ՀԱՅԱՍՏԱՆԻ ԳԻՏՈՒԹՅՈՒՆՆԵՐԻ ԱԶԳԱՅԻՆ ԱԿԱԴԵՄԻԱՅԻ ՏԵՂԵԿԱԳԻՐ
ИЗВЕСТИЯ НАЦИОНАЛЬНОЙ АКАДЕМИИ НАУК АРМЕНИИ

Մեխանիկա 56, №4, 2003 Механика

УДК 539.3

Localized Bending Waves in an Elastic Orthotropic Plate

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Локализованные изгибные волны в упругой ортотропной пластинке

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В работе приведены результаты исследования вопроса существования изгибных локализованных волн в окрестности свободного края прямогольной ортотропной пластинки.

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Լոկալիզացված ծռման ալիքները առաձգական օրթոտրոպ սալում

Աշխատանքում բերված են ուղղանկյուն օրթոտրոպ սալի ազատ եզրի շրջակայքում ծռման լոկալիզացված ալիքների գոյության խնդրի արդյունքները:

The aim of this work is the theoretical study of bending localized waves in a thin elastic orthotropic cantilever plate. These waves are spatially non-uniform bending perturbations varying in time, localized near the vicinity of limiting free surface of a plate and practically immaterial outside this relatively narrow zone. The first studies related to the localized bending waves in elastic plates were first presented in [1] and further developed in [2], where for semi-infinite plate the existence of surface bending waves near the free edge have been shown. For two semi-infinitive plates being in conditions of elastic contact, a similar problem was investigated in [3].

The dynamic problem is considered for elastic bending propagation waves in an orthotropic cantilever plate, one edge of which is free from mechanical stresses and restrictions.

In Cartesian system (x, y), where the plate occupy domain $x \in [0, a]$, $y \in [0, b]$ the equation for plate middle plane normal displacement $W(x, y)$ can be expressed as [4]

$$\frac{\partial^4 W}{\partial \alpha^4} + 2k \frac{\partial^4 W}{\partial \alpha^2 \partial \beta^2} + \frac{\partial^4 W}{\partial \beta^4} + \frac{3\rho(1 - \nu_1 \nu_2)}{h^2} \frac{\partial^2 W}{\partial t^2} = 0 \qquad (1)$$

In (1) $\alpha = xE_1^{-1/4}, \beta = yE_2^{-1/4}$, $k = (E_1 E_2)^{-1/2}[\nu_1 E_1 + 2(1 - \nu_1 \nu_2)G]$, $E_1, E_2, G, \nu_1, \nu_2$ are elastic constants, ρ is the bulk density of plate material, h and is the thickness of plate. In the case of an isotropic plate one has $k = 1$. For cantilever plate the equation (1) is supplemented by the following boundary conditions:

Free edge (the bending moment and the generalized transverse force are vanished)

$$\frac{\partial^2 W}{\partial \beta^2} + \nu \frac{\partial^2 W}{\partial \alpha^2} = 0; \quad \frac{\partial}{\partial \beta}\left[\frac{\partial^2 W}{\partial \beta^2} + (2k - \nu)\frac{\partial^2 W}{\partial \alpha^2}\right] = 0 \text{ at } \beta = 0 \quad (2)$$

Where the following notation are used

$$\nu = \frac{\nu_1 E_1}{\sqrt{E_1 E_2}} = \frac{\nu_2 E_2}{\sqrt{E_1 E_2}}$$

Simply supported edges (the displacement and bending moment are vanished)

$$W = 0; \quad \frac{\partial^2 W}{\partial \beta^2} = 0, \quad \text{at} \quad \beta = bE_2^{-1/4} \quad (3)$$

$$W = 0; \quad \frac{\partial^2 W}{\partial \alpha^2} = 0, \text{ at } \alpha = 0; \quad \alpha = aE_1^{-1/4} \quad (4)$$

In (1-4) a time-harmonic plane wave solution be considered

$$W(\alpha, \beta, t) = W_0(\beta)\sin(p_n \alpha)\exp(i\omega t) \quad (5)$$

where ω is the vibration frequency, $p_n = \pi n E_1^{1/4} / a$

Substituting (5) in (2-4) we have the following self-adjoint eigenvalue boundary problem for displacement function $W_0(\beta)$

$$\frac{1}{p_n^4}\frac{d^4 W}{d\beta^4} - \frac{2k}{p_n^2}\frac{d^2 W}{d\beta^2} = \lambda W \quad \beta \subseteq \left[\ 0,\ bE_2^{-1/4}\right]$$

$$\frac{d^2 W}{d\beta^2} - \nu p_n^2 W = 0; \quad \frac{d}{d\beta}\left[\frac{d^2 W}{d\beta^2} - (2k - \nu)p_n^2 W\right] = 0 \text{ at } \beta = 0 \quad (6)$$

$$W = 0; \quad \frac{d^2 W}{d\beta^2} = 0 \text{ at } \beta = bE_2^{-1/4}$$

Here $\lambda = \frac{3\rho(1-\nu_1\nu_2)\omega^2}{h^2 p_n^4} - 1$ are eigenvalues of the boundary problem (6).

Negative eigenvalues λ of the boundary problem, if they exist, define the localized mode of vibration [5]; positive eigenvalues correspond to periodic modes.

Using common procedure of self adjoint boundary value problem solution [2,6,7] we have the following equation determining the frequencies of localized mode of vibration for negative eigenvalues $\mu = -|\lambda|$

$$F^{(+)}(\mu) = F^{(-)}(\mu) \quad (7)$$

where

$$F^{(\pm)}(\mu)=\frac{\left(k-\nu\pm\sqrt{k^2-\mu}\right)^2}{\sqrt{k\pm\sqrt{k^2-\mu}}}\operatorname{th}\left(\gamma_n\sqrt{k\pm\sqrt{k^2-\mu}}\right);\gamma_n=(E_1/E_2)^{1/4}b\pi n/a$$

In the case of elongated plate $b/a>>1$ replacing function

$\operatorname{th}\left(\gamma_n\sqrt{k\pm\sqrt{k^2-\mu}}\right)\to 1$ we can obtain the results of [6].

Based on equation (7) the necessary and sufficient conditions are obtained regarding localized wave existence in depend of on anisotropy coefficients k,ν. It is shown that equation (7) may have only one root that correspond to localised bending wave.

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Received
7.10.2003