A.A. Tshantshapanyan, K.G. Dvoyan, and E.M. Kazaryan

Russian-Armenian State University, Yerevan, E-mail: achanch@server.physdep.r.am, dvoyan@web.am

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1. Introduction

Quantum dots (QDs) obtained by interrupted growth in strained semiconductor interfaces are currently under intense study by many experimental and theoretical groups [1]. These "selfassembled" QDs are mostly dislocation-free, coherent islands of deposited material on the surface of different semiconductors. The lattices mismatch on semiconductor interface forces the segregation of material whenever the epitaxial growth exceeds a critical layer thickness, resulting

in the growth of these so-called Stranski-Krastanov islands [2]. Special interest are the physical characteristics of the so-called "quantum lenses", or lenticular QDs [3]. More specifically, the energy spectrum of charge carriers inside QDs shaped as spherical segment or ellipsoid



Fig. 1. Thin coated ellipsoidal quantum lens.

was studied. In [4], a cylindrical quantum lens was considered having a cross-section close to a semi-circle. So far, however, the cylindrical QDs with a thin lenticular section were studied in [5].

The study of coated lensshaped quantum dots (or coated quantum lens (CQL)) will permit to model more realistic structures forming during manufacturing. Generally, during the QDs growth due to non-avoidable diffusion process of interface atoms, a coating interlayer (e.g., GaAlAs) forms between quantum lens (QL) material (e.g., GaAs) and semiconductor matrix (e.g., GaAs). This new interlayer transforms QL into CQL which significantly affects the distribution of quantum levels of the lens. Quantum lenses, especially CQL, have various commercial applications, in particular, in large two-dimensional focal plane arrays in the mid- and far-infrared (M&FIR) region having important applications in the fields of pollution detection, thermal imaging object location and remote sensing as well as IR imaging of astronomical objects (see US Patent # 6541788). In this paper we study electronic states in a thin ellipsoidal quantum lens.

2. Theory

Consider an impenetrable coated quantum lens shown in Fig. 1. The potential energy of a charged particle (electron or hole) inside the CQL has the following form:

$$U(X,Y,Z) = \begin{cases} 0, \frac{X^{2} + Y^{2}}{a_{1}^{2}} + \frac{(Z + Z_{01})^{2}}{c_{1}^{2}} \le 1, X, Y \in [-R_{0}, R_{0}], Z \in [0, L_{1}] \\ U_{0}, \frac{X^{2} + Y^{2}}{a_{1}^{2}} + \frac{(Z + Z_{01})^{2}}{c_{1}^{2}} > 1, \bigcup \frac{X^{2} + Y^{2}}{a_{2}^{2}} + \frac{(Z + Z_{02})^{2}}{c_{2}^{2}} \le 1, X, Y \in [-R_{0}, R_{0}], Z \in [0, L_{2}]. \end{cases}$$
(1)

$$\infty, Other areas$$

where a_1, c_1, a_2 , and c_2 are respectively two half-axis of two revolutional ellipsoids.

The particular shape of CQL suggests that motion of a charge in the Z direction should be

faster than in the plane normal to Z, a fact making possible the adiabatic approximation. The system Hamiltonian by means of dimensionless variables may be represented as a sum of "fast system" (H_1) and "slow system" (H_2) operators

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$$H = H_1 + H_2 + U(r, \varphi, z), (2),$$

where $H_1 = -\partial^2/\partial z^2$, $H_2 = -(\partial^2/\partial r^2 + \partial/r\partial r + \partial^2/r^2\partial \varphi^2)$ and the

following notations are introduced: $r = \rho/a_B$, $z = Z/a_B$, $H = \mathbf{H}/E_R$, with $E_R = \hbar^2/2\mu a_B^2$ being the effective Rydberg energy, $a_B = \kappa \hbar^2 / \mu e^2$ is the effective Bohr radius of an electron, μ is the effective mass of an electron, and κ is the dielectric constant of the medium. We seek the wave function (WF) of the problem as a product $\psi(r, \varphi, z) = e^{im\varphi} \chi(z; r) R(r)$, (3). When the rcoordinate of the "slow system" is fixed, the charge motion is localized inside a one-dimensional step-like effective potential well (see Fig. 2.), where $V_0 = U_0/E_R$,

$$L_{1}(r) = \frac{1}{R_{0}}\sqrt{R_{0}^{2}c^{2} - r^{2}\left(2L_{1}c - L_{1}^{2}\right)} - \left(c - L_{1}\right), \ L_{2}(r) = \frac{1}{R_{0}}\sqrt{R_{0}^{2}c^{2} - r^{2}\left(2L_{2}c - L_{2}^{2}\right)} - \left(c - L_{2}\right), \tag{4}$$

with $a = a_1/a_B$ and $c = c_1/a_B$. The Schrödinger equation for the "fast" subsystem has the form $H_1\chi(z;r) = \varepsilon_1\chi(z;r)$. After simple transformations, one can obtain the following equation for

