ՀԱՅԱՍՏԱՆԻ ԳԻՏՈՒԹՅՈՒՆՆԵՐԻ ԱԶԳԱՅԻՆ ԱԿԱԴԵՄԻԱՅԻ ՏԵՂԵԿԱԳԻՐ
ИЗВЕСТИЯ НАЦИОНАЛЬНОЙ АКАДЕМИИ НАУК АРМЕНИИ

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A PROBLEM OF LOW LEVEL STRESS IN COMPOUND PLATES

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Բաղադրյալ սալերի թերլարվածության խնդիրը

Ֆիզիկական և երկրաչափական պարամետրերի տարածության մեջ փնտրվում են բաղադրյալ սալի կոնտակտային մակերևույթի եզրի համար թերլարվածության տիրույթները, որոնց իմացությամբ, նախագծելիս, կարելի է հասնել նշված եզրի հուսալի ամրությանը:

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Задача малонапряженности составных плит

В пространстве физических и геометрических параметров ищутся области малонапряженности для края контактной поверхности составной плиты, с помощью которых при проектировании можно обеспечить надежную прочность указанного края.

In this paper domains of low stress level [1,2] for a contact surface edge are sought in the space of physical and geometrical parameters. If these domains are known, then it will enable not only to avoid concentrations hazardous for joint strength when designing the mentioned edge, but also to exclude completely stresses in it.

1. Problem Statement. It is assumed that a plate is made of incompressible materials hardening according to a power low, which are fully connected along a cylindrical surface perpendicular to the plate middle plane. The plate is bent by external transverse forces which however are not applied in the vicinity of the edge under study. Using cylindrical coordinates let us denote values in the domains $0 \le \theta \le \alpha$, $-h/2 \le z \le h/2$, and $-\beta \le \theta \le 0$, $-h/2 \le z \le h/2$, where h is a plate thickness, within the vicinity of the rib $r=0$ by indices 1 and 2, respectively (Fig. 1).

Let us assume that intensities of stress and deformations are related by the expression

$$\sigma_0 = k\varepsilon_0^m$$

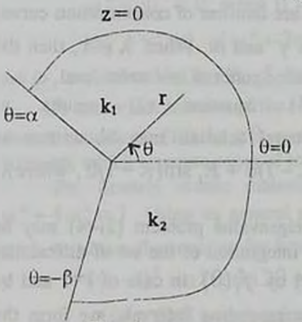

Fig. 1

where, as it is assumed within the frames of the classical theory of thin plate bending, transverse shear deformations are neglected, $\sigma_z = 0$, and $\varepsilon_z = -\varepsilon_r - \varepsilon_\theta$ are excluded from the material incompressibility condition. The hardening power $n = 1/m$ of the both materials are assumed to be equal, and the deformation module k to be different.

The principal stresses σ_r, σ_θ and $\tau_{r\theta}$ for each domain may be written as follows:

$$\sigma_{ij} = \frac{M_{ij}(r,\theta)}{2J} z|z|^{m-1}, \quad J = \frac{1}{m+2}\left(\frac{h}{2}\right)^{m+2}$$

2. Differential Equations and Boundary-and-Contact Conditions. Representing a deflection in the vicinity of a rib $r=0$ as

$$w_i = r^{\lambda+1} f_i(\theta,\lambda),$$

where f_i and λ are be-sought functions and parameter, respectively, moments and tangential generalized shear force will be expressed as follows:

$$M_{n}=-D_i r^{(\lambda-1)m}\left(\frac{1}{2}f_i''+\rho f_i\right)\chi_i,\quad M_{\theta i}=-D_i r^{(\lambda-1)m}(f_i''+\nu f_i)\chi_i$$

$$M_{r\theta i}=-\frac{D_i}{2}\lambda r^{(\lambda-1)m}f_i'\chi_i,\quad \chi_i=\left(\sqrt{f_2''+2\nu f_i''f_i+\lambda^2 f_i'^2+\Delta^2 f_i^2}\right)^{m-1}$$

$$V_{\theta i}=D_i r^{(\lambda-1)m-1}\left\{\left[(f_i''+\nu f_i)\chi_i\right]'+\eta f_i'\chi_i\right\},\tag{1}$$

$$\Delta=(\lambda+1)\sqrt{\lambda^2+\lambda+1},\ \rho=(\lambda+1)\left(\lambda+\frac{1}{2}\right),\ \nu=(\lambda+1)\left(\frac{\lambda}{2}+1\right),$$

$$\eta=\lambda\left[1+(\lambda-1)m\right],\quad D_i=\frac{k_i}{m+2}h^{m+2}$$

Substituting the expressions of moments into the balance equation expressed through moments, we will obtain an ordinary differential equation of the 4th order

$$\left[(f_i''+\nu f_i)\chi_i\right]''+\eta(f_i'\chi_i)'+(\delta f_i''+\mu f_i)\chi_i=0\tag{2}$$

where $2\delta=(\lambda-1)m\left[(\lambda-1)m-1\right],\quad 2\mu=(\lambda-1)m(\lambda^2-1)\left[1+(2\lambda+1)m\right]$

