensions of the device are a, b, and c where the parameter a is the quantum dot diameter, the parameter b is the barrier width separating the two quantum dots from the left and the right reservoirs and c is the barrier width between the two quantum dots. In order to show that the presented junction operates as a resonant tunneling device, we perform a numerical calculation of the conductance (Eq.1). The present authors [12] had investigated the dependence of the conductance G on the dimensions of the present device in both absorption and emission cases of the induced photons. So, we shall analyze the present results as follows:

the barrier height  $V_b$  is determined by using the Monte-Carlo simulation technique and its value was found to be  $\approx 0.47$  eV for the present mesoscopic device. This value has been determined the authors [12, 15, 16, 17]. The total capacitance *C* in the charging energy of the double quantum dots has been calculated previously by the authors [12, 15, 16, 17] and its value is  $\approx 0.1 \times 10^{-15}$  F. This value was found to be optimal for the present device [18]. The value of the effective mass of electrons in the present GaAs/AlGaAs is 0.067  $m_e$  [18], where  $m_e$  is the free electronic mass. The value for the amplitude of the induced AC-field (microwave field) was taken as 0.001 eV [16, 17, 18] and the number of tunneled electrons *N* was taken as a random number [12, 15, 16, 17]. The value of each quantum dot diameter was taken as 5 nm, and the values of barrier widths b, c were taken as 1.5 nm. Now, we shall discuss the obtained results as follows:

(i) The dependence of the conductance *G* with the bias voltage  $V_0$ , measured for both the cases of absorption and emission of one photon of different frequencies, *f*, is shown in Fig. 1 and Fig. 2, respectively. Two resonant peaks occur at certain values of  $V_0$ . In the absorption case, the peak heights decrease as the frequency of the imposed AC-field (microwave field) increases. This effect may be attributed to the negative pumping of the electrons, dynamical localization and photo-induced electrical domains (EFDS) produced by the high AC- field [10,19]. But, in the emission-case, the two peak heights increase as the frequency of the emitted photon increases. No shift for the peak positions occurs for the both cases [10, 19].



Fig. 1. The dependence of the normalized conductance G on the bias voltage  $V_o$  calculated for the absorption of one photon of different frequencies.



Fig. 2. The dependence of the normalized conductance G on the bias voltage  $V_0$  calculated for the emission of one photon of different frequencies.

(ii) The dependence of the conductance G with the bias voltage  $V_0$  measured for both the cases of absorption and emission of different numbers of photons n each of frequency (4GHz) is shown in Fig. 3 and Fig. 4, respectively. In the absorption case, the peak heights decrease as the number of the absorbed photons increases, but in the emission case, the peak heights increase as the number of emitted photons increases.



**Fig. 3.** The dependence of the normalized conductance *G* with the bias voltage  $V_0$  calculated for the absorption of different numbers of photons (f = 10 GHz). The inset: the peak height versus the number of the absorbed photons calculated for n = 1 up to n = 4.



Fig.4: The dependence of the normalized conductance G with the bias voltage  $V_0$  calculate for the emission of different numbers of photons (f = 10 GHz).

(iii) The dependence of the conductance *G* with the absolute temperatures T measured for the absorption of one photon of different frequencies is shown in Fig. 5. From the figure the conductance *G* decreases as the temperature increases. This agrees well with those published in literature [20-24]. The dependence of the logarithm of the conductance,  $\ln G$ , with 1/T is plotted in Fig. 6. Taking the slope of the curve and by using the Arrhenius relation ( $G = G_0 \exp(-E/k_B T)$ ), the activation energy of the electron is calculated for both cases absorption and emission of photons (see table 1).

Frequency of photon (GHz)	The activation energy of electron (meV)	
	Absorption case	Emission case
f = 4	0.281	0.298
f = 10	0.266	0.311
<i>f</i> = 15	0.254	0.322
f = 20	0.241	0.335

Table 1



Fig. 5. The dependence of the normalized conductance G with the absolute temperatures calculated for the absorption of one photon of different frequencies.



Fig. 6. The dependence of the logarithm of the normalized conductance on 1/T calculated for different frequencies.

It is noticed that the activation energy of the electron decreases as the frequency of the absorbed photon increases (absorption case). Also, the dependence of the conductance G on the absolute temperature T measured for the case of emission of one photon shows the same behavior

Fig. 7. The dependence of  $\ln G$  versus 1/T is plotted in Fig. 8. From the Table 1, the calculated value of the activation energy of the electron increases slightly as the frequency of the emitted photon increases.



Fig. 7. The dependence of the normalized conductance G on the absolute temperature calculated for the emission of one photon of different frequencies.



Fig. 8. The variation of  $\ln G$  with 1/T for the emission case of one photon.

Our results are found concordant with those in literature [24, 25].

## Conclusion

New features appear when imposing high AC-field (microwave field) on the junction such as the decrease in the conductance (the absorption-case) or the observed increase in conductance (the emission-case). Also the effect of the parameter c was studied which shows the coupling effect. Such quantum coherent electron device is promising for future high speed nanodevices and for photo-detectors at very high frequencies.

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