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the charge energy $(1/\mu_1)k_1 \operatorname{ctg}(k_1L_1(r) + \pi n) = (1/\mu_2)k_2 \operatorname{cth}(k_2(L_1(r) - L_2(r)))$, (5), with the introduced notations $k_1 = \sqrt{\varepsilon_1(r)}$, $k_2 = \sqrt{V_0 - \varepsilon_1(r)}$. Solution of this transcendental equation by numerical methods gives the following expression for the energy spectrum of a charge, with high accuracy degree:

$$\mathcal{E}_{1}(r) = \alpha_{n} + \beta_{n}^{2} r^{2} / 4, \quad n = 1, 2, ...,$$
 (6)

where α_n and β_n are parameters whose values depend on the quantities μ_2/μ_1 and V_0 , while *n* is the quantum number (QN). Expression (6) enters the Schrödinger equation for the "slow" subsystem as an effective potential. Solving that, we obtain the final expression for the charge energy

$$\varepsilon = \alpha_n + \beta_n (N+1), \ N = 0, 1, \dots$$
(7)

where $N = 2n_r + |m|$, with n_r and m being, respectively, the radial and magnetic QNs. In the case when $L_2(r) = 0$, the energy expression becomes

$$\varepsilon = \alpha'_n + \beta'_n (N+1), \tag{8}$$

with $\alpha'_n = \pi^2 n^2 / L_1^2$ and $\beta'_n = \pi n \sqrt{cL_1} / aL_1^2$. Note that in the case of spherical QL ($c = a = R_0$) one has $\beta'_n = \sqrt{2\pi n} / L_1 \sqrt{R_0^2 + L_1^2}$.

3. Conclusions

As one can see from the CC energy spectrum expression (7), the energy levels inside the CQL are equidistant. More correctly, each level of the "fast" subsystem has its own family of equidistant energy levels created by the "slow" subsystem. The obtained result is valid only for the low spectrum levels (i.e., small QNs), which is a consequence of the adiabatic approximation. Note that the CC levels are equidistant also in the case when $L_2(r) = 0$ (see Ref. [8]). As already mentioned, when the coordinate r in the "slow" subsystem is fixed, the charge motion is localized in the effective one-dimensional potential well (see Fig. 2). Due to unique configuration of the problem, realization of effective one-dimensional potential well parameters is possible, in which well-bottom levels are equidistant [6]. This effect was observed in [6] where quantum films (the so-called "modified" asymmetric quantum wells) were studied. However, in the case of these films, each level of one-dimensional motion, due to averaging over the "slow" subsystem, has a proper family of equidistant levels there over. Such a double equidistance (in "fast" and "slow" motions) opens a sufficiently wide opportunity to use the CQL ensemble as an active medium in quantum lasers (see, e.g., US Patent # 6541788).



Fig. 3. One-dimensional energy difference as a function of $L_1(r)$ at $R_0 = 5a_B$ and $L_2 = 2a_B$.



Fig. 4. Energy dependence of the one-dimensional "fast" subsystem on L_1 at fixed values of $V_0 = 50E_R$, $R_0 = 5a_B$, $L_2 = 2a_B \cdot 1$. $\mu_1 = \mu_2$, 2. $\mu_1 \neq \mu_2$.

Fig. 2 illustrates the possible realization of said equidistant levels. The first realization corresponds to the case when two levels of one-dimensional well lie below potential step (CC is localized in QL), while the third level is positioned above the step (CC is localized in coating). The situation is inverse when the QL height $L_1(r)$ is small. Fig. 3 presents the difference of onedimensional energies as a function of $L_1(r)$, at fixed values of the QL coating height and basic radius of CQL. Note that the energy difference may become zero. This means that three lower levels of the "fast" subsystem become equidistant. The energy dependence of the onedimensional "fast" subsystem on L_1 is shown in Fig. 4, under the conditions that the coating height L_2 , potential step height, and QL basic radius are fixed, while the effective masses of CC inside the QL and coating may be either equal or non-equal. One can see from the figure that the CC mass difference, as one could expect, is revealed most clearly for the levels positioned over the potential step. The levels are merged inside the QL. When L_1 increases, the energy of CC falls with oscillations. This may be explained by the fact that the number of levels below the step $(E < U_0)$ increases as L_1 grows, due to smaller SQ. In other words, each new level created inside QL restructures the family of levels positioned higher. Figs. 5 and 6 illustrate the dependence of the position of the first two equidistant energy levels for CC inside QL, respectively, on the QL and coating heights. One can see from these figures that CC energy falls off faster as a function of L_1 , which is a result of reduced SQ. Much weaker is the dependence on L_2 , because in the considered case the charge is present inside the QL and coating has a weaker effect. The

situation becomes different on the interface of QL transformation into coating (see Fig. 6), when the existence of coating essentially affects the CC energy values.

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Fig. 5. Dependence of the first two equidistant families of energy levels for a CC in QL on the value of L_1 at fixed values of $V_0 = 50E_R$, $R_0 = 5a_B$, $L_2 = 2a_R$, $\mu_1 = \mu_2$.



Fig. 6. Dependence of the first two equidistant families of energy levels for a CC in QL on the value of L_2 , at fixed values of $V_0 = 50E_R$, $R_0 = 5a_B$, $L_2 = 1.5a_B$, $\mu_1 = \mu_2$.

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