For freely supported edges of the plate we have $M_{\theta i}=w_i=0$ and the boundary conditions

$$f_i''=f_i=0,\ \text{when}\ \theta=\alpha;-\beta\tag{3}$$

are obtained from it. The conditions for deflection, inclination angle of deflection, bending moment and generalized shear force should be satisfied on the contact surface. It results in

$$f_1=f_2,\ f_1'=f_2',\ (f_1''+\nu f_2)\chi_1=\gamma(f_2''+\nu f_2)\chi_2,\quad \gamma=k_2/k_1,$$

$$[(f_1''+\nu f_1)\chi_1]'+\eta f_1'\chi_1=\gamma\{[(f_2''+\nu f_2)\chi_2]'+\eta f_2'\chi_2\},\ \text{when}\ \theta=0\tag{4}$$

The set of differential equations (2) with the boundary-and-contact conditions (3) and (4) is a three-point eigenvalue problem, which defines in principle $f_i(\theta)$ functions up to a common uncertain multiplier and $\lambda=\lambda(\alpha,\beta,\gamma,m)$ for given values of the parameters α,β,γ and m. The corresponding expression connecting the parameters α,β,γ and m are found considering the inverse problem where $\lambda=\lambda_*<1$ is assumed. This expression describes a hypersurface in the space of these parameters, and which leaves traces in α,β coordinate plane. These traces are families of concentration curves of the same power moments (stresses), depending on γ and m. When $\lambda=1$, then the limiting curves of finite moments are built and they define zones of low stress level.

For linearly elastic materials (m=1) equation (2) results in $f_i^{IV}+2(\lambda^2+1)f_i''+(\lambda^2-1)^2 f_i=0$, which general solution may be written as $f_i=A_i\cos(\lambda+1)\theta+B_i\sin(\lambda+1)\theta+C_i\cos(\lambda-1)\theta+E_i\sin(\lambda-1)\theta$, where λ is assumed to be a complex number in a general case.

3. Integral Method. The above mentioned eigenvalue problem (2)-(4) may be studied by a particular method which overcomes an integration of the set of differential equations. Multiplying the both sides of equation (2) by $f_1(\theta)$ in case of i=1 and by $f_2(\theta)$ in case of i=2 and integrating it over corresponding intervals, we form the identity:

$$\int_0^{\alpha}\{[(f_1''+\nu f_1)\chi_1]''+\eta(f_1'\chi_1)'+(\delta f_1''+\mu f_1)\chi_1\}f_1 d\theta+$$

$$+\gamma\int_{-\beta}^{0}\{[(f_2''+\nu f_2)\chi_2]''+\eta(f_2'\chi_2)'+(\delta f_2''+\mu f_2)\chi_2\}f_2 d\theta=0$$

Integrating it by parts, and transforming, we obtain

$$\int_0^{\alpha}(f_1''+sf_1''f_1+\mu f_1^2-\eta f_1'^2)\chi_1 d\theta+\gamma\int_{-\beta}^{0}(f_2''+sf_2''f_2+\mu f_2^2-$$
$$-\eta f_2'^2)\chi_2 d\theta+L_1|_0^{\alpha}+\gamma L_2|_{-\beta}^{0}=0 \tag{5}$$

where $s=\nu+\delta$, and

$$L_i=\{[(f_i''+\nu f_i)\chi_i]'+\eta f_i'\chi_i\}f_i-(f_i''+\nu f_i)f_i'\chi_i.$$

Noting that the sum of the last two terms for the ordinary boundary-and-contact conditions (5) is equal to zero, we finally obtain

$$\eta=\frac{\int_0^{\alpha}(f_1''^2+sf_1''f_1+\mu f_1^2)\chi_1 d\theta+\gamma\int_{-\beta}^{0}(f_2''^2+sf_2''f_2+\mu f_2^2)\chi_2 d\theta}{\int_0^{\alpha}f_1'^2\chi_1 d\theta+\gamma\int_{-\beta}^{0}f_2'^2\chi_2 d\theta} \tag{6}$$

choosing f_i functions in such a manner that they satisfy as many boundary-and-contact conditions as possible and substituting them into Eq.(6) and performing integration, we obtain a transcendental-algebraic equation with respect to λ, from which $\lambda=\lambda(\alpha,\beta,\gamma,m)$ is found.

4. Zones of Low Stress Level. Passing to the limit $\lambda\to 1$ in Eq.(2) and integrating, we obtain

$$[(f_i''+3f_i)\chi_i]'+f_i\chi_i=H_i=\text{const} \tag{7}$$

where $\chi_i=\left(\sqrt{f_i''^2+6f_i''f_i+f_i'^2+12f_i^2}\right)^{m-1}$ (8)

Denoting $f_i=\chi|E_i|^n\psi_i(\theta)$, where $\chi=\mathrm{sign}E_1$, from Eq.(7) we obtain

$$\psi_i'''+4\psi_i'=\frac{[(\psi_i''+3\psi_i)^2+\psi_i'^2+3\psi_i^2]^{\frac{3-m}{2}}}{m(\psi_i''+3\psi_i)^2+\psi_i'^2+3\psi_i^2} \tag{9}$$

The conditions (3) and (4), that interests us will be written as

$$\psi_i''=\psi_i=0,\ \text{when}\ \theta=\alpha,-\beta,\quad \psi_2=\gamma^n\psi_1, \tag{10}$$
$$\psi_2'=\gamma^n\psi_1',\quad (\psi_1''+3\psi_1)\omega_1=(\psi_2''+3\psi_2)\omega_2,\quad \text{when}\quad \theta=0$$

and ω_i are defined by Eq.(8), f_i being substituted by ψ_i.

A numerical solution of the set of differential equations (9) with the boundary-and-contact conditions (10) in $\alpha\beta$ coordinate plane determines curves of finite moments (stresses), which separate zones of low stress level from concentration ones (Fig.2).

For linearly elastic materials the differential equation (9) takes the form $\psi_i'''+4\psi_i'=1$. Using its general solution and satisfying the conditions (10) in case of $m=1$, we obtain the equation for limiting curves of finite moments

$$(\gamma-1)^2\sin 2\alpha\sin 2\beta+(\gamma-1)(\beta+\gamma\alpha)\sin 2(\alpha-\beta)-$$
$$-[(\gamma+1)(\beta+\gamma\alpha)+6\gamma(\alpha+\beta)]\sin 2(\alpha+\beta)=0 \tag{11}$$

Fig.2 shows these curves in $\alpha\beta$ plane for different γ. When $\gamma=1$ we obtain $\alpha+\beta=\pi/2$ and $\alpha+\beta=\pi$. It means that zone of low stress level for homogeneous plate is an area comprising the triangular domain with the apices $(0,\ 0)$, $(0,\pi/2)$ and $(\pi/2,0)$ and the segment of $\alpha+\beta=\pi$ straight line concluded between the coordinate

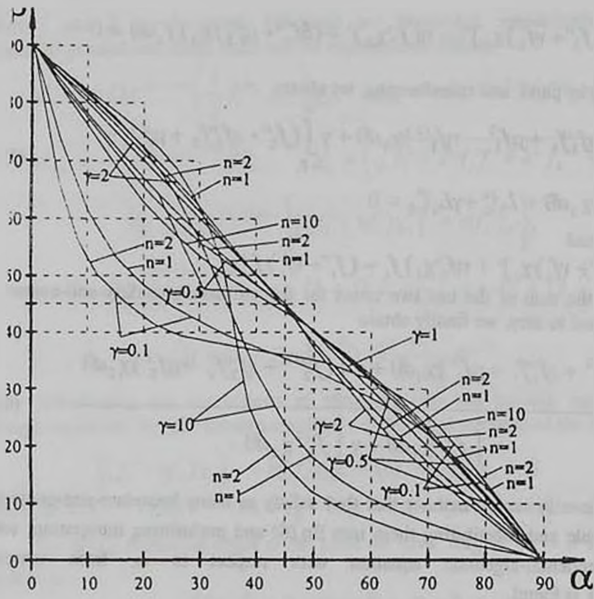

Fig. 2

axes. From the condition of shear forces finiteness we obtain that the zone of low stress level comprises the traingular area with the apices $(0, 0)$, $(0, \pi/3)$ and $(\pi/3, 0)$ and the segments of $\alpha+\beta=\pi/2$ and $\alpha+\beta=\pi$ straight lines concluded between the coordinate axes. These results fully correspond to the conclusions of B.G.Galyorkin [3], concerning a complete solution of the problem of sectorial plate with freely supported radial edges.

High concentration of shear forces compared to moments, which was mentioned by B.G.Galyorkin as long ago as 1920s, may be explained by non-perfection of the classical theory of plate bending. The study of crack edge of linearly elastic plates carried out by means of the revised theories [4-6] and comparison with the corresponding results of the classical theory [7,8] show that, first, shear forces at that edge are finite, and, second, concentrations of moments (stresses) are not changed.

Despite the usefulness of the principal conclusions obtained basing on the assumptions of the classical theory for study of compound plate joints strength, nevertheless study of the above mentioned problems taking account of plate transverse shears [9-11] are of principle interest. According to the approach stated here, zones of low stress level in case of mixed boundary conditions when one edge of plate is free and the other is freely supported, are built in [12].

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